1. Explain the role of convexity in proving the existence of a competitive equilibrium, then consider a two person exchange economy with agents who have the following preferences and endowments:

Agent I: \[ U^I = 2x^I + y^I; \quad \left( e^I_x, e^I_y \right) \]

Agent II: \[ U^II = \left( x^II \right)^2 + \left( 2y^II \right)^2; \quad \left( e^II_x, e^II_y \right) \]

where \((x^i, y^i)\) is the consumption vector and \(\left( e^I_x, e^I_y \right)\) is the endowment vector of agent i.

(a) Derive the demand curves for each agent.
(b) Does an equilibrium exist? Relate to the endowment vectors.
(c) Given the aggregate endowment vector, \(\{e^I_x, e^I_y\} = \{10,10\}\), find the set of Pareto efficient allocations.
(d) Which allocations, if any, can be supported as a competitive equilibrium? Explain and relate your answer to the Second Welfare Theorem.

2. How would your answers to each part of problem 1 change if preferences were given by:

Agent I: \[ U^I = 2x^I \]

Agent II: \[ U^II = x^II + 2y^II \]

3. Consider a two person, two good exchange economy where agents have the following preferences:

\[ U^I = \sqrt{x_1} + \sqrt{y_1}; \quad U^II = (1-\alpha)\ln(x_2) + \alpha \ln(y_2); \quad \alpha \in [0,1] \]

where \(\{x_i, y_i\}\) denotes the consumption vector of individual i. The endowment vectors for each agent are given by:

\(\left( \omega^I_x, \omega^I_y \right), \quad \left( \omega^II_x, \omega^II_y \right)\), where: \(\left( \omega^I_x + \omega^II_x \right) > 0, \quad \left( \omega^I_y + \omega^II_y \right) > 0\).

(a) Show that a competitive equilibrium exists provided \(\omega^I_x > 0\) or \(\alpha \omega^II_x > 0\).

(b) For the special case: \(\omega^I_x = 0, \quad \alpha = 0\) show that no equilibrium exists. Explain why this happens (is person I’s demand for good y continuous for all prices \(p_y \geq 0\)?)