Lapan

Econ 603
Spring 2003

Midterm Exam
Answer any **two** questions; answer all parts to each question.
Each question is worth 50 points.

1. Answer all parts

a) Consider the following two person (*A, B*), two good (*x,y*) exchange economy. Preferences are given by:

$$Person\quad A:\quad U^A = 4\ln\left(c_x^A\right) + c_y^A\,; \qquad \left(e_x^A, e_y^A\right) = \left(1, u\right)$$
$$Person\ B:\qquad U^B = Min\left(c_x^B, c_y^B\right); \qquad \left(e_x^B, e_y^B\right) = \left(5, 2\right)$$

where c_j^i is person *i*'s consumption, and e_j^i is person *i's* endowment, of good *j* (*i=A,B; j=x,y*).

i. Find the competitive equilibrium prices and consumption allocation. *Be sure to indicate when, if ever, a boundary solution occurs.* **(8 points)**

ii. Is the competitive equililbrium Pareto efficient, even when a boundary solution occurs? Explain. **(4 points)**

iii. Does a competitive equilibrium (CE) exist for all u? If you answer no, indicate when a CE does not exist and indicate what assumption, used to prove the existence of a CE, would not hold. (you do not need to construct an existence proof!) **(5 points)**

b) Prove that a competitive equilibrium is Pareto efficient. **(10 points)**

i. What step (if any) in your proof would fail to hold if preferences are non-convex? What step would fail to hold if there is a consumption tax, at the same rate, on all final goods? Explain **(6 points)**

c) Consider a simple exchange economy with two goods (*C, M*) and two people (*A,B*). Aggregate endowments are given by: $\left(\omega_c, \omega_m\right)$ and individual preferences are given by:

$$Person\ A:\quad U^A = \left(c^A\right)^2 \cdot m^A; \qquad Person\ B:\quad U^B = c^A \cdot \left(m^A\right)^2$$

i. Write down the equations needed to determine the set of Pareto efficient allocations (you do not need to solve, but you must have enough equations so that you could solve). **(6 points)**

ii. For a Pareto efficient allocation, as you increase the target utility level for person *B*, what happens to the shadow price of good *m* (the equilibrium price that would be required to support the allocation as a competitive equilibrium)? Explain. **(5 points)**

iii. Would society prefer the aggregate endowment vector $\left(\omega_c, \omega_m\right) = \left(8, 6\right)$ or the endowment vector: $\left(\omega_c, \omega_m\right) = \left(6, 8\right)$? Explain your answer and its significance. **(6 points)**

2. Answer all parts.

a) State the second welfare theorem, indicate its economic significance and give a sketch of how to prove it (a formal proof is not needed; just indicate the crucial steps in the proof). Identify the crucial *assumptions* used in the proof. **(10 points)**

b) Consider the following model with two goods and two firms. Each firm uses good two to produce good one: technologies are as follows:

Firm 1: $y_1^1 \le 4 \cdot \left\{ Max\left[\left(-3 - y_2^1 \right), 0 \right] \right\}^{1/2}$; $y_2^1 \le 0$;

Firm 2: $y_1^2 \le \left(-y_2^2 \right)$; $y_2^2 \le 0$

where y_l^j is firm *j*'s *netput* of good *l*. The endowment vector is: $\left(\omega_1, \omega_2 \right) = \left(0, 15 \right)$

i. Find the production possibility set for this economy. **(13 points)**
ii. Does profit maximization lead to efficient production? Can every efficient production point be supported through profit maximization? Be as specific as possible. **(7 points)**

c) Consider a three-good general equilibrium model with H consumers who have identical endowments, identical ownership of firms and all taxes are rebated equally (thus, individuals have identical income). They also have identical quasi-linear preferences given by:

$U^h = m^h + 3x_1^h + 3x_2^h - (1/2)\left(\left(x_1^h \right)^2 + \left(x_2^h \right)^2 - \left(x_1^h \right) \cdot \left(x_2^h \right) \right)$

There are H identical firms (which behave competitively) with the following cost function:

$C^j = q_1^j + \left(\left(q_2^j \right)^2 / 2 \right)$

where x_i^h is the consumption of good *i* by individual *h*, and q_i^j is production of good *i* by firm *j*. Good *m* is the numeraire, m^h is individual *h's* consumption of this good, and production costs $\left(C^j \right)$ denote the amount of the numeraire required to produce the corresponding output vector. Assume endowments of the numeraire are large enough to guarantee an interior solution in which all goods are consumed. Finally, suppose there is a consumption tax, t_1, on good 1. Given this consumption tax, the government is considering imposing a tax (or subsidy) t_2 on consumption of good 2.

i. Show how a tax/subsidy on good 2 affects the equilibrium price and quantity produced (consumed) of good 2. **(7 points)**

ii. Can you accurately measure the deadweight loss from the tax/subsidy on good 2 by using the supply and demand curves for good 2 (as is usually done in partial equilibrium analysis)? Discuss both the case when $t_1 = 0$ and when $t_1 > 0$. **(4 points)**

iii. Assume $t_1 = 2$. Find the optimal (second-best) tax/subsidy on good 2. **(9 points)**

3. Answer all parts.

a) Consider an exchange economy with two goods (F, M) and two people (A,B). Individual preferences are given by:

Person A: $U^A = \left(2f^A\right)^2 + \left(m^A\right)^2$; **Person B**: $U^B = \left(f^b + m^b\right)$

where $\left(f^i, m^i\right)$ is the consumption vector of individual i. Let $\left(\omega_f^i, \omega_m^i\right)$ denote the endowment vector of individual i, and assume the aggregate endowments of the two goods are equal $\left(i.e., \ \left(\omega_f^A + \omega_f^B\right) = \left(\omega_m^A + \omega_m^B\right)\right)$.

i. Find (and show graphically) the set of Pareto efficient allocations. Which allocations, if any, can be supported as a competitive equilibrium with transfers? **(10 points)**

ii. For what values of the endowment vectors will a competitive equilibrium exist? If a competitive equilibrium exists, will it be Pareto efficient? **(8 points)**

b) Consider an economy with two goods (F, C) and one primary factor, labor (L), which is in fixed supply. There are H households, each of whom is endowed with **2** units of labor. The households own equal shares in all firms (and thus have equal income), and have identical preferences given by:

$$U^h = c^h + 2f^h - \left(\left(f^h\right)^2 / 2\right) - \alpha Z^2$$

where $\left(f^h, c^h\right)$ is the household's consumption vector, and Z measures air pollution. Each good is produced under constant returns to scale, using only labor inputs. However, good F can be produced in two separate ways: one method, which has higher labor productivity, leads to pollution; while the second method has lower labor productivity, but does not result in pollution. Thus, technology for firms are given by:

Good C: $q_m^j \le L_m^j$

Good F: $q_f^j \le 2L_{f1}^j + L_{f2}^j; \quad z^j = L_{f1}^j$

Total pollution for the economy is: $Z = \sum_j z^j$;

And the labor constraint is: $\sum_j \left\{L_m^j + L_{f1}^j + L_{f2}^j\right\} \le 2H$

i. Find the symmetric Pareto efficient allocation for this economy (where everybody receives the same utility). As the economy grows in size (H increases), how does this equilibrium change? Explain. **(10 points)**

ii. Assuming no government policy, find the competitive equilibrium (CE) prices (for labor, good F and good C) and the competitive equilibrium allocation. Is it Pareto efficient (PE)? If you answer no, explain why the competitive equilibrium is not Pareto efficient. **(8 points)**

iii. What policy can the government implement to support the (symmetric) Pareto efficient allocation as a competitive equilibrium? Be specific, calculate the optimal value of this policy and show how it changes as the size of the economy grows (H increases). Explain this property of the solution. **(6 points)**

iv. Suppose the only policy available to the government is to tax output of good F (this tax **cannot** depend on how good F is produced). Find the optimal level of this tax (again, assume a symmetric allocation), and compare the level of this policy instrument to your answer from part (iii). With this tax, is the resulting allocation Pareto efficient? Explain. **(8 points)**