

Problem Set 1 – Due Friday, January 16, 2009

1. Consider an exchange economy with two agents who have the following preferences:

$$U^1(x_1^1, x_2^1) = (x_1^1) \cdot (x_2^1); \quad U^2(x_1^2, x_2^2) = \ln(x_1^2) + \alpha \ln(x_2^2)$$

where x_j^h is individual h 's consumption of good j . Each agent has endowment vector: (e_x^h, e_y^h) , $i = 1, 2$

with aggregate endowment $\{e_x^T, e_y^T\} = \{(e_x^1 + e_x^2), (e_y^1 + e_y^2)\}$

- (a) Given the initial endowments, find the competitive equilibrium price and consumption allocations for this economy.
- i. Given aggregate endowments, how does a transfer of resources (income) from person 1 to person 2 change equilibrium prices (relate your answer to the value of α).
- ii. Must an equilibrium (price and consumption allocation) exist for this economy for any endowment vector and all values of α ? If so, how can you be sure?
2. Consider the same basic preferences as in question 1 but assume $\alpha = 2$. Assume aggregate endowments $\{e_x^T, e_y^T\} = \{10, 10\}$ and assume an initial allocation such that $(e_x^1, e_y^1) = (5, 5)$
- (a) Calculate the initial utility levels for each person, then show the set of trades (or consumption levels) that would leave both people better off (at least as well off) as with this initial allocation.
- (b) Repeat the exercise of part (a) for any arbitrary allocation – that is, find the set of *Pareto efficient allocations*.
- (c) Would your answers to parts (a) or (b) change if person 1's preferences changed to: $U^1(x_1^1, x_2^1) = \ln(x_1^1) + \ln(x_2^1)$? Explain your answer.