

Midterm Exam

Answer any **two** questions; answer all parts to each question.

1. Answer All Parts.

- a) Consider an economy with two goods (M, F) and two people (A, B). The aggregate initial endowment vector is: $(e_m, e_f) = (10, 10)$. The production technology for this economy is:

$$y_m + y_f \leq 0$$

where, as usual, y_i is the netput of good i . Finally, preferences for the two people are:

$$U^A = (c_m^A)^{2/3} \cdot (c_f^A)^{1/3}; \quad U^B = (c_m^B)^{1/3} \cdot (c_f^B)^{2/3}$$

where (c_m^i, c_f^i) is the consumption vector of individual i .

- i. Derive the set of Pareto efficient allocations for this economy, and interpret the conditions needed to insure Pareto efficiency. Show how prices and aggregate output change as one moves along the utility possibility frontier, increasing person A's utility. {You should find an analytic solution. However, if you cannot derive it, give an intuitive explanation of how these variables will change as you redistribute utility from B to A}.
 - ii. Given the Social Welfare Function: $W = U^A \cdot U^B$, state the additional condition(s) that must be fulfilled at the social optimum, then find the social optimum.
 - iii. Suppose person A's utility function is modified to: $U^A = (c_m^A)^2 \cdot (c_f^A)$, while B's utility function is unchanged. Will this modify the set of Pareto efficient allocations? The social optimum allocation (that maximizes the social welfare function in (ii)). Explain. {You do **not** need to find the analytic solution for this part}.
- b) Consider a two good (X,Y), two factor (K,L) general equilibrium model where production exhibits constant returns to scale. Production functions (and dual cost curves) are given by:

$$Q_x = (3/2^{2/3}) K_x^{2/3} L_x^{1/3} \rightarrow TC(Q_x, W, R) = Q_x (R^{2/3} W^{1/3});$$

$$Q_y = (3/2^{2/3}) K_y^{1/3} L_y^{2/3} \rightarrow TC(Q_y, W, R) = Q_y (R^{1/3} W^{2/3});$$

where (W, R) are the input prices for labor and capital, respectively. Let Y be the numeraire good, so the three (relative) prices for this economy are $\{W, R, P_x\}$.

- i. Given the total supply of K and L, find the general equilibrium supply curves (*i.e.*, express output supply in terms of output prices and factor endowments). Discuss whether production efficiency holds.

- ii. Suppose agents have identical and homothetic preferences given by: $U = (C_x C_y)$, so that demand for final goods is given by: $D_x = (I/2P_x)$, $D_y = (I/2P_y)$, where I denotes total income. Find the equilibrium output price and input prices. {Hint: set the ratio of supplies to the ratio of demands}. For future reference, denote these equilibrium prices by $\{W^e, R^e, P_x^e\}$.
- iii. Next, suppose there is a technological innovation that increases productivity in sector X . Specifically, the production function (and dual cost curves) for sector X become:

$$Q_x = A(3/2^{2/3})K_x^{2/3}L_x^{1/3} \rightarrow TC(Q_x, W, R) = (Q_x/A)(R^{2/3}W^{1/3}); \quad A > 1$$

Given output prices, and assuming both goods are produced, how will this technological change affect output of each good, and factor prices? {A quantitative answer is required}. Do these results differ from what you would expect in a partial equilibrium model? Explain.

2. Answer all parts and subparts.

- a) Consider the following simplified general equilibrium model of $(L+1)$ goods. Each of H individuals have preferences given by:

$$U^h(m^h, x_1^h, \dots, x_L^h) = m^h + \phi^h(x_1^h, \dots, x_L^h); \quad h = 1, \dots, H$$

Let good M denote the numeraire good. Households are endowed with units of good M , and with fractional ownership in each of the J firms. The cost function for each firm is:

$C^j(y_1^j, \dots, y_L^j)$, where: $\bar{y}^j \geq 0$ denotes the output vector of the firm, and C^j denotes the amount of good M required to produce that output vector. Assume all goods are rival (private) goods, so the usual feasibility conditions hold.

- i. State, and interpret, the (Pareto) efficiency conditions for this economy (*i.e.*, state the mathematical conditions and interpret them economically). What efficiency condition would be violated if “poor” households (those with little endowments or little ownership in firms) received a per unit subsidy for purchase of x_1 ?
- ii. Given the subsidy in part (i), under what conditions can you properly measure the welfare impact of a **tax on good 2** with the standard partial equilibrium welfare measures in that market (those of producer and consumer surplus)? Assuming these conditions are *violated*, discuss when the use of this welfare measure will **understate** the true welfare cost of the tax. *Prove* your answer. {I will give partial, but not full, credit for a descriptive answer}.

