

Econ 603 Part II Homework #4  
Spring 2008  
Iowa State University

1. Consider two firms simultaneously deciding whether to build a polluting factory (e.g., each firm builds a paper mill along the same river). If one firm builds its factory, the government will not regulate the firm (since the resulting pollution is not very serious). However, if the two firms both build factories, there is a 50% chance that the government will impose pollution regulation on the factories (since there are now two factories and the pollution becomes too serious). We use a state variable  $S$  to denote the possibilities:  $S = R$  if the government regulates, and  $S = N$  otherwise. If only one factory is built, its net return is 100. If two factories are built and the government does not impose regulation, the net return of each factory is 63. If two factories are built and the government imposes regulation, the net return of each factory is  $-c$  (i.e.  $c$  is the net loss). If a firm does not build a factory, its net return is zero.

Before the decisions on factories are made, firm  $i$  receives a signal  $g_i$  about the likelihood of regulation in case both firms build,  $i = 1, 2$ . The signals  $g_1$  and  $g_2$  are conditionally independent (conditional on the true state  $R$  or  $S$ , the distributions of  $g_i$ ,  $i = 1, 2$  are independent). For  $i = 1, 2$ , signal  $g_i$  can take two values,  $g_i^R$  and  $g_i^N$ , and its probability distribution conditional on the true state  $S$  is given by

$$\Pr(g_i^R|R) = \Pr(g_i^N|N) = p_i$$

where  $p_1 = 0.7$  and  $p_2 = 0.9$ . Thus, the signals of firm 2 are more informative than those of firm 1. (Think of the case when each firm has its own connections to the government, and thus both firms *independently* investigate the likelihood the government will regulate. Firm 2 is better connected to the government. For simplicity, assume away any investigation costs.)

Each firm's observed signal is its private information, but the distributions of the signals in the above equation are common knowledge. After the firms observe their private signals, they simultaneously choose whether or not to build the factories.

- (1) Formulate the above situation as a Bayesian game.
- (2) Suppose  $c = 20$ . Find the pure strategy BNE of the game. Calculate the *ex ante* expected payoffs of both firms. Which one is higher? Why?
- (3) Suppose  $c = 50$ . Find the pure strategy BNE of the game. For each PSBNE, calculate the *ex ante* expected payoffs of both firms.

2. There are a buyer and a seller of a certain good, the values of both are iid uniform on  $[0, 1]$ . The values (denoted by  $\theta_i$ ) are private information, but the distribution is common knowledge. Now consider the following *double auction* setting: the seller (player 1) and buyer (player 2) each submits a sealed bid,  $b_i \geq 0$ . If  $b_1 \geq b_2$ , nothing happens. If  $b_2 > b_1$ , the seller sells the good to the buyer at price  $(b_1 + b_2)/2$ .

Solve for a (pure strategy) BNE of this game in which each player  $i$ 's strategy takes the form  $b_i(\theta_i) = \alpha_i + \beta_i\theta_i$ . Is the BNE *ex post* efficient? (Efficiency is defined as the following: the good should be sold if  $\theta_2 > \theta_1$ , i.e., if the buyer has a higher valuation of the good than the seller.)