Lapan

Econ 603
Spring 2006

Midterm Exam
Answer any **two** questions; answer all parts to each question.
Each question is worth 50 points.

1. Answer all parts.

a) Assume there are *J* firms, each with feasible production technology described by $Y^j \subset \Re^L$. The netput vector $\vec{y}$ (with the usual convention that $y_l > 0$ represents an output and $y_k < 0$ represents an input) is feasible for firm *j* if and only if $\vec{y} \in Y^j$.

i. How is the aggregate production set for the economy derived? Explain carefully. **(4 points)**

ii. Will profit maximization by firms lead to an aggregate production point that is efficient? **Prove your answer** and explain what role this result plays in the first welfare theorem. **(6 points)**

iii. What does it mean to say that any efficient production point can be supported through competitive profit maximization? Under what assumptions is it true that any efficient production point can be supported as a profit maximizing solution? What role does this result play in the second welfare theorem? **(4 points)**

b) Consider an economy with two goods and three firms. Production technology is given by:

Firm 1: $y_2^1 \le 4\left(-y_1^1\right)^{1/2}$; $y_1^1 \le 0$; ***Firm 2:*** $y_2^2 \le 4\left\{Max\left[\left(-y_1^2 - 4\right); 0\right]\right\}^{1/2}$; $y_1^2 \le 0$;

***Firm 3*:** $y_2^3 \le \left(-y_1^3/2\right)$; $y_1^3 \le 0$

where y_l^j is firm *j's* netput of good *l*. All firms use good one as an input to produce good 2. Also note that firm 2's technology implies that there will be no output if the input of good 1 is not larger than four – in essence, four is the quasi-fixed cost required by the firm to produce any output.

i. Let $\left(p_1, p_2\right)$ denote the prices of goods. Derive the industry supply curve for this economy. **(5 points)**

ii. Derive the aggregate production set for this economy. Are all efficient points supportable through profit maximization? If you answer no, explain why not **and** show which points are not supportable through profit maximization. Be as precise as possible. **(8 points)**

iii. Would your answer to part ii change if firm 3's technology were: $y_2^3 \le \left(-2y_1^3\right)$; $y_1^3 \le 0$? **(5 points)**

c) Consider an economy with the technology given in part (b) (with firm 3's technology given by: $y_2^3 \le \left(-y_1^3/2\right)$; $y_1^3 \le 0$ **).** Aggregate endowments for this economy are: $\omega^T = \left(\omega_1, \omega_2\right) = \left(20, 0\right)$. There are two individuals with preferences:

Person 1: $U^1 = \left(c_1^1\right)\cdot\left(c_2^1\right)^3$; **Person 2:** $U^2 = \left(c_1^2\right)^3\cdot\left(c_2^2\right)$

where c_l^h is person *h's* consumption of good *l*. Let I^T denote total income for this economy, and let s_h denote person *h's* share of total income $\left(s_1 + s_2 = 1\right)$. {Note that I^T includes the value of endowments plus profits of firms}.

i. Do the conditions required to prove the existence of a competitive equilibrium hold for this economy? Can we be certain whether such an equilibrium exists? **(3 points)**

ii. Find the values of $s_1 \in [0,1]$, if any, for which a competitive equilibrium exists. **(7 point)**

iii. Do the conditions required to prove the Second Welfare Theorem hold for this economy? Explain the economic implications of your answer. **(3 points)**

iv. Which *pareto efficient allocations, if any, can be supported as a competitive equilibrium*? Be as specific as possible. {HINT: use your results from existence of competitive equilibria to answer this part, rather than trying to solve explicitly for the *pareto efficient* allocations}. **(5 points)**

2. Answer all parts

a) Consider an exchange economy with L goods and H people. Let $\vec{\omega}^h = \left(\omega_1^h, ..., \omega_L^h\right)$ denote the endowment vector and $\vec{x}^h = \left(x_1^h, ..., x_L^h\right)$ the consumption vector of individual h. The individual's preferences are represented by the utility function $U^h\left(\vec{x}^h\right)$, which has all the standard properties, *including convexity*.

