1 The Cournot Model

A single good is produced by 2 firms. The cost of producing q units is $C_i(q) = cq$ for $i = 1, 2$. If the total output of the firms is Q then the market price is

$$\begin{aligned} P(Q) &= \max\{0, \alpha - Q\} \\ &= \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha \end{cases} \end{aligned}$$

where $0 \leq c < \alpha$.

Formally, the game is defined as follows: $\langle N, (A_1, A_2), (\pi_1, \pi_2) \rangle$ where

1. $N = \{1, 2\}$
2. $A_i = [0, \infty)$, for $i = 1, 2$
3. $\pi_i(q_1, q_2) = q_i P(q_1 + q_2) - cq_i$, for $i = 1, 2$.

We are interested in finding the Nash equilibria of this game. For this we first calculate the firms' best response correspondence. Consider firm 1. Its profit function can be written as

$$\pi_1(q_1, q_2) = \begin{cases} q_1(\alpha - c - q_2 - q_1) & \text{if } q_1 \leq \alpha - q_2 \\ -cq_1 & \text{if } q_1 > \alpha - q_2. \end{cases}$$

There are two cases to consider.

$q_2 \geq \alpha - c$: In this case it is easy to see that $\pi(q_1, q_2) < 0$ for all $q_1 > 0$ and $\pi(0, q_2) = 0$. Therefore the unique best response is $\mathcal{B}(p_2) = 0$.

$q_2 < \alpha - c$: In this case, it it easy to see that if the firm chooses $q_1 \in (0, \alpha - c - q_2)$ profits will be positive while if the firm chooses $q_1 \geq \alpha - q_2 - c$ profits will be non-positive. Consequently, the best response must be a quantity in the interval $(0, \alpha - c - q_2)$. Since in this interval the profit function is an inverted parabola, it is easy to see that the profit maximizing quantity is $q_1 = \frac{\alpha - q_2 - c}{2}$.

We can conclude then that firm 1's best response correspondence is given by

$$\mathcal{B}_1(q_2) = \begin{cases} 0 & \text{if } q_2 \geq \alpha - c \\ \frac{\alpha - q_2 - c}{2} & \text{if } q_2 < \alpha - c. \end{cases}$$

Similarly, firm 2's best response correspondence is given by

$$\mathcal{B}_2(q_1) = \begin{cases} 0 & \text{if } q_1 \geq \alpha - c \\ \frac{\alpha - q_1 - c}{2} & \text{if } q_1 < \alpha - c. \end{cases}$$

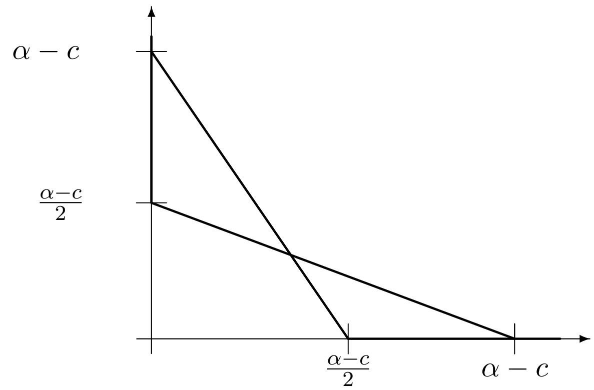

Figure 1: The best response correspondences.

A Nash equilibrium is a pair of qantities q_1^*, q_2^* such that $q_1 \in \mathcal{B}_1(q_2)$ and $q_2 \in \mathcal{B}_2(q_1)$. It is easy to see that that the only Nash equilibrium in the Cournot game is $q_1^* = q_2^* = \frac{\alpha-c}{3}$.