Economics 601 GianCarlo Moschini
Microeconomic Analysis I Fall 2008

Problem Set No. 5 Due by: Friday, October 10

5.1 Duality: recovering preferences

(a) A consumer with utility function $u(x_1, x_2)$ has an expenditure function that is written as

$$e(p,u) = \frac{u\, p_1 p_2}{p_1 + p_2}$$

Derive the direct utility function $u(x_1, x_2)$.

(b) Suppose that the indirect utility function that is dual to $u(x_1, x_2)$ is written as:

$$v(p,w) = \frac{w}{p_1 + p_2}$$

Derive the Marshallian demand functions.
Derive the expenditure function.
Derive the direct utility function. (Note: this part requires some care).

5.2 More duality

Consider the choice problem of a consumer with a linear budget constraint and strictly convex preferences defined on $\mathbb{R}^2_+$, and the standard notation whereby p_i denote prices, w is income, and u is a utility level. Suppose that the Hicksian demand function for the first good and the Marshallian demand function for the second good can be written as, respectively:

$$h_1(p_1, p_2, u) = \frac{\left(\sqrt{p_1} + \sqrt{p_2}\right)u}{\sqrt{p_1}}$$

$$x_2(p_1, p_2, w) = \frac{w}{\left(p_2 + \sqrt{p_1 p_2}\right)}$$

Derive the indirect utility function $V(p_1, p_2, w)$ that represents these preferences. Make sure you explain clearly the steps of your derivation.

5.3 Revealed preferences (MWG, 2.F.3)

In a two-good world, suppose that a consumer's observed purchases in two periods are as follows:

	Period I		Period II	
	quantity	price	quantity	price
good 1	100	100	120	100
good 2	100	100	y	80

Determine the range of y (i.e., the quantity of good 2 consumed in period II) that would allow you to conclude:

(a) That the consumer's behavior contradicts WARP (the weak axiom of revealed preferences).

For the next sub-questions, assume that WARP is satisfied.

(b) That the bundle in period I is revealed preferred to that of period II.

(c) That the bundle in period II is revealed preferred to that of period I.

(d) That good 1 is an inferior good (at some price).

(e) That good 2 is an inferior good (at some price).

5.4 Utility function and revealed preferences (MWG, 3.D.7)

In a two-good world, a consumer makes choices in two periods with budget sets of, respectively, B_{p^0,w^0} and B_{p^1,w^1}, where $p^0=(1,1)$, $w^0=8$, $p^1=(1,4)$ and $w^1=26$. The observed choice vector with B_{p^0,w^0} is $x^0=(4,4)$. When the budget set is B_{p^1,w^1} all we know is that the consumer picks a vector x^1 such that $p^1\cdot x^1=w^1$. [Hint: the answer to the following questions can be obtained with the aid of properly drawn diagrams.]

(a) Determine the region of permissible choices for x^1 so that (x^0,x^1) satisfy the WARP.

For the remainder of this problem, assume that preferences are represented by a differentiable utility function $u(x)$

(b) Determine the region of permissible choices for x^1 so that (x^0,x^1) are consistent with the maximization of preferences that are *quasilinear* with respect to the first good.

(c) Determine the region of permissible choices for x^1 so that (x^0,x^1) are consistent with the maximization of preferences that are *quasilinear* with respect to the second good.

(d) Determine the region of permissible choices for x^1 so that (x^0,x^1) are consistent with the maximization of preferences for which both goods are *normal*.

(e) Determine the region of permissible choices for x^1 so that (x^0,x^1) are consistent with the maximization of preferences that are *homothetic*.