i. Suppose the government bans trade in good 1, but allows trade in all other goods. **Define** a competitive equilibrium for this economy (denote the equilibrium price vector by $\vec{P}' = \left(P_2, ..., P_L\right)$). Will a competitive equilibrium exist? **(3 points)**

ii. **Given** that no trade in good 1 is allowed, will the resulting competitive equilibrium be Pareto efficient? **Prove your answer.** {To be precise, the question asks: given that the constraint $x_1^h = \omega_1^h$ must hold for all h, is there any feasible allocation satisfying this constraint that is *pareto superior* to the competitive equilibrium allocation which satisfies this constraint?}. **(8 points)**

iii. Finally, suppose the ban on trade in good 1 is eliminated so that trade in all L goods is allowed. Will the resulting competitive equilibrium, with trade in all goods, be "more efficient" then the competitive equilibrium without trade in good 1? Will *everybody* necessarily gain as a result of eliminating the ban on trade in good 1? Explain. {**As part of your answer, you must say what *you* mean by "more efficient"}.** **(5 points)**

b) Consider a two good *(X,Y)*, two factor *(K,L)* general equilibrium model where production exhibits constant returns to scale. Production functions (and dual cost curves) are given by:

$$Q_x = \left(4/3^{3/4}\right)\theta_x K_x^{1/4} L_x^{3/4} \rightarrow TC\left(Q_x, W, R\right) = \left(Q_x/\theta_x\right)\left(R\right)^{1/4}\left(W\right)^{3/4};$$

$$Q_y = \left(4/3^{3/4}\right)\theta_y K_y^{3/4} L_y^{1/4} \rightarrow TC\left(Q_y, W, R\right) = \left(Q_y/\theta_y\right)\left(R\right)^{3/4}\left(W\right)^{1/4}$$

where $\left(R, W\right)$ is the price of capital and labor, respectively (there are no factor taxes). Note that the terms θ_x, θ_y reflect what is called Hicks neutral technical progress in sectors x and y, respectively. Thus, an increase in θ_x increases output in sector x, given inputs, but leaves unchanged the ratio of marginal products (at given inputs) in that sector. Let $\left(\bar{K}, \bar{L}\right)$ denote the (fixed) factor endowments.

Turning to consumers (households), assume there are two types of households in this economy. Type I households (workers) own only labor, and hence their income depends on the wage rate (W); type II households (capitalists) own only capital, and hence their income depnds on the return on capital (R). All households have the same preferences, given by: $U = \left(c_x \cdot c_y\right)$

i. **Given output prices**, derive the general equilibrium supply curves. Since, at the firm level, marginal cost is constant (*i.e.*, independent of output level), does that imply these general equilibrium supply curves are infinitely elastic? Explain your answer. **(8 points)**

ii. *Given output prices,* show how an increase in productivity in sector *x* (*i.e.,* an increase in θ_x) affects output of each good and the return to each factor. Does everybody gain from this increased productivity? Do these results differ, in any way, from what you might expect in a partial equilibrium model? **(7 points)**

iii. Given consumer preferences, find the equilibrium prices (output prices and factor prices) for this economy as functions of technology $\left(\theta_x, \theta_y\right)$ and factor endowments $\left(\bar{K}, \bar{L}\right)$. **(6 points)**

iv. Suppose the government levies an ad valorem (percent) tax on good *x*, and that all the proceeds from this tax are distributed back equally to households. Show how this tax affects equilibrium output and factor prices and discuss what inefficiency, if any, this tax creates. Will **all** households necessarily be hurt by these taxes? Explain carefully. **(7 points)**

v. Assume there are no government taxes, but assume productivity in sector *x* $\left(\theta_x\right)$ depends positively on output of good *y*; thus, $\theta_x = h\left(Q_y\right), h'\left(Q_y\right) > 0$.
(1)Will the competitive equilibrium lead to efficient production (i.e., will the production point be on the societal production possibility frontier)? (2)Will the competitive equilibrium be *pareto efficient?* If it is not, what is the inefficiency and what government policy is required to correct the market failure (you do not need an analytic solution). **(6 points)**

