

Midterm Exam

Answer any two questions; answer all parts to each question (Each question is worth 50 points)

1. Answer all parts.

- a) Consider the following two person (A, B), two good (x,y) exchange economy. Preferences are:

$$\text{Person A: } U^A = \ln(c_x^A) + \ln(c_y^A); \quad \text{Person B: } U^B = (c_x^B \cdot c_y^B)$$

where c_l^i is person i 's consumption of good l . Endowments are: $(\omega_x^A, \omega_y^A) = (1, 16)$; and $(\omega_x^B, \omega_y^B) = (16, 1)$.

- i. Find the set of *Pareto efficient allocations* (include boundary points, if any). **(5 points)**
 - ii. Given the initial endowments, find and show graphically the contract curve (the Core) for this two person economy. Carefully label the end points of the contract curve. **(3 points)**
 - iii. Suppose the economy increases in size so that there are **four** people (two of type A and two of type B). Is the allocation $(c_x^A, c_y^A) = (5, 5)$, $(c_x^B, c_y^B) = (12, 12)$ in the core for the two person economy of part (ii)? Is this consumption allocation in the core for the four person economy? **Demonstrate your answer** (i.e., a simple yes or no does not suffice). **(6 points)**
 - iv. What is the competitive equilibrium consumption allocation for this economy? How does this allocation relate to the core? Be as specific as possible **(5 points)**
- b) Consider a two good (X,Y), two factor (K,L) general equilibrium model where production exhibits constant returns to scale. Production functions (and dual cost curves) are given by:

$$Q_x = 2K_x^{1/2}L_x^{1/2} \rightarrow TC(Q_x, W_x, R_x) = Q_x (R_x)^{1/2} (W_x)^{1/2};$$

$$Q_y = (4/3^{3/4})K_y^{3/4}L_y^{1/4} \rightarrow TC(Q_y, W_y, R_y) = Q_y (R_y)^{3/4} (W_y)^{1/4}$$

where (R_i, W_i) is the price paid for capital and labor, respectively, in sector i . Unless otherwise specified, assume factors are freely mobile between sectors and there is no differential taxation, so that $(R_x = R_y = R)$ and $(W_x = W_y = W)$. Let (\bar{K}, \bar{L}) denote the (fixed) factor endowments. Finally, assume consumers have identical and homothetic preferences given by:

$$U = \alpha \ln(c_x) + (1 - \alpha) \ln(c_y); \quad \alpha \in (0, 1)$$

- i. **Given output prices**, derive the general equilibrium supply curves and equilibrium factor prices (in terms of output prices and endowments). Does constant returns to scale imply general equilibrium supply curves are infinitely elastic and the production possibility frontier is linear? Explain. **(8 points)**
- ii. Given preferences, find the **competitive equilibrium** output and input prices. How does an increase in α (an increase in demand for x) affect the return to capital and labor? Explain your result. **(6 points)**

(question 1, continued)

- iii. **Given output prices**, show how an *advalorem* tax, τ , on capital used in sector X affects equilibrium output levels and factor prices (if R is the return received by capital owners, then the tax implies:

$$R_x = R(1 + \tau); \quad R_y = R). \text{ What inefficiency, if any, results from this tax? Be specific!} \quad (7 \text{ points})$$

- c) Consider a very simple two person (A, B), two good (x, y) exchange economy. Preferences are:

$$U^A = (c_x^A)^{1/2} + (c_y^A)^{1/2}; \quad U^B = \text{Min}(c_x^B, c_y^B)$$

Aggregate endowments for this economy are $(\omega_x^T, \omega_y^T) = (15, 10)$, and (ω_x^A, ω_y^A) denotes A 's endowment.

- i. Are preferences convex? Will an equilibrium exist for all (ω_x^A, ω_y^A) ? Explain your answer and, if you answer no, specify the set of endowments for which an equilibrium will exist. **(5 points)**
- ii. Find the set of *pareto efficient allocations* and indicate which, if any, are supportable as a competitive equilibrium with transfers? As a quasi-equilibrium with transfers? Explain your answer. **(5 points)**

2. Answer all parts.

- a) Consider an economy with H consumers. Each consumer is endowed with one unit of time, which is divided between work (L) and leisure (l). There is one consumption good, produced by a competitive firm under constant returns to scale. There is also a good, G , (e.g., a park) that is non-rival in consumption. This good is also produced under constant returns to scale using only labor. Preferences and technology are:

