Lapan

Econ 603
Spring 2004

Midterm Exam
Answer any **two** questions; answer all parts to each question.
Each question is worth 50 points.

1. Answer all parts

a) Prove that, under the standard assumptions, a competitive equilibrium is Pareto efficient. **(10 points)**

i. Suppose the utility of each person depends on his (her) own consumption bundle **and** on the level of pollution, where the amount of pollution depends on the aggregate output of good one. Would a competitive equilibrium still be Pareto efficient? **Show which step(s)** in your proof above, if any, would fail to hold. **(6 points)**

b) Consider a general equilibrium model with two final goods (*M*, *F*), two people (*A*,*B*) and one input (*L*). Let $\left(e_L^A, e_L^B\right)$ represent each individual's endowment of this input and assume: $e_L^A = 1, \quad e_L^B = 2$. In addition, assume person *A* is endowed with a technology that allows him to produce good *M*, and person *B* is endowed with a technology that allows her to produce good *F* (person *A* cannot produce *F* and *B* cannot produce *M*). Preferences and technology for each person are given by:

Person A: $U^A = \left(C_M^A\right)^2 \left(C_F^A\right); \quad Q_M^A \leq \theta_M \left(L_M\right)^{1/2}$;

Person B: $U^B = \left(C_M^B\right)^2 \left(C_F^B\right); \quad Q_F^B \leq \theta_F \left(L_F\right)^{1/2}$

where C_j^h is person *h*'s consumption of good *j*, Q_j^h is person *h*'s production of good *j*, $\theta_j > 0$ is a productivity parameter for good *j* and L_j represents the amount of input *L* used to produce good *j*.

i. Assume that trade between people is allowed in final goods (*M*, *F*) **but a law prohibits trade in the input** (hence we must have: $L_M = e_L^A,\ L_F = e_L^B$). Find the competitive equilibrium prices $\left(P_M, P_F\right)$, consumption vectors and utility allocation. Is this equilibrium Pareto efficient, **given the ban** on trade of inputs? **(9 points)**

ii. Assume the ban on trade of inputs is removed so that individuals may trade both in final goods and in the input (so the resource constraint now becomes: $\left(L_F + L_M\right) \leq \left(e_L^A + e_L^B\right) = 3$). Calculate the competitive equilibrium prices $\left(P_F, P_M, P_L\right)$, consumption vectors and utility allocation. How does allowing trade in the input affect "economic efficiency"? **(8 points)**

c) Consider the following two good (*M*, *F*), two person (*A*, *B*) exchange economy. Preferences are:

Person A: $U^A = \left[\left(2C_M^A\right)^2 + \left(C_F^A\right)^2\right]$; **Person B:** $U^B = \left[C_M^B + 2C_F^B\right]$

where C_j^h is person *h*'s consumption of good *j*. Let $\left(\omega_M^h, \omega_F^h\right)$ denote person *h*'s endowment vector, and assume total endowments are: $\left(\omega_M^T, \omega_F^T\right) = \sum_{h=A,B} \left(\omega_M^h, \omega_F^h\right) = (10,10)$

i. ***Briefly*** discuss what difficulties non-convex preferences may present in proving the existence of an equilibrium. **(5 points)**

ii. For the exchange model given above, show the set of endowments for person *A* for which a *competitive equilibrium* will not exist. **(6 points)**

iii. Using the Edgeworth box, show the set of *Pareto efficient* allocations that can be supported as a competitive equilibrium. **(6 points)**

2. Answer all parts.

a) State the second welfare theorem and give a sketch of how to prove it (a formal proof is not needed). What are the crucial *assumptions* used in the proof? **(10 points)**

b) Consider the following general equilibrium model. There are two individuals (A, B) and two goods (M,F). Each individual's utility depends on consumption of these goods; in addition, each individual possesses a production set that allows production of each of these goods. Thus:

Individual A: $U^A = \left(C_M^A\right)\cdot\left(C_F^A\right);\quad q_M^A + \left[\left(q_F^A\right)^2 \big/ 2\right] \le 200$

Individual B: $U^B = \left(C_M^B\right)\cdot\left(C_F^B\right);\quad q_M^B + \left[\left(q_F^B\right)^2 \big/ 2\right] \le 100$

where C_j^h is individual *h*'s consumption of good *j* and q_j^h is individual *h*'s production of good *j* (*h=A,B; j=M,F*). The resource constraint for each good is: $\left(C_j^A + C_j^B\right) \le \left(q_j^A + q_j^B\right),\quad j = M, F$

i. Find the set of Pareto efficient allocations for this economy (consumption and production vectors for each individual) and the corresponding utility possibility frontier. **(7 points)**

ii. Assume a competitive economy where individual's can buy or sell goods at market prices. Given prices $\left(P_f, P_m\right)$ each individual chooses production and consumption plans to maximize utility, subject to his (her) budget constraint. Assuming no government policy, find the competitive equilibrium prices, and consumption and production vectors for each person. Is this equilibrium Pareto efficient? {NOTE: by definition, an individual's net sales of good *j* is: $s_j^h = \left(q_j^h - c_j^h\right)$ } **(7 points)**

