Problem Set 4 - Due April 14, 2003

1. Consider the standard two good \((X, Y)\), two factor \((K, L)\) model, where both goods are produced under constant returns to scale and good \(X\) is relatively intensive in factor \(K\). Consumer households are endowed with the two factors, which they sell to firms. Household preferences depend upon consumption of the two goods (plus, in parts where so noted, upon consumption of one of the factors). Use this model to answer the following questions:

   a) What inefficiency would arise if a tax were levied on labor input used in both sectors?
      i. How would your answer change if individual’s utility also depend on leisure (so the quantity of labor supplied to firms was endogenous)? Explain your answer.

   b) What inefficiency occurs if labor employed in sector \(X\) is taxed, but labor employed in sector \(Y\) is not taxed?
      i. Given output prices, how will this tax affect factor prices? How do your predictions differ from those of a partial equilibrium model?

   c) Assuming no other distortion, how would a subsidy on output of good \(X\) affect the equilibrium and efficiency? What efficiency conditions are violated? Explain.
      i. If there were a tax on labor employed in sector \(X\), how would a subsidy on output of good \(X\) affect economic efficiency? How, if at all, does the presence of the labor tax affect your conclusions regarding the efficiency of the output subsidy? Explain.

   d) Suppose good \(Y\) is produced by a monopolist, while good \(X\) is produced under perfect competition. What inefficiency arises from this market structure? Does “underproduction” by the monopolist imply production is inside the production possibility frontier? Explain.
      i. Given the monopoly in sector \(Y\), would a subsidy or tax in sector \(X\) improve efficiency? If so, which should you use and why?

2. Given \(L\) goods and \(J\) firms, each with production set \(Y^j \subset R^L\), let \(\bar{y}^j\) denote the netput vector of firm \(j\) \((y^j_l > 0\) means good \(l\) is an output, while \(y^j_k < 0\) implies good \(k\) is an input of the firm).

   a) Given the technology for each firm (i.e., its production set), how is the aggregate production set for the economy derived? What does it mean for an aggregate netput vector to be an efficient production vector?
      i. Does the fact that each firm chooses a netput vector that is efficient in its own production technology imply the aggregate netput vector is efficient (with respect to the aggregate production set)? If you answer no, provide an example to illustrate your answer.

   b) Prove that competitive profit maximization by all firms, at the same price vector, leads to aggregate production efficiency.
      i. If a tax were levied on the sale of good 1, would this lead to an aggregate production vector that was not efficient (not on the boundary of the aggregate production set)? In answering, distinguish between the case where: (1)good 1 was an output for all firms; and (2)good 1 was an input for some firms and an output for other firms.
3. Consider a model with two goods and three firms. Let \( y^l_j \), \( l \in \{1, 2\} \) and \( j \in \{1, 2, 3\} \) denote the netput of good \( l \) by firm \( j \). Suppose the three firms have the following production technologies:

Firm 1: \( y^1_1 \leq 2\left(1 - y^2_2\right)^{1/2} - 1 \), \( y^1_1 \leq 0 \); Firm 2: \( y^2_1 \leq \left(1 - y^2_2\right)^{1/2} - 1 \), \( y^2_2 \leq 0 \)

Firm 3: \( y^3_2 \leq 2 \left[(1 - y^3_1)^{1/2} - 1\right] \), \( y^3_1 \leq 0 \)

Thus, firms 1 and 2 use good 2 to produce good 1, whereas firm 3 uses good 1 to produce good 2. Note that no firm can produce output without using some input (that is, there is no netput vector with strictly positive components that is an element of any firm’s production set).

a) Define the aggregate production set, \( Y \). Is it possible that a strictly positive vector could be an element of that set? Explain and relate your answer to the magnitude of “\( B \”).

b) For the functions given above, derive the aggregate production set.

c) Show that profit maximization leads to production efficiency and derive the net supply curves for the economy.

i. Would inefficiency occur if there were a tax on all sales of good 2? If so, what inefficiency occurs? Show how this tax affects the derivation of the supply curves (if at all).

d) Show that every efficient production point can be supported through profit maximization.

e) What role did convexity of the individual production sets play in parts (c) and (d)? Explain.