Lapan

Econ 603
Spring 2008

Midterm Exam

Answer any two questions; answer all parts to each question (Each question is worth 50 points)

1. Answer all parts.

a) Assume there are J firms, each with feasible production technology described by $Y^j \subset \Re^L$. The netput vector $\vec{y}$ (with the usual convention that $y_l > 0$ represents an output and $y_k < 0$ represents an input) is feasible for firm j if and only if $\vec{y} \in Y^j$.

i. Define the aggregate production set for this economy and define what it means for a production point to be efficient in this set. If each individual firm chooses a production plan that is efficient in its production set, will the aggregate production point be efficient in the aggregate production set? Explain your answer. **(6 points)**

ii. Does profit maximization by individual firms lead to an efficient aggregate production point? **Prove your answer** and indicate the key assumptions used in your proof. **(6 points)**

iii. What does it mean to say that any efficient production point can be supported through competitive profit maximization? Under what assumptions is it true that any efficient aggregate production point can be supported as a profit maximizing solution? What role does this result play in the Second Welfare Theorem? **(4 points)**

b) Consider a simple example with two firms and two goods. Technology is given by:

Firm 1: $y_1^1 \le A \cdot \left(y_1^2\right)^{\alpha} \left(-y_2^1\right)^{1/2}$; $y_2^1 \le 0$; $A > 0, \alpha \ge 0$;

Firm 2: $y_1^2 \le B\left(y_1^1\right)^{\beta} \left(-y_2^2\right)^{1/2}$; $y_2^2 \le 0$; $B > 0, \beta \ge 0$;

As usual, y_l^j is firm *j's* netput of good *l*. Also, assume the initial endowment vector for the economy is $\omega^T = (0, 30)$.

i. Assuming $\alpha = \beta = 0$, find the aggregate production set and the production possibility frontier for this economy. Will individual profit maximization lead to an aggregate production point that is efficient? Can every efficient production point be supported through profit maximization? If so, assuming $A = 6, B = 8$, show how $\left(y_1^T, y_2^T\right) = (40, -16)$ can be supported via profit maximization ($y_l^T \equiv \left(y_l^1 + y_l^2\right)$). **(7 points)**

ii. Let $\alpha = \beta = (1/4)$. Set up the optimization problem and write the set of equations used to derive the aggregate production set. Will individual profit maximization (where the two firms make independent decisions) ever lead to an efficient aggregate production point? If so, precisely when? Be careful and be specific. **(6 points)**

c) Use the technology from part (b) with $A = B = 1$, and $\alpha = \beta = (1/4)$. Assume there are two individuals, with the following identical preferences:

$$U^i = \ln\left(c_1^i\right) + \ln\left(c_2^i\right)$$

i. Find the competitive equilibrium production point. Will this production point be on the boundary of the aggregate production set? **(5 points)**

ii. Will the competitive equilibrium be Pareto efficient? Explain your answer and, if you answer no, specify what policy is required to achieve a Pareto efficient allocation as a competitive equilibrium (an intuitive, not a numerical, answer is required here). **(5 points)**

d) Consider a simple two primary factor (*K*, *L*), two good economy with the following technology:

$$Q_1 = \left[(K_1)^2 + (L_1)^2\right]^{1/2}; \quad Q_2 = K_2 + L_2; \quad (K_1 + K_2) \le 10; \quad (L_1 + L_2) \le 15$$

i. Derive and sketch the production possibility frontier for this economy. Since production functions are homogeneous of degree one, does that guarantee the aggregate production set is convex? Explain your answer. **(11 points)**

2. Answer all parts.

a) Consider a two good (*X*,*Y*), two factor (*K*, *L*) general equilibrium model. Production functions, which exhibit constant returns to scale, and resource constraints are given by:

$$Q_x = 2K_x^{1/2}L_x^{1/2}; \quad Q_y = \left(K_y + L_y\right); \quad K_x + K_y \le \bar{K}; \quad L_x + L_y \le \bar{L}$$

The input endowments for the economy are: $\left(\bar{K}, \bar{L}\right) = (36, 108)$.

i. Derive the production possibility frontier for this economy. Are all points on the frontier supportable through profit maximization? If so, what are the prices $\left(P_x, P_y, W, R\right)$ that support these points? **(8 points)**

ii. Suppose individuals have identical and homothetic preferences given by:

$$U^h = \left(c_x^h\right)^{\alpha}\left(c_y^h\right)^{1-\alpha}; \quad \alpha \in (0,1)$$

Find the *Pareto efficient production point* (*your answer should depend on* α) (**5 points)**

iii. Next, consider a competitive market economy in which the government provides a 50% subsidy on capital used in sector *X* (there is no subsidy on capital used in sector *Y* and no tax or subsidy on labor). Thus, factor prices are such that:

$$W_x = W_y \equiv W; \quad R_y \equiv R, \quad R_x = (1 - s)R_y, \quad s = (1/2).$$

