

Econ 603 Part II Homework #5
Spring 2008
Iowa State University

1. MWG, 9.B.6; 9.B.14.

2. (Holdup game) Two players play the game of splitting a pie. First, player 2 takes an action that affects the size c of the pie. He may exert little effort, resulting in a small pie of size c_L , or great effort, resulting in a large pie of size c_H . The costs of the little and great effort levels to him are L and H respectively, with $L < H$.

After the size of the pie is determined, player 1 makes a proposal to divide up the pie between 1 and 2. If 2 accepts the proposal, the pie is divided up accordingly, but if 2 rejects the proposal, neither of them receives the pie and the game ends. (This part is thus an ultimatum game.)

Regardless of how the pie is divided, the effort of 2 in the first stage is spent and cannot be recovered.

(1) Formulate the holdup game precisely, i.e., write out the elements of the extensive game. Solve for the SPNE of the game.

(2) Suppose now that if 2 rejects the proposal, he can make another proposal to divide up the pie. If 1 accepts, the pie is divided up accordingly, and if 1 rejects, neither of them receives anything and the game ends. Further, it takes one period to make and reject a proposal, and players have a discount factor of δ . Find the SPNE of the game.

3. Armies 1 and 2 are fighting over an island initially held by a battalion of army 2. Army 1 has K battalions and 2 has L . Whenever the island is occupied by one army, the opposing army can launch an attack. The outcome of the attack is that the occupying battalion and one of the attacking battalions are destroyed; the attacking army wins and, so long as it has battalions left, occupies the island with one battalion. The commander of each army is interested in maximizing the number of surviving battalions (say, with each battalion worth 1 util) but also regards the occupation of the island as worth more than one battalion but less than two. (If after an attack neither army has any battalion left, then the payoff of each commander is 0.)

Analyze this situation as an extensive game, and find the SPNE *outcome* as a function of K and L .

4. (Three way duel) Three gangsters armed with pistols, Al, Bob and Curly, are in a room with a suitcase of money. Al is the least accurate, with a 20% chance of killing his target. Bob has a 40% probability. Curly is slow but sure; he kills his target with 70% probability. For each, the value of his own life outweighs the value of any amount of money. Survivors split the money.

Suppose that each gangster has one bullet and the order of shooting is Al, Bob and Curly. Assume also that each gangster must try to kill another gangster when his turn comes. What is an equilibrium strategy combination and what is the probability that each of them dies in your equilibrium?