Econ 603 Part II Homework #3

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Iowa State University

1. Consider the following variation of the Battle of Sexes game:

		2 A	B
1	X	5,1	0,0
	Y	4,4	1,5

Table 1: Variation of BoS

Find all possible correlated equilibria for this game. In particular, restrict yourself to the symmetric equilibria, with the probabilites of outcomes given in the following table:

		$q_2(\cdot)$ A	B
$q_1(\cdot)$	X	$(1-\gamma-\rho)/2$	ρ
	Y	γ	$(1-\gamma-\rho)/2$

In other words, identify values of γ and ρ so that the outcome probabilities in the above table can be supported as an outcome of a correlated equilibrium.

2. Consider the following two payoff tables: the first one depicts a Prisoners Dilemma situation while the second one modifies the payoffs.

		2 Work	Laze
1	Work	5,5	-3,6
	Laze	6,-3	-1,-1

Table 2: Prisoner's Dilemma I

Using these payoffs, consider the static Bayesian game: players 1 and 2 each chooses "Work" or "Laze" simultaneously. There is a 1/2 probability that the payoffs are as in PD I and a 1/2 probability that the payoffs are as in PD II.

(1) Suppose both players observe which payoff table it is before playing the Bayesian game. Solve for all of the Bayesian Nash equilibria in this game (including mixed strategies).

(2) Suppose player 1 observes which payoff table it is before playing the Bayesian game but player 2 does not. Solve for all of the Bayesian Nash equilibria in this game (including mixed strategies).

3. Consider an auction with two bidders. Bidder i's valuation of the object is given by

$$v_i = s_i + \frac{1}{2}s_j,$$

		2	
		Work	Laze
1	Work	5,5	-3,2
	Laze	2,-3	-1,-1

Table 3: Prisoner's Dilemma II

for $i, j = 1, 2$. Bidder i (respectively j) knows the value of s_i (respectively s_j), but not the value of s_j (respectively s_i). However, it is common knowledge that s_i, and s_j are i.i.d. uniform on $[0, 1]$. The auction format is first price sealed bid auction.
(1) Write out the Bayesian game for the auction setting.
(2) Solve for the symmetric, increasing, and differentiable Bayesian equilibrium of the game.

4. Consider consumers' demand for a product of possibly two quality levels, $q_L = 0$ and $q_H = 1$. There is a continuum of consumers with different preferences for quality: each consumer consumes *one* unit of the product, with a willingness to pay given by $1 + \theta q$, where θ is uniformly distributed on $[0, 1]$. Thus, the net payoff of a consumer with θ buying the product of quality q' at price p' is $1 + \theta q' - p'$.
There are two kinds of firms, low-quality firms producing only the low-quality product, and high-quality firms producing only the high-quality product. The marginal cost of production is 1, and is the same for both types of firms.

(a). Suppose there are only two firms in the market, and they engage in Bertrand competition (i.e., compete in prices). The quality of the firms is common knowledge among the firms and the consumers.
(i) Show that $p_L = 1$ is a weakly dominant strategy for a low-quality firm.
(ii) Suppose one firm is a low-quality firm and the other is a high-quality firm. Given $p_L = 1$, find p_H in the unique Nash equilibrium, and calculate the profit of the high-quality firm. Denote this profit as π_H.
(iii) Suppose both firms are of high-quality. Find the unique (pure strategy) Nash equilibrium, and calculate the profit of each firm.

(b). Suppose again there are two firms, but the quality of each firm is its private information. Others only know that each firm is of high quality with probability 1/2, and the two firms' types are independent. Each firm can claim to be of either quality level (e.g., through labeling its product). Based on their claims, they will compete in the same way as specified in Part (a). For example, if two firms claim to be of high quality, they will engage in competition as in (a)(iii), regardless of whether one or both firms lied or not. Assume that the consumers blindly believe the firms' claims. If a low-quality firm lies and claims to be of high quality, it may be detected: if detected, the firm has to pay a hefty fine of $F \geq 1/8$. (The firm still gets to keep its profit earned from lying.) The detection technology is as follows: if both firms lie, they will be detected with probability one. If only one firm lies, it will be detected with probability η.
In this game, we study whether the firms lie or not (e.g., the labeling choices of the firms).
(i) Formulate the situation as a Bayesian game. Write out the game elements including the types, strategies, beliefs, and the payoff structure. (Hint: use results in Part (a) for the payoff structure.)
(ii) Show that in a Bayesian Nash equilibrium, no high-quality firm has an incentive to claim to be of low quality.

(iii) Find the critical detection probability $\bar{\eta}$ such that for $\eta \geq \bar{\eta}$, no firm lies in the symmetric Bayesian Nash equilibrium. (That is, low-quality firms will claim to be of low quality.)
(iv) If $\eta < \bar{\eta}$, show that in any *symmetric* Bayesian Nash equilibrium, a low-quality firm lies with a positive probability. Find the mixed strategy of the low-quality firms in the symmetric Bayesian equilibrium. *Hint: in a Bayesian equilibrium, it can never happen that both low type firms lie with probability one.*

5. (War of attrition under incomplete information) Consider a variant of the war of attrition game (homework 1) but with incomplete information. There are two players, $i = 1, 2$, with each player i choosing a number a_i in $[0, \infty)$ (e.g., how long to stay in a market). The choices are made simultaneously. The payoff functions are given by

$$u_i(a_i, a_j, \theta_i) = \begin{cases} -a_i & \text{if } a_j \geq a_i \\ \theta_i - a_j & \text{if } a_j < a_i \end{cases},$$

where θ_i is player i's type, his private information. It is common knowledge that θ_i and θ_j are i.i.d. according to $F(\cdot)$ on $[0, \infty)$. Find the symmetric, strictly increasing and differentiable (pure strategy) Bayesian Nash equilibrium of this game.