

Problem Set No. 9

Due by: Friday, November 7

1. Derive the profit maximizing output supply and input demand functions, and find the profit function, for each of the following production functions:

(a) $f(z_1, z_2) = \sqrt{z_1 + z_2}$

(b) $f(z_1, z_2) = \sqrt{\text{Min}\{z_1, z_2\}}$

(c) $f(z_1, z_2) = z_1^\alpha z_2^\beta$

[Note: case (c) was partially analyzed in class, where we also determined the restrictions that α and β must satisfy for profit maximization; part (b) is a special case of problem 8.4 assigned last week.]

2. Suppose the cost function of a competitive firm is:

$$C(w, q) = \begin{cases} (q^2 + 1)(w_1^\alpha w_2^{1-\alpha}) & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

- (a) Let p denote the price of output, and suppose that $w_1 = w_2 = 1$. If $p = 4$, how much will the firm produce? If $p = 1$, how much will the firm produce?
- (b) What is the profit function for this firm?
3. (Varian) Consider the production function given by: $f(z_1, z_2) = \min\{(2z_1 + z_2), (z_1 + 2z_2)\}$. Derive the conditional input demand functions and the cost function implied by this technology.
4. Definition: An input z_i is said to be *inferior* if $\partial h_i(w, q)/\partial q < 0$.
- (a) Can you think of a real-world example of an inferior input? Illustrate the case of z_1 being inferior in a (z_1, z_2) diagram.
- (b) Suppose that the technology is homothetic. Are inferior inputs possible in this case?
- (c) Suppose that z_i is inferior. How does a decrease of price w_i affect the marginal cost?
5. A firm has been contracted to supply an enzyme to an ISU lab for two consecutive periods. Enzyme production technology is given by the production function $q = X \cdot L$, where q is enzyme output, X is a raw material, and L is labor. The price of X is r (for both periods) and the price of L is w (for both periods). The quantity of output to be supplied is q_1 for the first period and q_2 for the second period. Enzyme is perishable, and must be produced in the period it is delivered. The firm aims at minimizing total costs for the two periods, but its optimal plans must account for the fact that labor hired in the first period cannot be fired and must be used in the second period as well. However, additional labor may be hired in the second period alone, if necessary.
- (a) Set up the cost minimization problem of the firm, and derive optimality conditions.
- (b) Derive the input demand functions of the firm.
- (c) Does demand for material inputs in the second period depend on q_1 ? Discuss.