

Midterm Exam

Answer any **two** questions; answer all parts to each question.

1. Answer all parts

a) **Prove** that a competitive equilibrium is Pareto efficient. Indicate: (i) Where in your proof, if at all, you used the assumption preferences and technology are convex?; (ii) What steps in your proof, if any, would fail to hold if there were a monopoly supplier of a final product?

b) Consider an exchange economy with three people (A, B, C) and three goods (S, M, F). Individuals have identical preferences given by:

$$U^i = m^i \cdot (s^i + f^i)$$

where (s^i, m^i, f^i) is person i 's consumption vector. The individuals' endowment vectors are:

$$(\omega_m^A, \omega_s^A, \omega_f^A) = (8, 0, 0); \quad (\omega_m^B, \omega_s^B, \omega_f^B) = (2, 5, 1); \quad (\omega_m^C, \omega_s^C, \omega_f^C) = (2, 1, 5)$$

- i. Find equilibrium prices, individual consumption vectors and individual utility, **assuming trade in good F is not allowed** (e.g., there may be large transportation costs associated with trade in F). Is this allocation constrained Pareto efficient – that is, given that F cannot be traded, is there any way to make all people better off than in this equilibrium? Explain. {HINT: In solving, watch for a corner solution for one person}.
 - ii. Find the equilibrium assuming there is costless trade in all goods (e.g., a new technology is discovered that allows costless shipping of F). Is this equilibrium Pareto efficient? Is this equilibrium *Pareto superior* to the equilibrium above (*i.e.*, is everybody better off)? If not, would it be possible to redistribute income (endowments) so that everybody is better off in this equilibrium than in the one above? If so, show how.
- c) Use the same model as in part (b), and assume that **all** goods can be traded (or reallocated) among individuals. Also, assume society possesses the following Social Welfare Function:

$$W = \ln(U^A) + \ln(U^B) + \ln(U^C)$$

- i. First, state the conditions for an allocation that maximizes Social Welfare. Is this allocation unique? Is the allocation Pareto efficient?
- ii. Find the consumption allocation that maximizes social welfare.
- iii. Suppose person A 's utility function changed to: $U^A = (m^A)^2 \cdot (s^A + f^A)^2$, while the utility functions of persons B and C were unchanged. Would this change in person A 's utility function change the set of Pareto efficient allocations? Would it change the allocation that maximizes Social Welfare? Explain, and if you answer yes to the last question, find the new allocation that maximizes social welfare.

2. Answer all parts

a) State the second welfare theorem, indicate its importance and give a sketch of how to prove it (a formal proof is not needed; just indicate the crucial steps in the proof). In particular, identify the crucial *assumptions* in this proof, including the role of the convexity assumption.

i. What is the economic significance of the *Second Welfare Theorem*?

b) Next, consider an economy with two goods (y_1, y_2) , two firms and two people. The firm's production technologies are given by:

$$\text{Firm 1: } y_{1,1} + 10y_{2,1} + \left(\frac{y_{2,1}^2}{2}\right) \leq 0; \quad y_{2,1} \leq 0; \quad \text{Firm 2: } y_{1,2} + 4y_{2,2} \leq 0, \quad y_{2,2} \leq 0$$

where $y_{i,j}$ is firm j 's netput of good i . The initial endowment vector for the economy is:

$$(\omega_1^T, \omega_2^T) = (0, 16). \text{ Individuals have identical preferences, which are given by:}$$

$$\text{Person } h: U^h = c_{1,h} + 12c_{2,h} - \left(\frac{c_{2,h}^2}{2}\right) \text{ where } c_{i,h} \text{ is person } h \text{'s consumption of good } i.$$

i. Derive the production (consumption) possibility set for this economy.

ii. Show how each Pareto efficient allocation can be supported as a competitive equilibrium (watch for corner solutions!).

c) Use the same production structure as in (b) above, but assume that, for each firm, production of good 1 produces a by-product ("pollution") that hurts consumers. Specifically:

$$\text{Firm 1: } y_{1,1} + 10y_{2,1} + \left(\frac{y_{2,1}^2}{2}\right) \leq 0; \quad y_{2,1} \leq 0; \quad z_1 = y_{1,1}$$

$$\text{Firm 2: } y_{1,2} + 4y_{2,2} \leq 0, \quad y_{2,2} \leq 0; \quad z_2 = (y_{1,2}/2)$$

where z_i is firm i 's output of pollution. Assume there are N identical people with preferences given by:

$$U^h = c_{1,h} + 12c_{2,h} - \left(\frac{c_{2,h}^2}{2}\right) - \phi z; \quad z = (z_1 + z_2); \quad 1 > N\phi > 0$$

i. Find the symmetric Pareto efficient allocation (i.e., where all consumers receive the same consumption bundle). Will the resulting production allocation be on the production possibility frontier you identified in part (b)? Why or why not?

ii. Will a competitive equilibrium be Pareto efficient? If not, what policy – if any – could be used to make the competitive equilibrium Pareto efficient? Be specific.

