Let $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ be a strategic game. Let the following be exogenously given:

- A finite probability space $(\Omega, \pi)$
- For each player $i \in N$ a partition $P_i$ of $\Omega$
- For each player $i \in N$ a function $\sigma_i : \Omega \rightarrow A_i$ which is measurable with respect to $P_i$.

**Definition 1** Player $i \in N$ is Bayes rational at $\omega \in \Omega$ if his expected payoff at $\omega$, $E(u_i(\sigma)|P_i)(\omega)$ is at least as large as the amount $E(u_i(\sigma_{-i}, a_i)|P_i)(\omega)$ that he would have got had he chosen action $a_i \in A_i$ instead of $\sigma_i(\omega)$.

**Theorem 1** If each player is rational at each state of the world, then the distribution of the action $n$-tuple is a correlated equilibrium distribution.

**Proof**: We need to show a correlated equilibrium whose induced probability distribution over the action profiles coincides with the distribution of $\sigma$. We claim that the correlated strategy we are looking for is the function $\sigma : \Omega \rightarrow A$ itself. Let $\tau_i : \Omega \rightarrow A_i$ be a function that is measurable with respect to $P_i$. Since $i$ is Bayes rational at $\omega$

$$E(u_i(\sigma)|P_i)(\omega) \geq E(u_i(\sigma_{-i}, a_i)|P_i)(\omega) \quad \forall a_i \in A_i.$$ 

That is,

$$\sum_{\omega' \in P_i(\omega)} \frac{\pi(\omega')}{\pi(P_i(\omega))} u_i(\sigma_{-i}(\omega'), \sigma_i(\omega')) \geq \sum_{\omega' \in P_i(\omega)} \frac{\pi(\omega')}{\pi(P_i(\omega))} u_i(\sigma_{-i}(\omega'), a_i) \quad \forall a_i \in A_i.$$ 

In particular, for $a_i = \tau(\omega) = \tau(\omega')$ for all $\omega' \in P_i(\omega)$,

$$\sum_{\omega' \in P_i(\omega)} \frac{\pi(\omega')}{\pi(P_i(\omega))} u_i(\sigma_{-i}(\omega'), \sigma_i(\omega')) \geq \sum_{\omega' \in P_i(\omega)} \frac{\pi(\omega')}{\pi(P_i(\omega))} u_i(\sigma_{-i}(\omega'), \tau(\omega')).$$ 

Multiplying both sides by $\pi(\omega)$ and adding over all $\omega$ we get

$$\sum_{\omega \in \Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \sigma_i(\omega)) \geq \sum_{\omega \in \Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \tau(\omega)).$$ 

$\square$