- b) Use the same model as in part (a), except assume that good L is “pollution” that affects everyone: $x_L^h = X$ for all h , where $X = \sum_j y_L^j$, denotes pollution. Since pollution is a bad,

$(\partial U^h / \partial x_L^h) < 0$. To further simplify, assume that: $y_L^j = \phi_j(y_1^j)$, $\phi_j' > 0$ - that is, firm j 's pollution emissions are an increasing function of its output of good *one*. **{NOTE: unlike part (ai), there is no subsidy to poor people for purchases of good one}.**

- i. Derive the efficiency conditions for this economy and compare to part (ai). Is the competitive equilibrium Pareto efficient? If not, derive the optimal government policy required to make the competitive equilibrium Pareto efficient.
- ii. Suppose the only feasible policy is to tax or subsidize good *two*. Determine whether the appropriate policy is a tax or subsidy, and discuss whether a Pareto efficient allocation can be achieved with this policy. {For simplicity, assume each person's utility function is separable, so that: $U^h(m^h, x_1^h, \dots, x_L^h) = m^h + \sum_{i=1}^L \{\phi_i^h(x_i^h)\}$; $h = 1, \dots, H$. }

- c) Consider a country with two regions, A and B. In each region there are two people (for a total of four people). All individuals have identical and homothetic preferences given by:

$$U^h = c_1^h c_2^h c_3^h; \quad h = 1, \dots, 4; \quad \text{where } c_i^h \text{ is individual } h\text{'s consumption of good } i$$

Individuals one and two live in region A, while three and four live in B. The individuals differ in their endowment vectors as follows (e_i^h is individual h 's endowment of good i):

$$\text{Region A: } (e_1^1, e_2^1, e_3^1) = (5, 1, 3); \quad (e_1^2, e_2^2, e_3^2) = (4, 5, 0);$$

$$\text{Region B: } (e_1^3, e_2^3, e_3^3) = (2, 1, 6); \quad (e_1^4, e_2^4, e_3^4) = (1, 5, 3);$$

- i. Assume that initially, people within the same region can trade in all goods with each other, but trade between regions A and B is only allowed in goods **one and two**. Find the equilibrium prices (and utility levels) in each region and indicate whether the equilibrium is Pareto efficient.
- ii. Suppose an agreement between regions A and B now allows trade in all three goods. Find the resulting equilibrium prices and indicate whether the equilibrium is Pareto efficient. Does everybody benefit from this agreement? Explain and relate your answer to the relationship between Pareto efficiency and Pareto superior allocations.

3. Answer all parts

- a) **Prove** that a competitive equilibrium is Pareto efficient (the First Welfare Theorem). Show what step(s) in your proof would fail to hold if there is: (i) a monopolist producer of a final product consumed by households; (ii) a production externality that affected other firms.
- b) *Briefly* discuss the importance of the convexity assumption in proving each of the following: (i) the Second Welfare Theorem; (ii) the existence of a competitive equilibrium; and (iii) the relationship between profit maximization and production efficiency. Then consider the following model of two goods (M, F) and two inputs (K, L) in fixed supply. Production technology is given by:

$$Q_m = \left[(L_m)^2 + (K_m)^2 \right]; \quad Q_f = 2L_f + K_f; \quad L_m + L_f \leq \bar{L}; \quad K_m + K_f \leq \bar{K}$$

- i. Derive the production possibility set, and show which outputs are supportable through profit maximization.
- ii. Given preferences of a representative agent: $U = M^\alpha \cdot F^{(1-\alpha)}$, what are the values of α for which a competitive equilibrium exists?
- c) Consider the production side of a general equilibrium model with L goods and F firms. Let Y^f denote the feasible production set for firm f , and $\bar{y}^f \in Y^f$ denote a feasible netput vector for the firm, where: $\bar{y}^f = (y_1^f, y_2^f, \dots, y_L^f)$

- i. Let Y denote the aggregate production set. How is this aggregate production set defined? When can we use the supply curves derived (from profit maximization) from this aggregate production set to represent the sum of individual supply curves? Explain.
- ii. Assume there are two firms and two goods; each firm's production technology is:

$$\text{Firm I: } y_{11} + (10 + \mu y_{12}) y_{21} + \frac{(y_{21})^2}{2} \leq 0; \quad y_{21} \leq 0;$$

$$\text{Firm II: } y_{12} + y_{22} \leq 0; \quad y_{22} \leq 0$$

where y_{ij} is firm j 's netput of good i .

1. Assuming $\mu = 0$ derive the aggregate production set. Will profit maximization lead to aggregate production efficiency?
2. Rederive the aggregate production set when $\mu > 0$. Will profit maximization lead to aggregate production efficiency? If not, for given total inputs of good 2, explain what inefficiency results from the competitive solution.