Econ 618:
Correlated Equilibrium

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1 Basic Concept of a Correlated Equilibrium

MSNE assumes players use a random device privately and independently, that tells them which strategy to choose for any given play. The Bayesian environment, we have discussed so far assumes that signals are random, private and independent. The question that we address here is what happens if the signals are random but public (not private) and not independent across the agents and further, the agents decide their action based on these signals. The specific issue we are interested in here is, can the agents do better than their MSNE payoffs.

Example 1: Consider the following game:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>(5,1)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>D</td>
<td>(4,4)</td>
<td>(1,5)</td>
</tr>
</tbody>
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The two PSNEs are \((U, L)\) and \((D, R)\) with asymmetric payoffs \((5, 1)\) and \((1, 5)\). Using methods discussed earlier, the game has a unique MSNE, \(((1/2, 1/2), (1/2, 1/2))\) with expected payoff = \((1/4).5 + (1/4).0 + (1/4).4 + (1/4).1 = 2.5\).

Now suppose a referee steps in and tells each of them to use the following strategy depending on the outcome of a "public" coin-flip (whose outcomes therefore are observable by both players):
Player 1 is to play $U$ if $h =$ head occurs and $D$ if $t =$ tail occurs. Player 2 is to play $L$ if $h$ occurs and $R$ if $t$ occurs.

First, note that these are pure, not mixed strategies. As in the Bayesian environment discussed earlier, a pure strategy is a map that tells a player which action to take if a certain uncertain event occurs. Second, note that the above pure strategies are best responses to each other. If $h$ occurs, player 1 plays $U$ and $L$ is player 2’s BR, if $t$ occurs, player 1 plays $D$ and $R$ is player 2’s BR. Similarly, if $h$ occurs, player 2 plays $L$ and $U$ is player 1’s BR, if $t$ occurs, player 2 plays $R$ and $D$ is player 1’s BR. Thus players have no incentive to unilaterally deviate from what the referee has told them to do - implying, they will play as they are told and achieve what is achievable without any ”binding contracts”. Thirdly, the players’ decisions are not independent. They are tied to a public signal and the actions chosen are correlated via this signal, although the signal itself is random - if player 1 chooses $U$, for sure, player 2 chooses $L$ and so on. Fourthly, the random event itself and the associated device has no direct impact on the payoffs except through the actions chosen by the players.

Note, that by correlating their actions with a public random device, the players receive a higher expected payoff compared to their MSNE payoff. Under this device, the outcome of the game is $(U,L)$ with prob $1/2 = \text{Prob}(h)$ and $(D,R)$ with prob $1/2 = \text{Prob}(t)$. So, the expected payoff is $(1/2).5 + (1/2).1 = 3 > 2.5$ for each player. Also, note that the reason for this higher expected payoff is that the undesirable event $(U,R)$ never happens with this device while it could occur with prob $(1/4)$ under a MSNE. The pair of pure strategies, \{Play $U$ if $h$ and $D$ if $t$; Play $L$ if $h$ and $R$ if $t$\}, is described as a ”correlated equilibrium”.

Some economists have described such publicly observable random events which have no direct effect on an agent’s payoff but at the same time enable them to coordinate their actions and achieve a different set of outcomes (compared to what they can achieve without these random events), ”sunspots”. The word ”extrinsic uncertainty” has also been used to describe the same concept - a random event which has no direct effect on the fundamentals of an economy but can cause agents to coordinate their actions and through this has an impact.

To continue with our earlier comparisons of the expected payoffs under the MSNE and the correlated equilibrium, note that under an MSNE, a somewhat ”desirable” but non-NE outcome $(D,L)$ with payoffs $(4,4)$ occurs with prob $(1/4)$ but it never occurs with the above device. However, we
can change all that by using another device and a different pair of correlated strategies to achieve yet another correlated equilibrium.

