Problem Set 3 – Due Friday, Feb. 6, 2004
(turn in your problem set at the beginning of the discussion section)

1. Consider a two person, two good exchange economy in which agents have the following preferences:

\[ U^i = (x_i)^\delta + (y_i)^\delta, \quad \delta > 0; \quad U^{ii} = \beta \ln(x_2) + (1-\beta)\ln(y_2); \quad \beta \in [0,1) \]

where \( \{x, y\} \) denotes the consumption vector of individual \( i \). The endowment vectors for each agent are given by:

\[ (\omega_i^i, \omega_i^{ii}), \quad (\omega_x^i, \omega_y^i), \text{ where: } (\omega_i^i + \omega_x^i) > 0, \quad (\omega_i^{ii} + \omega_y^i) > 0. \]

(a) Assume \( \delta < 1 \). Show that a competitive equilibrium exists provided \( \omega_x^i > 0 \) or \( \beta \omega_y^i > 0 \).

(b) Assume \( \delta > 1 \). Show under what conditions a competitive equilibrium exists.

(c) Assume \( \delta < 1 \). For the special case: \( \omega_x^i = 0, \beta = 0 \) show that no equilibrium exists.

Explain why this happens (is person I’s demand for good \( x \) continuous for all prices \( p_x \geq 0 \)?)

2. Consider a Robinson Crusoe economy. A single input (\( L \), labor) is used to produce a single output (\( q \), food) according to the following technology:

\[ q = AL + (B/2)L^2; \quad A > 0 \]

The individual is endowed with zero units of food, and one unit of time, which can be divided between labor (\( L \)) and leisure (\( R \)). The person’s preferences are given by:

\[ U = \phi \ln (R) + q \quad (L + R) \leq 1; \quad L \geq 0, R \geq 0 \]

a) Derive the production possibility frontier (\( ppf \)) for this economy (in \( q-R \) space). Relate the curvature of the \( ppf \) to the value of \( B \).

b) Given preferences, find the optimal production, consumption point and depict it graphically. Can you have a corner solution (with \( R=0 \); with \( R=1 \)?)

c) Let \( W \) denote the wage rate, and \( P \) the price of output. Assuming that Robinson Crusoe acts as a competitive profit maximizer in deciding how much output to produce (labor to hire) and a utility maximizer in deciding his consumption bundle, derive the relevant supply and demand curves for this economy. Do there exist prices such that markets clear? (Relate your answer to the magnitude of \( B \)). Remember that, in deriving demands, total income includes not only the value of the labor endowment but the profits earned by the firm which the person owns.
i. If, in part b you find a corner solution is optimal, can it be supported as a competitive equilibrium? Explain carefully.

d) If a competitive equilibrium exists, is it Pareto efficient?

3. Consider a two good, two factor model. Technology is given by:

\[ q_1 \leq \lambda (z_{11})^{\phi_1} (z_{21})^{\phi_1(1-\beta)} ; \quad q_2 \leq \theta (z_{12})^{\phi_2} (z_{22})^{\phi_2(1-\delta)} \]

\[ \phi > 0; \quad \beta \in (0,1), \quad \delta \in (0,1) \]

where \( z_{ij} \) is input \( i \) used in good \( j \), and \( \{\lambda, \theta\} \) denote Hicks-neutral productivity parameters. Assume people have identical and homothetic preferences given by:

\[ U^h(c_{1h}, c_{2h}) = \alpha_1 \ln(c_{1h}) + \alpha_2 \ln(c_{2h}) \]

where \( \{c_{1h}, c_{2h}\} \) is the consumption vector of individual \( h \).

a) Derive the dual cost curves for each good; what restrictions on \( \phi \) are required to guarantee competitive behavior is feasible?

b) Assuming \( \beta = \delta \), derive the production possibility frontier. What restriction on \( \phi \) guarantees the production possibility set is convex (the frontier concave)? What does (strict) convexity of the set imply in terms of the marginal rate of transformation? How does your answer concerning the curvature of the production possibility frontier change if \( \beta \neq \delta \)?

For the remainder of this question, assume \( \phi = 1, \quad \beta = (2/3), \quad \delta = (1/3) \)

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<td>c)</td>
<td>Assuming competitive behavior, use your results from part (a) to solve for factor prices in terms of output prices and the technology parameter.</td>
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<td>d)</td>
<td>Given total resource endowments, find the general equilibrium supply curves (in implicit form).</td>
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<td>e)</td>
<td>Given output prices, show how changes in the endowment of input one affects factor prices and the general equilibrium supply curves.</td>
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<td>f)</td>
<td>Repeat e) for an increase in productivity in sector one.</td>
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<td>g)</td>
<td>Given preferences as above (and hence the corresponding demands), find the equilibrium price vector, and resource allocation, for this economy and show how a productivity increase in sector 1 affects this equilibrium.</td>
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<td>h)</td>
<td>Suppose there is a tax on input two used in either sector. What inefficiency, if any, is created by this policy? Will production occur on the production possibility frontier?</td>
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<td>i)</td>
<td>Modify (h) to assume that input two is taxed only when used in sector two. What inefficiency, if any, is created by this policy? Will production occur on the production possibility frontier?</td>
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