
2 (War of Attrition) Two players fighting over an object. The values of the object to them are respectively $v_1$ and $v_2$. Each player chooses a time at which he intends to give up. When $i$ gives up first, $j$ gets the object (and the time at which $j$ intended to give up becomes irrelevant). If both give up at the same time, each gets half of the object. Fighting is costly: each prefers a short fight. That is, letting a player’s action $t_i$ be the time he intends to give up, the payoffs are

$$u_i(t_i, t_j) = \begin{cases} 
-t_i & \text{if } t_i < t_j \\
0.5v_i - t_i & \text{if } t_i = t_j \\
v_i - t_j & \text{if } t_i > t_j 
\end{cases}$$

for $i, j = 1, 2$.

(1) Find the pure strategy Nash equilibria of the above strategic game.

(2) Suppose now that each player attaches no value to the time spent waiting for the other player to concede, but the object loses value as time passes (e.g., a melting ice cream cone). In particular, assume that the value of the object to player $i$ after $t$ units of time is $v_i - t$. Further, $i$ gets the object at time $t_j$ if $t_j < t_i$: if $j$ gives up, $i$ gets the object immediately. (Of course, if $t_i = t_j$, each gets half of the object.) Specify the game and find the pure strategy NE of this game.