Example 2: Consider the same strategic form game as in Example 1 but a different random device. Assume that the current device has 3 equi-probable outcomes \(\{A, B, C\}\), so that each of these events can occur with prob \((1/3)\). But the players cannot observe the realization perfectly. Player 1 knows whether \(A\) has occurred but cannot distinguish between \(\{B, C\}\) (thus knows that \(A\) has not happened) and player 2 knows whether \(C\) has occurred but cannot distinguish between \(\{A, B\}\) (thus knows that \(C\) has not happened). You can even imagine that the device is not exactly public, that the outcome is observable only by the referee, that the players trust the referee and that the referee provides them with this partial information. The referee also tells them to use the following strategy: \{Player 1 play \(U\) if \(A\) and \(D\) if \(\{B, C\}\); Player 2 play \(R\) if \(C\) and \(L\) if \(\{A, B\}\}\).

Note that the \(P(B|\{B, C\}) = 1/2 = P(C|\{B, C\})\) and \(P(A|\{A, B\}) = 1/2 = P(B|\{A, B\})\). Under these, the strategies are BR with respect to each other. If player 1 has information \(A\), he knows that player 2 has information \(\{A, B\}\) and will play \(L\). Hence \(U\) is player 1’s BR. If player 1 has information \(\{B, C\}\), he expects player 2 to play \(R\) with prob \((1/2)\) and \(L\) with prob \((1/2)\). In this case both \(U\) and \(D\) are player 1’s BR, so he plays \(D\). Thus player 1 has no incentive to unilaterally deviate from the given strategy. Similarly, if player 2 has information \(C\), he knows player 1 has information \(\{B, C\}\) and will play \(D\). So \(R\) is player 2’s BR. If player 2 has information \(\{A, B\}\), he expects player 1 to play \(U\) with prob \((1/2)\) and \(D\) with prob \((1/2)\). In this case both \(R\) and \(L\) are player 2’s BR and he chooses \(L\). Thus player 2 has no incentive to deviate.

Further, note that under these strategies, if the outcome of the random device is \(A\), the outcome of the play is \((U, L)\), if the outcome of the random device is \(B\), the outcome of the play is \((D, L)\), and if the outcome of the random device is \(C\), the outcome of the play is \((D, R)\). Each of these outcomes has prob \((1/3)\) given the nature of the device. And so the expected payoff of each player is \((1/3).5 + (1/3).4 + (1/3).1 = 3.33\) which is greater than the expected payoff under the first device.

Note that under this second device, the ”somewhat” desirable outcome \((D, L)\) has a positive weight, the bad outcome \((U, R)\) never happens. Most importantly, as shown by Fig 1, the payoff of
3.33 is outside the convex hull of the Nash payoffs.

2 Definition and characterization of a correlated equilibrium

The key components of a correlated equilibrium are (A) a correlating device and (B) a set of strategy maps.

(A). Formally, a correlating device is a triple \( (\Omega, \{H_i\}_{i \in N}, p) \), where \( \Omega \) denotes a (finite) set of states corresponding to the possible outcomes of the device. \( p \) is the probability measure over \( \Omega \) and the prior for each player. \( H_i \) is an information partition for player \( i \).

An information partition for player \( i \), \( H_i \) is a collection of disjoint subsets of \( \Omega \) such that the union of these subsets is \( \Omega \). We shall describe an individual subset of an information partition as an information set. A subset of \( \Omega \) or an element of the information partition defines a possible event resulting from the use of the device. Within a given information partition, the realization of a specific information set tells the player what has occurred, or what his information is about the result of the use of the device. It is clear therefore that within a given information partition, an element of \( \Omega \) cannot belong to two different information sets, as this would imply that a particular outcome has both "occurred" and "not occurred".

Given any generic random device with three possible outcomes, \( \Omega = \{A, B, C\} \), a player \( i \) will have one of the following possible information partitions.

1. \( H_i = \Omega = \{\{A, B, C\}\} \) under which the partition consists of only one subset element, \( \{A, B, C\} \). Thus all three outcomes belong to the same information set, which says that player \( i \) has no information as to what has occurred. He cannot distinguish between the three states or outcomes.

