Problem Set 2 - Due Wednesday, January 28, 2004

1. Consider an exchange economy with two goods and two agents. The endowment vector of each consumer is: \( e_1 = (e_{11}, e_{21}) \) and \( e_2 = (e_{12}, e_{22}) \). Further, suppose preferences are quasi-linear, and are given by: \( U^i = c_{ii} + h_i (c_{2i}) \), \( i = 1, 2 \); \( h_i' > 0 > h_i'' \) where \( c_{ji} \) is person \( i \)'s consumption of good \( j \).

   a) Find the set of Pareto efficient allocations. At an interior solution (both people consume both goods), how does consumption of each good, and the marginal rate of substitution, vary as we move along the efficiency locus? Does your answer change for Pareto efficient allocations that are not interior? Explain.

   b) Draw the utility possibility frontier for this problem. On the diagram indicate the utility allocations that correspond to interior solutions and those that correspond to corner solutions.

   c) Find the competitive equilibrium for this economy, and show how changes in the endowment vector (obtained by transferring goods from person 1 to person 2) affect equilibrium prices and utility. Relate your answer to part (a) above (and discuss the role of interior versus corner solutions).

   d) Let the endowment of individual one increase \( \hat{e}_1 > e_1 \), while that of person two is unchanged. Show how this affects competitive equilibrium prices and the utility of each person. Can person one be worse off as a result of this change? If so, why can this happen? (Relate your answer to whether \( e_{11} \) or \( e_{21} \) increases).

   e) For this part, consider a transfer of endowments from person two to person one so that total endowments are unchanged: \( \hat{e}_2 = e_1 + z; \hat{e}_2 = e_2 - z; \) \( z \geq 0 \) \( (z \in \mathbb{R}_+^2) \). Given quasi-linear preferences, how does this transfer affect equilibrium prices? Can person one be worse off due to this transfer?

   f) Finally, consider the transfer as in part (e), but drop the assumption of quasi-linear preferences. In a competitive equilibrium, could the transfer lower the equilibrium utility of person one? Prove your answer.

2. Consider an exchange economy with two agents who have the following preferences:

   \( U^i = \alpha_i \ln (x_i) + \beta_i \ln (y_i); \) \( i = 1, 2 \); \( (\alpha_i + \beta_i) = 1 \)

   Each agent has endowment vector: \( (e_{1i}, e_{2i}), i = 1, 2 \) with aggregate endowment \( \{e^x_i, e^y_i\} = \{(e_{1i}^x + e_{2i}^x), (e_{1i}^y + e_{2i}^y)\} \)

   a) Given endowments, find the competitive equilibrium for the economy.
i. Given total endowments, show how a transfer of endowments from individual 1 to individual 2 affects equilibrium prices when: $\alpha_1 = \alpha_2$ and when $\alpha_1 \neq \alpha_2$. Does total demand depend upon the distribution of income? Explain and compare your results to the case of quasi-linear preferences from question 1.

(b) Find the set of Pareto Efficient allocations and derive the utility possibility frontier.

i. How does a change in the utility allocation change the corresponding consumption allocation?; the associated MRS? Explain and relate your answer to the cases in which $\alpha_1 \neq \alpha_2$ and $\alpha_1 = \alpha_2$.

(c) Illustrate the Second Welfare Theorem by showing how each Pareto Efficient Allocation can be supported as a competitive equilibrium.

3. Explain the role of convexity in proving the existence of a competitive equilibrium, then consider a two person exchange economy with agents who have the following preferences and endowments:

Agent I: $U^I = x^I + 2y^I$; $\left( e_x^I, e_y^I \right)$

Agent II: $U^II = (2x^{II})^2 + (y^{II})^2$; $\left( e_x^{II}, e_y^{II} \right)$

where $\left( x^I, y^I \right)$ is the consumption vector and $\left( e_x^I, e_y^I \right)$ is the endowment vector of agent $i$.

(a) Derive the demand curves for each agent.

(b) Does an equilibrium exist? Relate to the endowment vectors.

(c) Given the aggregate endowment vector, $\left\{ e_x^T, e_y^T \right\} = \{10,10\}$, find the set of Pareto efficient allocations.

(d) Which allocations, if any, can be supported as a competitive equilibrium? Explain and relate your answer to the Second Welfare Theorem.