Lapan

Econ 603
Spring 2009

Midterm Exam

Answer any **two** questions; answer all parts to each question. Each question is worth 50 points.

1. Answer all parts.

a) Assume there are *J* firms, each with feasible production technology described by $Y^j \subset \Re^L$. The netput vector $\vec{y}$ (with the usual convention that $y_l > 0$ represents an output and $y_k < 0$ represents an input) is feasible for firm *j* if and only if $\vec{y} \in Y^j$.

i. How is the aggregate production set for the economy derived? What does production efficiency mean and does individual production efficiency guarantee aggregate production efficiency? Explain carefully. **(3 points)**

ii. Will profit maximization by firms lead to an aggregate production point that is efficient? **Prove your answer,** indicating clearly the assumptions used. Is convexity of technology used in the proof? What role does this result play in the first welfare theorem? **(6 points)**

iii. What does it mean to say that any efficient production point can be supported through competitive profit maximization? Is convexity required to prove this result (you do not need to supply the proof)? What assumptions are required to prove the statement and what role does this result play in the second welfare theorem? **(4 points)**

b) Consider an economy with three goods and two firms. Production technology is given by:

Firm 1: $\frac{(y_{3,1})^2}{2} + \frac{(y_{2,1})^2}{8} + y_{1,1} \le 0; \quad y_{1,1} \le 0;$ ***Firm 2:*** $\frac{(y_{3,2})^2}{8} + \frac{(y_{2,2})^2}{2} + y_{1,2} \le 0; \quad y_{1,2} \le 0;$

where $y_{l,j}$ is firm *j's* netput of good *l*. Both firms use good one as an input to produce goods 2 and 3 (they are multi-product firms).

i. What conditions must be fulfilled to guarantee the netput vectors of individual firms lead to aggregate production efficiency? (You can assume an interior solution.) **(3 points)**

ii. Derive the aggregate production set for this economy (in 3 good space). Are all efficient points supportable through profit maximization? Explain. **(6 points)**

iii. Assume the total endowment of good 1 is ω_1^T and that consumers do not derive any utility from consuming good 1. **Find the production possibility frontier (in 2 good space) and derive the general equilibrium supply curves.** **(5 points)**

c) Consider an economy with the technology given in part (b). Suppose there are two individuals $\{A,B\}$ in the economy; each person has the endowment vector; $(\omega_1^h, \omega_2^h, \omega_3^h) = (5,0,0)$ (so that aggregate endowments of good one are 10, and there are no initial endowments of the other two goods. **Person A** owns 100% of firm 1, and **person B** owns 100% of firm 2. Individual preferences are given by:

Person A: $U^A = \left(c_2^A\right)^2 \cdot \left(c_3^A\right)$; **Person 2:** $U^B = \left(c_2^B\right) \cdot \left(c_3^B\right)^2$;

where c_l^h is person *h's* consumption of good *l*.

i. Do the conditions required to prove the existence of a competitive equilibrium hold for this economy? Explain. **(3 points)**

ii. Calculate the equilibrium price vector, consumption levels, and production levels for this economy. **(8 point)**

iii. Do the conditions required to prove the Second Welfare Theorem hold for this economy? If you want to support a *Pareto efficient allocation* that makes person *A* better off than she is under the competitive equilibrium, how can this be achieved (within a market economy) and how will the policy affect equilibrium prices? Be as specific as possible. **(4 points)**

d) Consider the following policies that are designed to help person A. **Discuss** what inefficiencies, if any, each of these policies causes. Be as specific as possible (but no analytics are required)

i. The government imposes a 50% profits tax on both firms. All the revenue from the profits tax is given to person A. **(4 points)**

ii. The government provides a 100% subsidy for the outputs of firm 1, which is owned by person A. {For example, if the price vector faced by firm 2 is $\left(p_{1,2}, p_{2,2}, p_{3,2}\right)$, then the price vector faced by firm 1 is: $\left(p_{1,1}, p_{2,1}, p_{3,1}\right) = \left(p_{1,2}, 2p_{2,2}, 2p_{3,2}\right)$, where – by definition - $p_{l,f}$ is the price firm *f* receives (pays) for output (input) of good *l*.}. The subsidy is financed by an equal lump sum tax on both people. **(4 points)**