3. Answer all parts.

a) Consider a two good (X, Y) , two factor (K, L) general equilibrium model where production functions (and dual cost curves) are given by:

$$Q_x = \left(3/2^{2/3}\right)\theta K_x^{2/3} L_x^{1/3} \rightarrow TC(Q_x, W_x, R_x) = (Q_x/\theta)(R_x)^{2/3} (W_x)^{1/3};$$

$$Q_y = \left(3/2^{2/3}\right)K_y^{1/3} L_y^{2/3} \rightarrow TC(Q_y, W_y, R_y) = Q_y (R_y)^{1/3} (W_y)^{2/3}$$

where (R_i, W_i) is the price paid for capital and labor, respectively, in sector i , and θ is a productivity parameter. Assume factors are freely mobile between sectors and there is no differential taxation, so that $R_x = R_y = R$ and $W_x = W_y = W$. Let (\bar{K}, \bar{L}) denote the aggregate factor endowments. Finally, let (C_x^h, C_y^h) denote the consumption vector of agent h , and assume there are \bar{L} identical agents with the following preferences:

$$U^h = (C_x^h \cdot C_y^h)^{1/2}$$

- i. Find the competitive equilibrium goods prices, factor prices, outputs and utility of each agent as functions of the endowment vector and the technology parameter (θ) .
- ii. Suppose that, with research and development (R&D), productivity in sector X can be increased. Let: $\theta = (1 + K_\theta)^{1/2}$, where K_θ is the amount of capital used in R&D. Given total endowments, (\bar{K}, \bar{L}) , the amount of resources available to **produce goods** is: $(K^n, L^n) = ((\bar{K} - K_\theta), \bar{L})$. Find the socially optimal level of investment in R&D.
- iii. Do you think that a competitive equilibrium would achieve the social optimum you found in part (ii)? Justify your answer.

b) Answer the following questions.

- i. *Answer using the model from part a) above.* Suppose the government imposes a payroll tax (tax on wage income) that is the same in both sectors. The proceeds of the tax are distributed to poor households. What inefficiency, if any, does this tax cause? Would your answer change if individuals derived utility from leisure? Explain.
- ii. Consider a three good world (C, M, F) ; each good is produced using only labor inputs. Output of good F creates pollution (Z) , which reduces productivity in sector M . Thus:

$$Q_c = C(L_c); \quad Q_f = F(L_f), \quad Z = G(L_f); \quad Q_m = M(L_m, Z); \quad (\partial M/\partial Z) < 0$$

Individual's utility depends only on consumption of the three goods (and not on pollution). How does the pollution affect the price and output of good M ? If the only feasible policy is a tax or subsidy on good M , what determines the sign of the optimal policy? Explain your answer.

(Question 3, part c, on next page. If you answer Question 3, be sure to do all parts, including c).

- c) Consider an exchange economy with two goods (F, M) and two people (A,B). Individual preferences are given by:

$$\text{Person A: } U^A = (2f^A)^2 + (m^A)^2; \quad \text{Person B: } U^B = (f^B)^2 + (2m^B)^2$$

where (f^i, m^i) is the consumption vector of individual i . Let (ω_f^i, ω_m^i) denote the endowment vector of individual i , and assume the aggregate endowments of the two goods are equal (i.e., $(\omega_f^A + \omega_f^B) = (\omega_m^A + \omega_m^B)$).

- i. Find (and show graphically) the set of Pareto efficient allocations. Which allocations, if any, can be supported as a competitive equilibrium with transfers?
- ii. What condition(s) on the individual endowment vectors are required to ensure existence of a competitive equilibrium? If a competitive equilibrium exists, will it be Pareto efficient?
- iii. Do you think the addition to the economy of a third individual, with preferences: $U^C = (f^C)^2 + (m^C)^2$, is likely to increase or decrease the probability that a competitive equilibrium exists? Explain your answer (you do **not** have to provide a rigorous answer to this part; just give an intuitive discussion).