2. \( H_i = \{\{A\}, \{B\}, \{C\}\} \) under which the partition consists of three subset elements, \( \{A\}, \{B\}, \{C\} \). which says that player \( i \) has perfect information. He knows exactly whether \( A \) has occurred or \( B \) has occurred or \( C \) has occurred.

3. \( H_i = \{\{A, B\}, \{C\}\} \) under which the partition consists of two subset elements, \( \{A, B\} \), and \( \{C\} \). Thus the player has partial information. He knows whether \( C \) has occurred or whether one of the outcomes \( A \) or \( B \) has occurred. He cannot distinguish between \( A \) and \( B \). You may also say that at the throw of the device, the player knows whether "C" or "not C" has happened.
4. \( H_i = \{\{A\}, \{B, C\}\} \) under which the partition consists of two subset elements, \( \{A\} \), and \( \{B, C\} \). The player has partial information - he knows whether \( A \) has occurred or whether one of the outcomes \( B \) or \( C \) has occurred. He cannot distinguish between \( B \) and \( C \).

5. \( H_i = \{\{A, C\}, \{B\}\} \) under which the partition consists of two subset elements, \( \{A, C\} \), and \( \{B\} \). The player knows whether \( B \) has occurred or whether one of the outcomes \( A \) or \( C \) has occurred. He cannot distinguish between \( A \) and \( C \).

A correlated equilibrium is specific to a device and a given set of information partitions, one for each player. Given the device \( \Omega \) and an information partition \( H_i \) for player \( i \), the information sets (or the specific subsets of \( \Omega \)) making up the partition are indexed, \( H_i = \{h^k_i\} \). In Example 2 of the previous section, \( h^1_1 = \{A\} \) and \( h^2_1 = \{B, C\} \). For each \( h^k_i \) with positive prior prob, player \( i \)'s posterior beliefs about \( \Omega \) are given by \( p(\omega|h^k_i) = \frac{p(\omega)}{p(h^k_i)} \) for \( \omega \in h^k_i \) and 0 otherwise.

(B). A pure strategy of the expanded game for which a correlated equilibrium is defined, is a map \( s_i, h^k_i \in H_i \rightarrow A_i \), the set of pure actions for the strategic game. Note that \( s_i \) assigns to each information set in the partition \( H_i \), an element of \( A_i \). Thus \( s_i \) must assign the same pure action from \( A_i \), to two different elements of \( \Omega \) if they belong to the same information set under \( H_i \). Thus the "allowed" set of pure strategies for the expanded game must be adapted to the information partition for each player. This is important to keep in mind because, sometimes, we may find it more convenient to define a pure strategy as a map from \( \omega \in \Omega \rightarrow A_i \) rather than as a map from \( h^k_i \in H_i \rightarrow A_i \). When we define a pure strategy as a map from \( \omega \in \Omega \rightarrow A_i \), we shall keep in mind that only the "adapted" strategies are to be considered and allowed, not any possible map (Note this slight difference with the framework we discussed under Bayesian games). In Example 2, the pure strategy map for player 1: (play \( U \) if \( A \) or \( B \); play \( D \) if \( C \)), is not adapted because it does not assign the same action to elements of the same information set \( \{B, C\} \). The pure strategy map: (play \( U \) if \( A \) or \( B \) or \( C \)) is however adapted.

A correlated equilibrium corresponding to a device \( (\Omega, \{H_i\}_{i\in N}, p) \) is a profile of pure adapted strategy maps \( s^*_1, s^*_2 \ldots s^*_N \), such that for every \( i \) and every alternative "adapted" pure strategy \( \tilde{s}_i \), the
following is true:

\[
\sum_{\omega \in \Omega} p(\omega) u_i(s_i^*(\omega), s_{-i}^*(\omega)) \geq \sum_{\omega \in \Omega} p(\omega) u_i(\bar{s}_i(\omega), s_{-i}^*(\omega))
\]