Preferences:
$$U^h = (c^h \cdot l^h)^{1/2} + \alpha^h (G^h)^{1/2}; \quad (l^h + L^h) = 1; \quad c^h \geq 0; \quad l^h \in [0, 1]$$

Technology:
$$Q_c \leq L_c; \quad Q_g \leq \beta L_g, \quad \beta > 0$$

Resource Constraints:
$$L_c + L_g \leq \sum_h (L^h); \quad \sum_h (c^h) \leq Q_c; \quad G^h \leq Q_g, \quad h = 1, \dots, H$$

- i. State the condition for the optimal provision of a public good, then find the symmetric *pareto efficient* allocation (where all consumers receive the same utility from their *private* consumption vector (c^h, l^h)). **(6 points)**
- ii. Consider a competitive market economy in which the government raises the revenue to pay for provision of the public good by taxing individuals. Can the optimal solution be supported with lump sum taxes? Can it be supported if lump sum taxes are not available? Explain. **(3 points)**
- iii. Suppose the only policy available to the government is an income tax (consumption and income taxes are identical in this model). Let t be the tax rate on wage income, so:

Household budget constraint:
$$p_c c^h \leq W L^h (1 - t);$$

Government budget constraint:
$$W L_g \leq t W \left[\left(\sum_h L^h \right) - L_g \right]$$

Profits of C producer:
$$\pi_c = P_c Q_c - W L_c$$

Find the competitive equilibrium and the optimal level of the income tax (assuming the goal is to maximize the sum of individual utilities). Compare the level of provision of the public good found here with that found in part (i). Explain the difference, if any. **(6 points)**

(question 2, continued)

b) State the First and Second Welfare Theorems. What role does convexity play in each theorem? **(3 points)**

i. Prove the first welfare theorem. **(7 points)**

ii. Indicate where, if at all, your proof would fail if the utility (preferences) of one consumer depended upon the consumption choices of another consumer. **(3 points)**

c) Consider an economy with two goods and two firms. Production technology is given by:

$$\text{Firm 1: } y_2^1 \leq (-3y_1^1) \text{ for } -10 \leq y_1^1 \leq 0; \quad y_2^1 \leq 30 \text{ for } y_1^1 \leq -10;$$

$$\text{Firm 2: } y_2^2 \leq 0 \text{ for } -9 \leq y_1^2 \leq 0; \quad y_2^2 \leq 30(-y_1^2 - 9)^{1/2} \text{ for } y_1^2 \leq -9$$

where y_l^j is firm j 's netput of good l . Both firms use good one as an input to produce good 2. Firm 2's technology implies that there will be no output if the input of good 1 is not larger than nine– in essence, nine is the quasi-fixed cost required by the firm to produce any output.

i. Let (p_1, p_2) denote the price vector. Set up the profit maximization problem and derive and sketch the output supply (input demand) curves for each firm. *Does individual profit maximization lead to an efficient production point? Does it lead to aggregate profit maximization?* Explain. **(5 points)**

ii. Derive the aggregate production set for this economy. **Are all efficient points supportable through profit maximization?** If you answer no, indicate which ones are not supportable. **(7 points)**

iii. Let aggregate endowments be $\omega^T = (54, 0)$. Assume there are two individuals with preferences:

$$\text{Person 1: } U^1 = (c_1^1)^{5/6} (c_2^1)^{1/6}; \quad \text{Person 2: } U^2 = (c_1^2)^{1/2} (c_2^2)^{1/2}$$

Let Y^T denote total income and s_i denote person i 's share of total income ($s_1 + s_2 = 1$). For what income distributions (what values of s_1), if any, will a competitive equilibrium exist? A specific answer is required but, if you cannot find it, then discuss when an equilibrium is most likely to exist. **(6 points)**

iv. Does the second welfare theorem apply to this economy? Are any - or all - pareto efficient allocations supportable as a competitive equilibrium? Be specific. **(4 points)**

(question 3 on next page)

3. Answer All Parts.

- a) Consider a two person (A,B), “one good”, two period exchange model with uncertainty. Let ω_t^h denote person h 's endowment in period t . Person A's endowment in each period is known with certainty, and is given by: $\omega^A = (14, 6)$. Person B's first period endowment is **6**, but his second period endowment depends on the weather. If the weather is “DRY”, his endowment will be **8**; if the weather is “WET”, his endowment will be **20**. Both A and B agree each weather state is equally likely. Thus B's endowment, depending on weather, is:

If DRY, (occurs with probability $1/2$) $\omega^B = (6, 8)$; If WET, (occurs with probability $1/2$) $\omega^B = (6, 20)$

Individuals have identical preferences, and both behave as expected utility maximizers. Their realized utility, which depends on actual consumption in each period is:

$$U^h = 20c_1^h + \left\{ 20c_2^h - \left[\frac{(c_2^h)^2}{2} \right] \right\}$$

Since there is no storage, actual consumption in each state must be no larger than endowments for that state:

Resource constraints: $c_1^A + c_1^B \leq 20$; If DRY: $c_2^A + c_2^B \leq 14$; If RAINY: $c_2^A + c_2^B \leq 26$

Finally, period one consumption decisions must be made before the weather for period two is known.

- i. Find the set of *pareto efficient allocations* for this economy (an allocation specifies consumption in period 1 for each person and consumption in period 2 for each person in each weather state; the criterion is expected utility, and you may restrict attention to interior solutions where $c_i^h > 0 \forall i, h$). **(6 points)**
 - ii. Suppose the only market that exists is a capital market where individuals can borrow or lend; however, period two payments **cannot** be contingent on the realized weather. Let p_t denote the price in each period so that for each unit borrowed (or lent) in period one, (p_1/p_2) units must be repaid (or received) in period two. Find the competitive equilibrium and indicate whether it is *pareto efficient*. **(6 points)**
 - iii. Finally, assume a complete set of markets exists. Find the equilibrium prices and the individual consumption vectors. Is this equilibrium *pareto efficient*? Is it *pareto superior* to part (ii)? **(6 points)**
- b) Consider an economy with two goods and **three** firms. Each firm uses inputs of good one to produce good two according to the following technology (written in netput notation):

Firm j : $y_2^j \leq A_j(-y_1^j) - (1/10)(y_1^j)^2$; $y_1^j \leq 0 \forall j$; $A_1 = 4$; $A_2 = 3$; $A_3 = 2$

The aggregate endowment vector for the economy is $(\omega_1^T, \omega_2^T) = (0, 50)$

- i. Does production efficiency at the firm level guarantee the aggregate production point is on the boundary of the industry production set? Justify your answer. **(2 points)**
- ii. Derive the economy's production set and the consumption possibility set, given endowments. **(5 points)**
- iii. Does individual profit maximization lead to an efficient production point? Can every efficient production point be supported through profit maximization? Justify your answer. **(3 points)**

(question 3, continued)

Next, suppose the technology for firms 1 and 2 are as above, but firm 3's technology is given by:

Firm 3: $y_2^3 \leq (2 - \alpha z)(-y_1^3) - (1/10)(y_1^3)^2$; $y_1^3 \leq 0$; $z = (-y_1^1)$

For $\alpha = 0$ this is the same technology as above; for $\alpha > 0$ it implies firm 1's decisions affect firm 3's technology (production set). In what follows assume $\alpha = (1/15)$.

- iv. Using the new specification for firm 3's technology, re-derive the economy's production set and compare to your answer to part (ii). For a given total input of good 1, how does the input allocation to each sector change? **Be as specific as possible in your answer.** (6 points)
- v. Will profit maximization lead to production efficiency? Will a competitive equilibrium be efficient? Relate your answer to the equilibrium relative price of good 2. If not, what policy is needed to support the efficient allocation? **Be specific in your answer.** (6 points)
- c) Consider a simple exchange economy with two goods (X, Y) and two people (A, B) . Aggregate endowments are given by: (ω_x, ω_y) and consumption by (c_l^h) , $l = x, y$; $h = A, B$. Individual preferences are given by:

Person A: $U^A = 2\ln(c_x^A) + \ln(c_y^A)$; Person B: $\ln(c_x^B) + 2\ln(c_y^B)$

- i. Write down the equations needed to determine the set of Pareto efficient allocations (you do not need to solve, but you must have enough equations so that you could solve). (3 points)
- ii. Suppose the current endowment vector is $(\omega_x, \omega_y) = (6, 8)$ but that there exists a project that would alter this endowment vector to $(\omega_x, \omega_y) = (8, 6)$. Should society approve the project? What additional information, if any, do you need to know to answer this question? (3 points)
- iii. Answer part (ii) assuming there is a central planner who makes the decision, and his goal is to maximize the social welfare function: $W = 2U^A + U^B$. (4 points)