

Problem Set No. 5

Out: Wednesday, April 20, 2005

Due: Wednesday, April 27, 2005

1. Consider the following three-player game:

	<i>L</i>	<i>R</i>	
<i>T</i>	0, 0, 3	0, 0, 0	
<i>B</i>	1, 0, 0	0, 0, 0	
	<i>A</i>		

	<i>L</i>	<i>R</i>	
<i>T</i>	2, 2, 2	0, 0, 0	
<i>B</i>	0, 0, 0	2, 2, 2	
	<i>B</i>		

	<i>L</i>	<i>R</i>	
<i>T</i>	0, 0, 0	0, 0, 0	
<i>B</i>	0, 1, 0	0, 0, 3	
	<i>C</i>		

- (a) Show that the pure strategy equilibrium payoffs are $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 0)$.
- (b) Show that there is a correlated equilibrium in which player 3 chooses B and players 1 and 2 play (T, L) and (B, R) with equal probabilities.
2. Consider the following game (Chicken):

	<i>Stop</i>	<i>Continue</i>	
<i>Stop</i>	1, 1	0, 2	
<i>Continue</i>	2, 0	-1, -1	

- (a) What are the Nash equilibrium payoffs?
- (b) What is the set of all its correlated equilibrium distributions?
- (c) Use a computer program that solves linear programming problems to find the correlated equilibrium that maximizes the sum of the players' payoffs (don't even try to find it by paper and pencil).
- (d) Use a computer program that solves linear programming problems to find the correlated equilibrium that minimizes the sum of the players' payoffs.

3. Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; he assigns probability α to person 2 being strong. Person 2 is fully informed. Each person can either fight or yield. Each person obtains a payoff of 0 if he yields (regardless of the other person's action) and a payoff of 1 if he does not yield and his opponent does yield. If both people fight then their payoffs are $(-1,1)$ if person 2 is strong and $(1,-1)$ if person 2 is weak. Formulate this situation as a Bayesian game and find its Nash equilibria if $\alpha < \frac{1}{2}$ and if $\alpha > \frac{1}{2}$.
4. Each of two individuals receives a ticket on which there is an integer from 1 to 5 indicating the size of a prize he may receive. The individuals' tickets are assigned randomly and independently; the probability of an individual receiving each of the possible numbers is positive. Each individual is asked independently and simultaneously whether he wants to exchange his prize for the other individual's prize. If both individuals agree then the prizes are exchanged; otherwise, each individual receives his own prize. Each individual's objective is to maximize his expected monetary payoff. Model this situation as a Bayesian game and show that in any Nash equilibrium the highest prize that either individual is willing to exchange is the smallest possible prize.