

Problem Set No. 8

Due by: Friday, October 31

8.1. Consider a profit maximizing firm with a monotonic production function  $q = f(z_1, z_2)$  where  $q$  is output and  $z_i$  are inputs. Let  $p$  and  $w_i$  denote output and input prices, respectively. Assume that the SOSCs hold at the profit maximizing solutions  $(q^*, z_i^*)$ . Use the standard comparative statics approach to show that the profit maximizing output supply satisfies  $\frac{\partial q^*}{\partial p} > 0$ .

8.2. (MWG) We say that the production set  $Y$  is a **convex cone** if, for any two netput vectors  $y, y' \in Y$  and scalars  $\alpha, \beta \geq 0$ , then we have  $[\alpha y + \beta y'] \in Y$ . Show that the production set  $Y$  is **additive**, and satisfied **nonincreasing returns to scale**, if and only if it is a convex cone.

8.3. (Varian) Determine whether the following input requirements sets are *regular*, *monotonic*, and *convex*. (Throughout, assume that  $a > 0$  and  $b > 0$ ). Sketch out the shape of the given input requirement sets.

$$(i) \quad V(q) \equiv \{z_1, z_2 \mid az_1 + bz_2 - \sqrt{z_1 z_2} \geq q\}$$

$$(ii) \quad V(q) \equiv \{z_1, z_2 \mid z_1 + \min(z_1, z_2) \geq 2q\}$$

$$(iii) \quad V(q) \equiv \{z_1, z_2 \mid az_1 + bz_2 \geq q, z_2 > 0\}$$

8.4. Consider the following 2-input production function:

$$f(z_1, z_2) = \left( \min\{z_1, z_2\} \right)^a, \quad a > 0$$

(a) Determine the profit-maximizing input demand and output supply functions.

(b) What additional restriction must the parameter  $a$  satisfy for a solution to the profit maximization problem to exist?

8.5. Consider the following production function:  $f(z) = \frac{k}{1 + z_1^{-\alpha} z_2^{-\beta}}$ ,  $\alpha > 0$ ,  $\beta > 0$ , and  $k > 0$ .

Note that  $k$  is an upper bound on the level of output (so that  $0 \leq q < k$ ).

(a) Compute the “elasticity of scale”  $e(z)$  for this production function.

(b) Show that this production function displays “variable returns to scale.” Identify the output levels for which the technology displays locally increasing, constant and decreasing returns to scale.