2. Answer all parts

a) Consider an economy with *L* goods, *H* individuals and *J* firms. The production sets of firms are given by $Y^j \subset \Re^L$, each individual *h* has an endowment vector $\vec{\omega}^h \in \Re_+^L$, and owns a fraction $\theta_j^h,\ j = 1,...,J$ of firm *J*; $\sum_h \theta_j^h = 1$ for all *j*.

i. Define a competitive equilibrium for this economy and **prove** that a competitive equilibrium is *Pareto efficient*. Is it necessary to assume preferences and technology are convex to prove this result? What assumptions **are** used in the proof? **(7 points)**

ii. Would the proof hold if firms had different technology - for example, if firm 1 had a superior technology to firm 2? If you answer no, show what step in the proof fails to be valid. **(3 points)**

iii. Would the proof hold if person 1's utility depends upon the consumption vector of person 2? If you answer no, show what step in the proof fails to be valid. **(3 points)**

b) Consider a three good, one factor (labor) general equilibrium model. Technology for the three goods is given by:
$q_1 \le 2L_1$; $q_2 \le \phi(z) L_2$; $q_3 \le L_3$; where: *z* represents pollution and $\phi = (1 - \mu z)$

The specification indicates that the productivity of firm 2 is affected by pollution. There are H identical people, each endowed with **10** units of labor. Preferences and the resource constraints are:

$$U^h = \ln\left(c_1^h\right) + \ln\left(c_2^h\right) + c_3^h; \quad \sum_h c_i^h \le q_i, \quad i = 1,2,3; \quad \sum_i L_i \le L^T = 10H$$

i. Assume the pollution is caused by production of good 1: $z = q_1$. **Find** the competitive equilibrium and indicate whether it is Pareto efficient. {**Throughout this question assume that μ is small enough so that an interior solution always occurs}.** **(5 points)**

ii. What policy, or policies, could make the competitive equilibrium *Pareto efficient*? Name several policies. **(2 points)**

iii. Suppose the only feasible policy is a **tax or subsidy to sector 2**, the sector which is harmed by the negative externality. **Calculate** this second best tax or subsidy. **(5 points)**

iv. Would your answer to part (iii) change if preferences were $U^h = \left(c_1^h \cdot c_2^h\right)^{1/4} + c_3^h$, rather than the original preferences? If so, how would your answer be affected (an analytic response is not required, rather an intuitive discussion of why preferences matter suffices). **(4 points)**

c) Consider the two good (F,M), two factor (L,K) general equilibrium model studied in class. Let the production technologies, and the dual cost curves, be given by:

$$Q_f = \theta K_f^{1/3} L_f^{2/3},\; Q_m = \theta K_m^{2/3} L_m^{1/3},\; \theta \equiv 3 \cdot 2^{-2/3}; \quad TC_f = Q_f W^{2/3} R^{1/3};\; TC_m = Q_m W^{1/3} R^{2/3}$$

Assume there are H identical consumers, each with factor endowments $\left(\bar{L}, \bar{K}\right)$, and with preferences given by:

$$U = \left(c_f^h\right)^{\alpha} \left(c_m^h\right)^{1-\alpha}; \quad \alpha \in (0,1)$$

i. Derive the general equilibrium supply curves for the economy and show how factor prices are determined, given output prices. **(6 points)**

ii. Given output prices, show how an increase in the endowment of capital affects factor prices and the supply of each good. **(3 points)**

iii. Find the competitive equilibrium price vector, as functions of the endowment vector and preferences (α). **(5 points)**

iv. Suppose wages paid to labor in sector M are taxed, but labor in F is not taxed; as an example, agricultural firms may be exempt from the wage tax. If W is the net wage to workers, firms in sector M pay the wage rate: $W_m = (1+\tau)W$, where τ is the tax rate. The cost function for M producers is, of course: $Q_m W_m^{1/3} R^{2/3}$.
(1)What inefficiencies, if any, does this tax create?
(2)Given output prices, calculate how this wage tax affects factor prices and output levels. Does your answer differ from what you would expect with a partial equilibrium model? **(7 points)**