3. Answer all parts.

a) Consider an exchange economy with two goods (X,*Y*) and two people *(a, b*). Household preferences are:

$$U^a = 3\ln\left(c_x^a\right) + \ln\left(c_y^a\right); \quad U^b = \ln\left(c_x^b\right) + 3\ln\left(c_y^b\right)$$

where c_l^h is household *h's* consumption of good *l*. Let aggregate endowments be: $\left(\omega_x^T, \omega_y^T\right)$

i. Write down the equations needed to find the set of Pareto efficient allocations (you do not need to solve, but you must have enough equations to be able to solve). **(5 points)**

ii. For a Pareto efficient allocation, as you increase the target utility level for person *B*, what happens to the shadow price of good *m* (the equilibrium price required to support the allocation as a competitive equilibrium)? Explain. **(5 points)**

iii. Suppose society must choose between two projects; if project I is chosen, the aggregate bundle available for consumption is $\left(\omega_x^I, \omega_y^I\right) = (5,10)$; if project II is chosen, the aggregate consumption bundle is $\left(\omega_x^I, \omega_y^I\right) = (10,5)$. Which bundle (project) should be chosen? Explain your answer and discuss its significance. **(5 points)**

b) Consider an exchange economy with two goods (X,*Y*) and two people *(a, b*). Household preferences and total endowments are given by:

$$U^a = 2c_x^a + c_y^a; \quad U^b = \left[\left(c_x^b\right)^2 + \left(c_y^b\right)^2\right]^{1/2}; \quad \omega_x^T = \omega_y^T = 10$$

i. Derive the set of Pareto efficient allocations. Which allocations, if any, can be supported as a competitive equilibrium? **(5 points)**

ii. Suppose person *a's* endowments are given by: $\left(\omega_x^a, \omega_y^a\right) = (\mu, 5) \rightarrow \left(\omega_x^b, \omega_y^b\right) = (10 - \mu, 5)$. Find the values of μ for which a competitive equilibrium exists. As μ increases (within this range), what happens to the equilibrium relative price of good *x* and the utility level of each person? Be precise. **(8 points)**

c) Consider a simple model with three goods and *H* identical consumers. Goods 2 and 3 are produced using good 1 as an input. Technology and preferences are given by:

$$y_2^1 + 2y_1^1 \le 0, \; y_1^1 \le 0; \quad y_3^2 + y_1^2 \le 0, \quad y_1^2 \le 0; \quad U^h = c_1^h + \phi\left(c_2^h, c_3^h\right) + \theta Z; \quad \theta < 0; \; Z = \lambda y_2^1$$

where y_j^k is the netpot of good *j* for firm *k*, c_j^h is person *h's* consumption of good *j*, and Z represents pollution, which is generated by output of good 2. The function $\phi(...)$ is strictly concave in its arguments. The aggregate endowment vector is: $\left(\omega_1^T, \omega_2^T, \omega_3^T\right) = (10H, 0, 0)$.

i. Will a competitive equilibrium be *pareto efficient*? If not, specify the optimal government policy needed to make the competitive equilibrium *pareto efficient* and indicate how this policy changes as the size of the economy (*H*) increases. Be as specific as possible (you may assume an interior solution and that all people are treated identically). **(6 points)**

ii. Suppose that, for political reasons, the only feasible policy is to **tax or subsidize good *3*.** What determines the sign of this second-best policy (i.e., whether you should tax or subsidize)? Give an intuitive answer and relate your answer to some property of the function $\phi(...)$. **(7 points)**

d) Consider a simple two good (X, Y), two person exchange economy (A,B). Preferences are given by:

$$U^A = Min\left(c_x^A, c_y^A\right); \quad U^B = \left(c_x^B\right)^{1/2} + \left(c_y^B\right)^{1/2}$$

and aggregate endowments by: $\left(\omega_x^T, \omega_y^T\right) = (20, 30)$

i. Using the Edgeworth box, sketch the set of pareto efficient allocations. Are all supportable as competitive equilibria? Explain, and if you answer no, indicate which assumption required to prove the Second Welfare Theorem is violated. **(5 points)**

ii. Let $\left(\omega_x^B, \omega_y^B\right)$ denote *B*'s endowment vector (so $\left(\omega_x^A, \omega_y^A\right) = \left(20 - \omega_x^B, 30 - \omega_y^B\right)$). Will a competitive equilibrium exist for all endowment vectors? If not, for which endowments is there no competitive equilibrium, and what assumption used to prove the existence of an equilibrium is violated? **(4 points)**