Lapan

Econ 603
Spring 2005

Midterm Exam
Answer any **two** questions; answer all parts to each question.
Each question is worth 50 points.

1. Answer all parts

a) Consider the following two person (*A*, *B*), two good (*x,y)* exchange economy. Preferences are:

$$Person \quad A: \quad U^A = \ln\left(c_x^A\right) + \ln\left(c_y^A\right); \qquad Person\ B: \qquad U^B = \left(c_x^B + c_y^B\right);$$

where c_j^i is person *i*'s consumption of good *j*. Let total endowments be $\left(e_x^T, e_y^T\right) = (10,4)$ and let $\left(e_x^i, e_y^i\right)$ represent the (for now) unspecified endowment vector of person *i*.

i. Find the set of *Pareto efficient allocations* (include boundary points, *if any*). **(6 points)**

ii. Given the total endowments, find the competitive equilibrium as a function of the endowment vector of person A. Can it be a boundary point? If so, will it be Pareto efficient? **(5 points)**

iii. Given the specific endowment vector: $\left(e_x^A, e_y^A\right) = (9,1)$; $\left(e_x^B, e_y^B\right) = (1,3)$ show graphically the contract curve (the Core): i.e., the set of Pareto efficient allocations such that no one is worse off than if they consume their own endowments. Label the end points of the contract curve (and give their coordinates). **(4 points)**

iv. Choose either one of the end points of the contract curve from part (aiii). Modify the economy so that there are now 4 people, 2 of each type (each with endowments as in (aiii)). Show that the end point you have chosen is **not** in the core of this larger economy – *i.e.,* show how 3 of the 4 people can find a feasible allocation that makes all of them better off than at this end point (and does not confiscate any goods from the fourth person). **(5 points)**

b) Prove that a competitive equilibrium is Pareto efficient. **(9 points)**

i. What step (if any) in your proof fails to hold if preferences are non-convex? What step(s) might fail to hold if one person's utility depends upon the consumption bundle of another person? Explain. **(6 points)**

c) Consider a slight variant of the two person (A,B), two good (x,y) exchange economy of part (a). Preferences are modified to:

$$Person \quad A: \quad U^A = \ln\left(c_x^A\right) + \ln\left(c_y^A\right); \quad Person\ B: \qquad U^B = \left(\left(c_x^B\right)^2 + \left(c_y^B\right)^2\right)^{1/2};$$

As earlier, total endowments are $\left(e_x^T, e_y^T\right) = (10,5)$ and $\left(e_x^i, e_y^i\right)$ is person *i's* endowment vector.

i. Find each person's demand curve and contrast it with the demands from part (a). **(6 points)**

ii. Briefly discuss the role convexity of preferences plays in proving the *existence* of a competitive equilibrium for an exchange economy. If preferences are not convex, does that mean that a competitive equilibrium *does not* exist? **(4 points)**

iii. For this economy, find the set of endowments – if any – for which a competitive equilibrium does exist. **(5 points)**

2. Consider a two-good (*x,y),* two household (*A,B*) economy. Each household is endowed with **10 units of time,** which can used in the production of either good. Production technology, and the resource constraints, are given by:

$$Q_x = \left(2L_x^A + L_x^B\right); \quad Q_y = \left(3L_y^A + L_y^B\right) \; ; \; L_x^h + L_y^h \leq 10; \quad h = A, B$$

where L_j^h denotes the use of the labor of household *h* to produce good *j* Clearly, the specification implies that household *A's* labor is more productive than that of household *B*. Preferences are given by:

Household A: $U^A = \phi\left(c_x^A\right)^{4/5}\left(c_y^A\right)^{1/5}; \quad U^B = \phi\left(c_x^B\right)^{1/5}\left(c_y^B\right)^{4/5}; \quad \phi \equiv \left(5/4^{4/5}\right)$ where c_j^h is household *h*'s consumption of good *j*. There are four prices in this economy: one for each good, and one for each type of labor.

a) Find the competitive equilibrium prices, production and consumption allocation for this economy. Is it Pareto efficient? **(8 points)**

b) State the second welfare theorem and the critical assumptions required to prove it. Explain the role of the assumptions used in the proof (no proof is required). **(7 points)**

   i. Find the set of Pareto efficient allocations for this economy (I am most interested in you showing how the optimal production point changes as you move along the utility possibility frontier, starting from an allocation where *B* receives all the goods to one in which *A* receives all the goods) **(10 points)**

   ii. Show how, for this economy, each Pareto efficient allocation can be supported and show how equilibrium prices and production change as you redistribute utility from *B* to *A*. **(5 points)**

c) Assume that each individual's preferences (utility) depend upon her/his own *leisure,* and her/his consumption of goods: $U^h\left(c_x^h, c_y^h, l^h\right)$, where l^h is leisure, and the time budget constraint is now: $l^h + L_x^h + L_y^h \leq 10$.