3. Answer All Parts

a) Consider an exchange economy with *2* goods and *2* people (*A*,*B*). Each person's endowments are $\left(\omega_1^h, \omega_2^h\right), \quad h = A, B;$ where total endowments are $\left(\omega_1^T, \omega_2^T\right) = (10,10)$. Preferences are given by:

$$Person\ A: \quad U^A = \left(c_1^A + 2c_2^A\right); \qquad Person\ B: \quad U^B = \left(2c_1^B + c_2^B\right)$$

i. Are these preferences convex? Will a competitive equilibrium exist for this economy for all endowment vectors? **(3 points)**

ii. **Find** the set of *Pareto efficient* allocations for this economy (using the Edgeworth box should help you). Are these *Pareto efficient* allocations interior (people consume both goods)? Are all Pareto efficient allocations supportable as a competitive equilibrium with transfers? **(5 points)**

iii. Find the competitive equilibrium (if and when it exists) as a function of the endowment vector $\left(\omega_1^A, \omega_2^A\right), \quad \left(\omega_1^B, \omega_2^B\right) = \left(\left[10 - \omega_1^A\right], \left[10 - \omega_2^A\right]\right)$ **(3 points)**

b) Consider the same exchange economy as in part (a) except that preferences are modified to:

$$Person\ A: \quad U^A = \left[\left(c_1^A\right)^2 + \left(2c_2^A\right)^2\right]^{1/2}; \qquad Person\ B: \quad U^B = \left(2c_1^B + c_2^B\right)$$

i. Discuss the importance of convexity assumptions in proving existence of equilibrium and in proving the second welfare theorem. {You do NOT need to prove either theorem}. **(3 points)**

ii. **Find** the set of *Pareto efficient* allocations for this economy and compare to the *Pareto efficient allocations* found in part (a) (using the Edgeworth box should help you). **(5 points)**

iii. **Which** of these Pareto efficient allocations are supportable as a competitive equilibrium with transfers? Be specific and compare to your answer to part (a). **(3 points)**

iv. **Find** the set of initial endowments $\left(\omega_1^A, \omega_2^A\right)$ for which a competitive equilibrium exists and find the equilibrium prices corresponding to these endowments (total endowments remain 10 of each good). **(5 points)**

v. If the initial allocation is $\left(\omega_1^A, \omega_2^A\right) = \left(\omega_1^B, \omega_2^B\right) = (5,5)$, how would a transfer of one unit of good 1 from person *B* to person *A* affect equilibrium prices, and the equilibrium consumption vector and utility level of each person? Be specific. **(2 points)**

c) Consider an economy with *H* people; each person's utility depends upon three things: consumption c^h of a private consumption good, leisure l^h, and consumption of a (non-rival) public good $\left(G^h\right)$:

$$U^h\left(c^h, l^h, G^h\right) = \left(c^h \cdot l^h\right)^{1/2} + \alpha^h \ln\left(G^h\right)$$

Each individual is endowed with **one unit of time,** which can be split between leisure and working; when the person works, she is indifferent as to whether she works producing good C or good G.

Production technology and resource constraints, given the non-rival property for *G*, are given by:

Production technology: $Q_c = 2L_c$; $Q_g = 2L_g$;

Resource Constraints: (1) $\sum_h \left(c^h\right) \le Q_c$; (2) $G^h \le Q_g \quad \forall h$; (3) $\left[L_c + L_g + \sum_h l^h\right] \le H$

Of course, L_c, L_g are total hours worked in sectors c and g respectivity.

i. Set up the central planner's problem for determining a *Pareto efficient* allocation, derive the first order conditions, and interpret the condition for optimal provision of the public good. **(4 points)**

ii. **Solve** for the *Pareto efficient* allocation {consumption, leisure and level of public good} in which each individual derives the same utility from consumption of private goods. **(5 points)**

iii. Since provision of the public good uses real resources, explain how – within the context of a market economy – a benevolent government would acquire the resources required to produce the public good. **(3 points)**

iv. Assume a market economy and assume the only way for the government to finance provision of the public good is to tax wage income, and then to use this revenue to hire the workers required to produce the public good. Let *W* denote the (before tax) wage rate, τ the per cent tax on wage income, so that total government tax revenue would be: $TR = \tau W\left[\sum_h \left(1 - l^h\right)\right]$.

(1)Show how household decisions (consumption, leisure) and government provision of the public good depend on τ; and then
(2)Derive the optimal tax rate and the resulting provision of the public good. **(6 points)**

v. Compare the provision of the public good when financed with taxes (part (iv)) to the *Pareto efficient* solution found in part (ii). Which yields larger provision of the public good? Explain why this difference occurs. **(3 points)**