iii. Show what policy the government can use to support *any* Pareto efficient allocation as a competitive equilibrium. Relate this result to the 2nd welfare theorem. **(5 points)**

iv. Suppose the **only** policy the government can use is to tax net sales of each good (*i.e.*, the government cannot tax that portion of output a person produces and consumes himself/herself). The proceeds of this tax can then be redistributed to the person the government wants to help. **Can this policy be used to support any *pareto efficient* allocation?** If not, what inefficiency does this policy cause and what is the lowest level to which person *A's* utility can be reduced using this policy? Explain. (NOTE: you do not have to explicitly solve for the equilibrium as a function of the tax rate). **(7 points)**

c) Using the model of part (b), suppose the social welfare function is given by: $W = U^A \cdot U^B$

i. Find the production and consumption allocations, and the utility level for each person that would maximize social welfare (you can use your results from part b here). **(7 points)**

ii. Suppose person *A*'s utility function changed to $U^A = \left(C_M^A\right)^{1/2}\left(C_F^A\right)^{1/2}$ but everything else were unchanged. *How would this change* in person *A*'s utility function change the set of Pareto efficient allocations and the utility possibility frontier? Be specific. How would this change alter the allocation that maximized social welfare? (you do not need to resolve, just indicate the direction of change in consumption and production vectors for each person). **(7 points)**

3. Answer all parts.

a) Consider a three-good general equilibrium model with H consumers who have identical endowments, identical ownership of firms and all taxes are rebated equally (thus, individuals have identical income). They also have identical quasi-linear preferences given by:

$$U^h = m^h + 3x_1^h + 3x_2^h - (1/2)\left(\left(x_1^h\right)^2 + \left(x_2^h\right)^2 + 2\lambda\left(x_1^h\right)\cdot\left(x_2^h\right)\right) - \phi Z; \quad |\lambda| < 1$$

There are also H identical firms (which behave competitively) with the following cost function:

$$C^j = \left(q_1^j\right)^2 + \left(\left(q_2^j\right)^2 / 2\right)$$

where x_i^h is the consumption of good *i* by individual *h*, q_i^j is production of good *i* by firm *j* and *Z* is an index of air pollution. Good *m* is the numeraire good, m^h is individual *h's* consumption of this good, and production costs $\left(C^j\right)$ denote the amount of the numeraire good required to produce the corresponding output vector. Assume endowments of the numeraire are large enough to guarantee an interior solution in which all goods are consumed. Finally, assume that air pollution depends on the aggregate output of good 1: $Z = \sum_j q_1^j$.

i. Find the competitive equilibrium. Is it *pareto efficient?* Explain your answer. **(8 points)**

ii. Find the *symmetric pareto efficient allocation* (in which all consumers receive the same consumption vector). Explain what policy the government can implement to support this allocation as a competitive equilibrium. **Be specific.** **(8 points)**

iii. Suppose that, for political reasons, the only feasible policy the government can use is to tax (or subsidize) good 2. First, *discuss* whether this policy can achieve the *pareto efficient allocation*, then **derive** the optimal tax (subsidy) on good 2. (If you cannot solve for the optimal tax/subsidy, at least indicate whether the government should use a tax or a subsidy and explain your rationale.) **(9 points)**

b) Consider the following production model with two goods and three firms. Each firm uses good two to produce good one: technologies are as follows:

Firm 1: $y_1^1 \le 6\left(-y_2^1\right)^{1/2}$; $y_2^1 \le 0$;

Firm 2: $y_1^2 \le 6\left[Max\left\{0;\left(-y_2^2-3\right)\right\}\right]^{1/2}$; $y_2^2 \le 0$;

Firm 3: $y_1^3 \le \left(-y_2^3\right)$; $y_2^3 \le 0$

where y_l^j is firm *j*'s *netput* of good *l*. Note that the technology for firm 2 implies that there is a fixed cost of 3 units of good 2 to establish the firm. The aggregate endowment vector is: $\left(\omega_1,\omega_2\right)=\left(0,30\right)$.

i. Find the production possibility set for this economy (watch the non-convexity!) **(10 points)**

ii. Assuming competitive profit maximization, find each firm's supply curve for good 1 and the aggregate supply curve (as functions of the prices of goods 1 and 2). Does profit maximization lead to efficient production? Be specific. **(5 points)**

iii. Can every efficient production point be supported through profit maximization? Be as specific as possible. **(5 points)**

iv. Given preferences, can you tell whether a competitive equilibrium would exist for this economy? Given preferences, would every *pareto efficient* allocation be supportable as a competitive equilibrium (in answering, assume there are two consumers). Explain the relationship between your answers to these two questions. **(5 points)**