This is to say that the adapted strategy maps must be best responses to each other. Note that the above maximization problem uses unconditional probs or priors. As in the case of the Bayesian game, we may find it more convenient to use the conditional version of the maximization problem, to verify or compute a correlated equilibrium. The conditional version of the maximization problem says that for each player \(i\), the map \(s_i^*\) must satisfy,

\[
\sum_{\omega \in h_k} p(\omega|h_k) u_i(s_i^*(\omega), s_{-i}^*(\omega)) \geq \sum_{\omega \in h_k} p(\omega|h_k) u_i(a_i, s_{-i}^*(\omega))
\]

for each \(h_k \in H_i\) and for all \(a_i \in A_i\). Note the similarity of both versions of the maximization problem with the versions we discussed for Bayesian equilibrium. The only difference is the here in the first version, the pure strategies must be adapted. We use the conditional version of the maximization problem to verify a correlated equilibrium for the Battle of the sexes game.

**Example 3**: Correlated equilibrium in the Battle of the Sexes

Let the device be a public coin flip with two outcomes, \(\Omega = \{h = \text{head}, t = \text{tail}\}\), and \(p(h) = p(t) = 1/2\). The information partitions are \(H_1 = H_2 = \{\{h\}, \{t\}\}\). That is each player has perfect information about the outcome of the device. Define two pure strategies as follows: \(s_1 : h \rightarrow B, \ t \rightarrow S\) and \(s_2 : h \rightarrow B, \ t \rightarrow S\). Note that each strategy is adapted for the information partition of the respective player. We now verify that this pair of strategies form a correlated equilibrium.

Suppose \(\{h\}\) has occurred. Then for player 1, under this strategy the expected payoff (the expected payoff is in effect a sure payoff because each information set is a singleton under this device) is \(U_1(B,B) = 2 > U_1(S,B) = 0\). Similarly if \(\{t\}\) has occurred, for player 1 the expected payoff is \(U_1(S,S) = 1 > U_1(B,S) = 0\). Thus the strategy \(s_1\) is optimal for player 1 if player 2 plays \(s_2\). Similarly we can show that \(s_2\) is optimal for player 2 if player 1 plays \(s_1\). Thus the strategy pair is a correlated equilibrium.

The device essentially allows the two players to choose the same action under any eventuality and from the structure of the game we know that both prefer this outcome to going their separate
ways, although they have different tastes. Thus the outcome \((B,B)\) occurs with prob \((1/2)\) and the outcome \((S,S)\) occurs with prob \((1/2)\). The expected payoff under the correlated equilibrium is \((1/2).2 + (1/2).1 = 3/2\) for each player whereas the expected payoff under the MSNE is \((2/3)\). I leave you to check whether the payoff under the correlated equilibrium is outside the convex hull of the set of NE or not.

Finally, note that the above is one possible correlated equilibrium using the public coin-flip as the device. It is possible to identify other correlated equilibria using other devices.

### 3 The structure of the set of correlated equilibria

A correlated equilibrium is specific to a correlating device \(\Omega\), with an associated information partition \(H_i\) for each player. There are an infinite number of possible ways to draw up a \(\Omega\). Thus in principle, there can be an infinite number of correlated equilibria. Under the circumstances, what can we say about the structure of the set of correlated equilibria. There are two important results on this issue.

**Theorem 1** Any MSNE of a strategic game can be attained as a correlated equilibrium of an expanded game.

An informal proof is discussed below as it is important to understand why this result is true. An immediate implication of this lemma is that the set of correlated equilibria is at least as large as the set of MSNEs (remembering that the MSNEs also include the PSNEs). Note however that the converse is not true as all the examples above show - that is all correlated equilibrium may not be MSNEs. Thus the full implication of the lemma is that the set of correlated equilibria is generally larger than the set of MSNEs.

**Proof:** To prove the Lemma, we have to demonstrate the existence of a device \(\Omega\) with a prior distribution \(p\), an information partition \(H_i\) for each player \(i\), and a profile of pure strategy maps which are best responses to each other and each of which induces the same distribution on the pure action set of a player, as does the MSNE in question.
Define $\Omega = \times_{i \in N}A_i = A$. Thus each state of nature or outcome of the device is a joint strategy profile of the strategic game. You can think of the device as a multifaceted die with a specific joint strategy profile written on each facet.