Find the general equilibrium supply curves and derive and sketch the output locus (in Q_x, Q_y space) that corresponds to these supply curves. Compare this locus to the *production possibility frontier* found in part (i). What inefficiencies, if any, does this subsidy cause in a market economy? Do these inefficiencies occur at all competitive allocations? Be specific. **(9 points)**

iv. Given the subsidies in part (iii) and the preferences of part (ii), find the competitive equilibrium and indicate whether it will be *Pareto efficient*. {Be careful; relate your answer to the value of the parameter α }. **(7 points)**

v. Suppose it is not politically feasible to change the capital subsidy. Given this subsidy, is there any other policy (tax output, tax labor, etc.) that could improve welfare? Be specific and carefully justify your answer. Will this policy restore Pareto efficiency? Explain. **(7 points)**

b) State the First and Second Welfare Theorems and indicate the role, if any, that convexity plays in the proof of each theorem. **(3 points)**

i. Prove the First Welfare Theorem. **(7 points)**

ii. Indicate where, if at all, your proof would fail to hold if there were a tax on a final product (i.e., a good produced by firms and consumed by households). With this tax, would production still occur on the production possibility frontier? **(4 points)**

3. Answer All Parts

a) Consider a two good (*x*, *y*), two person (*A*, *B*) exchange economy. Preferences are given by:

$$Person \quad A: \quad U^A = 2\left(c_x^A\right)^{1/2} + \left(c_y^A\right)^{1/2}; \qquad Person \quad B: \quad U^B = Min\left(c_x^B, c_y^B\right)$$

Endowments are: $\left(\omega_x^A, \omega_y^A\right)$ and $\left(\omega_x^B, \omega_y^B\right)$, where $\left(\omega_x^A, \omega_y^A\right) + \left(\omega_x^B, \omega_y^B\right) = (20, 30)$

where c_l^i is person *i*'s consumption of good *l* and ω_l^i is person *i's* endowment of good *l* $\left(l = x, y; i = A, B\right)$.

i. What assumptions are needed to prove that a competitive equilibrium exists and are those assumptions satisfied for this economy? **(3 points)**

ii. Given the endowments, find the competitive equilibrium (when it exists). Is there any endowment vector(s) for which a competitive equilibrium does not exist? **(5 points)**

iii. Find the set of Pareto efficient allocations. Are all such allocations supportable as competitive equilibria? If you answer no, state what assumption used in the proof of the Second Welfare Theorem does not hold. **(5 points)**

b) Consider a two good (*x*, *y*), two person (*A*, *B*) exchange economy. Preferences are given by:

$$Person \quad A: \quad U^A = \left\{ \left(c_x^A\right)^2 + \left(c_y^A\right)^2 \right\}; \quad Person \quad B: \quad U^B = \left\{ c_x^B + 2c_y^B \right\}$$

Endowments are: $\left(\omega_x^A, \omega_y^A\right)$ and $\left(\omega_x^B, \omega_y^B\right)$, where $\left(\omega_x^A, \omega_y^A\right) + \left(\omega_x^B, \omega_y^B\right) = (30, 30)$

i. Find the set of Pareto efficient allocations. Can all Pareto efficient allocations be supported as a competitive equilibrium? If not, which cannot be supported and what assumption used to prove the second welfare theorem is violated? **(7 points)**

ii. Find the set of endowment vectors for which a competitive equilibrium exists and, when it exists, find the competitive equilibrium prices and consumption allocation. **(6 points)**

c) Consider a 3 good, 2 primary factor (*K*, *L*) economy with *H* identical consumers. Production technology for each good exhibits constant returns to scale and is given by:

$$Q_1 \le L_1; \quad Q_2 \le 2\left(K_2 \cdot L_2\right)^{1/2}; \quad Q_3 \le 2\left(K_3 \cdot L_3\right)^{1/2}; \quad L_i \ge 0, \quad K_i \ge 0, \quad i = 1, 2, 3$$

Each consumer is endowed with **1** unit of capital and **10** units of labor (so aggregate endowments of the primary factors are: $\left(\omega_L^T, \omega_K^T\right) = (10H, H)$, and each consumer has the following preferences:

$$U^h = c_1^h + 4\ln\left(c_2^h\right) + 4\ln\left(c_3^h\right) + \phi Z; \quad Z = Q_2; \quad \phi \le 0; \quad h = 1, 2, ..., H$$

For $\phi < 0$, there is a production externality whereby the utility of each consumer is adversely impacted by **aggregate** production of good two. The resource constraints for the economy are:

$$\sum_{i=2}^{3} K_i \le H; \quad \sum_{i=1}^{3} L_i \le 10H; \quad \sum_{h=1}^{H} c_l^h \le Q_l, \quad l = 1, 2, 3$$

(Assume throughout this problem that parameters are such that all solutions are interior, with all goods produced).

i. Find the competitive equilibrium and discuss whether it is *Pareto efficient*. How would lump sum transfers of income (say, from odd numbered to even numbered people) affect equilibrium prices and output? **(6 points)**

ii. Assuming $\phi < 0$, find the symmetric *Pareto efficient* allocation (all people get the same utility). Given that technology is constant returns to scale, discuss whether per capita utility (in the *Pareto efficient* allocation) changes with population size and, if it does, discuss why. **(6 points)**

iii. For $\phi < 0$, what government policy, if any, is required to support the (symmetric) *Pareto efficient* allocation as a competitive equilibrium? Be specific (indicate the magnitude of this policy). **(4 points)**

iv. Assume $\phi < 0$ and that the only feasible policy is a tax or subsidy on good 3. *Discuss* whether such a policy can improve welfare and then **find** the optimal (second best) tax or subsidy on good 3. **(8 points)**