   i. Does the second welfare theorem still apply to this model? If so, how can each *pareto efficient allocation be supported*? **(you did not need to solve – I am looking for a short verbal answer).** **(5 points)**

   ii. Suppose the only feasible mechanism for redistributing income is a tax on earned income that can not discriminate among people (i.e., all people with the same earned income pay the same taxes); for example, the tax system might be linear: $T^h = -\theta + \tau\left(W^h L^h\right)$, where T^h is the net tax paid by household *h,* θ is a transfer to each household, and τ is the marginal tax rate. *Can every pareto efficient allocation be supported using this mechanism? If not, what inefficiency does the tax scheme cause?* **(again, you did not need to solve – I am looking for a short verbal answer).** **(5 points)**

d) Finally, suppose we revert to assuming people do not value leisure but instead we modify the technology for producing good *x* as follows (all else is unchanged):

$$Q_x = \left[\left(2L_x^A\right)^2 + \left(L_y^B\right)^2\right]^{1/2}$$

   i. Are the conditions required to **prove** the second welfare theorem satisfied? **(4 points)**

   ii. For this model, can **every** Pareto efficient allocation be supported as a competitive equilibrium? Can ***any*** Pareto efficient allocation be supported as a competitive equilibrium? Justify your answer. {Hint: use your results from part (b) to help you answer.} **(6 points)**

3. Answer All Parts.

a) Suppose there are three firms that make up an industry. Each firm uses the same input (e.g., labor) to produce a homogeneous output (e.g., shoes). Using the "standard micro" convention, measure inputs and outputs as both positive variables (so it is less confusing), so that the production set for each firm is given by:

$$q_i \le f_i\left(L_i\right); \quad f_i' > 0 > f_i''; \quad L_i \ge 0$$

(the production function is the boundary of this set). Define the "industry" production function as the boundary of the *feasible* industry production set; this "industry" production function shows the relationship between the total labor used in the industry and the **maximum** output obtainable, i.e.: $q^T \le g\left(L^T\right); \quad L^T = \sum_i L_i$

i. Does production efficiency at the firm level guarantee the aggregate production point is on the boundary of the industry production set? Justify your answer. **(4 points)**

ii. For the special functions given below, derive the industry production function (or set): **(6 points)**
$q_i \le \alpha_i \left(L_i\right)^{1/2}; \quad \alpha_1 = 4, \quad \alpha_2 = 2; \quad \alpha_3 = 1$

iii. Let *p* denote the price of output (*q*) and *W* the wage rate (for labor). Does individual profit maximization guarantee aggregate profit maximization? Under profit maximization, will the production choices made by individual firms lead to aggregate production efficiency? Explain **(5 points)**

iv. Derive the maximum profit function for this industry (i.e., the maximized profits of all firms as a function of prices) and the industry supply curve. Does the usual relationship between the profit function and the behavioral relations (output supply, factor demand) hold for this industry profit function? **(5 points)**

b) Consider a two good *(X,Y)*, two factor *(K,L)* general equilibrium model where production exhibits constant returns to scale. Production functions (and dual cost curves) are given by:

$$Q_x = \left(3/2^{2/3}\right) K_x^{1/3} L_x^{2/3} \rightarrow TC\left(Q_x, W, R\right) = Q_x \left(R_x\right)^{1/3} \left(W_x\right)^{2/3};$$

$$Q_y = \left(3/2^{2/3}\right) K_y^{2/3} L_y^{1/3} \rightarrow TC\left(Q_y, W, R\right) = Q_y \left(R_y\right)^{2/3} \left(W_y\right)^{1/3}$$

where $\left(R_i, W_i\right)$ is the price paid for capital and labor, respectively, in sector *i*. If factors are freely mobile between sectors and there is no differential taxation, then $R_x = R_y$ and $W_x = W_y$. Finally, let $\left(\bar{K}, \bar{L}\right)$ denote the (fixed) factor endowments and assume: $\bar{K} = \bar{L}$

i. **Given output prices**, derive the general equilibrium supply curves. Since, at the firm level, marginal cost is constant (*i.e.*, independent of output level), does that imply these general equilibrium supply curves are infinitely elastic? Explain your answer. **(8 points)**

ii. **Given output prices**, show how a tax on labor used in sector *Y* affects equilibrium output levels and factor prices (for wages, distinguish between the wages paid by firms in each sector). How do your results concerning the impact of the tax (on wages, output,…) differ from what you would expect to find in a partial equilibrium model? Explain. **(7 points)**

(question 3 continued on next page)

***(question 3, continued*)**

c) Consider a model with three final products and a single input in fixed supply (labor). The production technology and resource constraints are given by:

$$Q_i \le L_i, \quad i = 1,2,3; \quad \sum_i L_i = L^T$$

There are H households (each endowed with the same quantity of labor), and with preferences

$$U^h = c_1^h + \left(\beta c_2^h - \frac{\left(c_2^h\right)^2}{2} \right) + \left(\gamma c_3^h - \frac{\left(c_3^h\right)^2}{2} \right) - \frac{\mu_h Z^2}{2}; \;\; \beta > 1, \gamma > 1, \, \mu_h > 0 \;\forall h$$

where c_j^h is household *h*'s consumption of good *j*, and *Z* is (air) pollution, which affects all people (though not necessarily to the same extent). Air pollution is caused by production of good 3: $Z = \theta Q_3, \quad \theta > 0$.

i. Find the Pareto efficient production levels (you may restrict the utility allocations to one in which all people consume all goods – i.e., to strictly interior allocations). **(8 points)**

ii. Will a competitive equilibrium be efficient? If not, what policy can a government use to enable a competitive equilibrium to achieve Pareto efficiency? Be specific (that is, describe not only the policy but the quantitative (numerical) level of that policy). **(7 points)**