1. Consider a four-good economy in which every consumer has preferences given by:

\[ U(\bar{x}_i) = x_{0i} + \alpha (x_{1i} + x_{2i}) - (1/2) [ (x_{1i})^2 + (x_{2i})^2] + \lambda x_{1i} \cdot x_{2i} + \beta x_{3i} - (1/2)(x_{3i})^2; \ |\lambda| < 1; \ i = 1,...,I \]

where \( x_{\ell i} \) denotes consumption of good \( \ell \ (\ell = 0,1,2,3) \) by consumer \( i \) (there are \( I \) identical consumers). Assume there are \( J \) firms, each of which produces goods 1, 2 and 3, using good 0 as an input. The cost function for each firm \( j \), is given by:

\[ C_j(q_{1j},q_{2j},q_{3j}) = q_{1j} + (1/2)(q_{2j}^2 + q_{3j}^2); \ j = 1,...,J \]

where \( q_{\ell j} \) denotes output of good \( \ell \ (\ell = 1,2,3) \) by firm \( j \), and \( C_j \) denotes the input of good 0 required to produce this output vector. Let good 0 be the numeraire (\( P_0 \equiv 1 \)).

For simplicity, assume that \( J=I \). Each consumer has the endowment vector: \( (e_0,0,0,0) \) (i.e., is endowed with \( e_0 \) units of good one, and nothing of the other goods), and each consumer has the same fractional ownership in each of the firms (and thus receives their profits as part of income).

Let \( (P_{1f}^c, P_{2f}^c, P_{3f}^c) \) denote the price vector firms receive for their output, and \( (P_{1c}^t, P_{2c}^t, P_{3c}^t) \) denote the price paid by consumers (\( P_{\ell}^c = P_{\ell}^f (1+t_{\ell}) \), where \( t_{\ell} \) is the % tax on good \( \ell \). \}

\{for all parts, assume an interior solution in which all goods are consumed\}.

a) Derive the indirect utility function (and expenditure function) for consumers and the (maximum) profit function for firms.
b) Derive the market supply curve and the market demand curve for each good.
c) Given taxes, calculate the market equilibrium prices, output levels and the welfare of each consumer (all profits and tax revenue are redistributed equally to consumers).
d) Find the Pareto efficient allocation and compare to the market equilibrium (part b). Under what conditions is the market equilibrium efficient?
e) Suppose the tax on good 2 increases. Show graphically how, in a partial equilibrium setting, you would measure the deadweight loss from this tax increase.
f) Will this area (part e) measure the true change in welfare for consumers (i.e., the amount of money needed to compensate them for the tax increase)? If not, will it overestimate or underestimate the true impact on consumers? In answering, consider the cases where:
   (i) \( t_1 = 0; \) (ii) \( t_1 > 0; \) (iii) \( t_1 < 0 \)  Relate your answer to the sign of \( \lambda \).
g) Assume, for political reasons, that \( t_1 > 0 \). Given that \( t_1 \) cannot change, how will a tax (or subsidy) on good 3 affect economic efficiency? If your answers to (f) and (g) differ, explain why you get different answers.