Recall from previous discussions that a given mixed strategy profile induces a prob distribution on the pure joint strategy space. If the MSNE in question is a profile of mixed strategies $\alpha^i = (\alpha^i_1, \alpha^i_2, \cdots, \alpha^i_N)$, then assign to each “state” $a = (a^1_1, a^2_2, \cdots, a^N_N)$, the prior probability $p(a) = \prod_{i \in N} \alpha^i(a^i_k)$. This is the prob distribution over $\Omega = A$.

An information partition $H_i$ for player $i$ is defined as follows. Suppose $a = (a^1_1, a^2_2, \cdots, a^N_N)$ and $a' = (a^1_1', a^2_2', \cdots, a^N_N')$ are two states in $\Omega$ (joint action profile for the strategic game). For player $i$, the two outcomes $a$ and $a'$ belong to the same information set under $H_i$, if the $i$th component of $a$ and $a'$ are identical. That is $h_i(a^i_k) = \{a' \in \Omega | a^i_k = a^i_k\}$. Thus fix a pure action for player $i$, say $a^i_k$. Collect all outcomes of $\Omega$ (joint action profiles of the strategic game) which have the $i$th component as $a^i_k$ and put them in one information set and call it $h_i(a^i_k)$. Note that such $h_i(\cdot)$s are disjoint. The collection of such information sets is $H_i$.

Finally, we define a suitable strategy map for each player. Define the map as, $s_i(a) = a^i_k$. That is to say, once the device is thrown and a facet, which is a joint action profile from the strategic game, shows up, the $i$th player chooses the action named in the $i$th component. Such a strategy map is adapted. Given the fact that the probs that we are using constitute the MSNE, this strategy map solves the maximization prob above - the prob of choosing a specific action, say $a^i_k$, under this strategy map is exactly the prob assigned under the MSNE.

**Example 4:** MSNE of the Battle of the Sexes as a correlated equilibrium

We want to show that the MSNE: $\{\{2/3, 1/3\}, \{1/3, 2/3\}\}$ can be sustained as a correlated equilibrium. Using the steps in the lemma, $\Omega = \{(B, B), (B, S), (S, B), (S, S)\}$. The prior prob distribution on $\Omega$ is $\{p(B, B) = 2/9; p(B, S) = 4/9; p(S, B) = 1/9; p(S, S) = 2/9\}$. The information partition for player 1 is $\{h_1(B) = \{(B, B), (B, S)\}; h_1(S) = \{(S, B), (S, S)\}\}$. The strategy map for player 1 is $s_1(B, B) = B, s_1(B, S) = B, s_1(S, B) = S$ and $s_1(S, S) = S$. Note that the prob with which player 1 chooses action $B$ under this map is $(2/9) + (4/9) = (2/3)$ and the prob with which player 1 chooses action $S$ under this map is $(1/9) + (2/9) = (1/3)$. From our knowledge of the MSNE, we know that this strategy maximizes payoffs for player 1 if player 2 chooses the analogously.
A second theorem provides some additional insight into the structure of the set of correlated equilibria. It says,

**Theorem 2** Any convex combination of correlated equilibrium payoff profiles of a game is a correlated equilibrium payoff profile of the game. (In other words, any such convex combination can be derived as a correlated equilibrium payoff profile corresponding to some correlating device.)

We shall not discuss the steps of the proof here because of the notational complexity. See O-R if you are interested. However the method of the proof is very similar to that of the previous Lemma. Suppose we know that \( K \) correlated equilibria exist with \( K \) associated devices and information partitions. To demonstrate that a convex combination of these \( K \) equilibrium payoffs can be derived as a correlated equilibrium, we need to demonstrate the existence of a state space \( \Omega \), a prob distribution, an information partition and an equilibrium strategy map for each player, for the specific convex combination of the payoffs. This can be done and this time we take the superset of all the \( K \) state spaces as the space \( \Omega \).

Although we do not discuss the steps of the proof, the lemma has important implication. Consider the strategic game of examples 1 and 2. The game has three MSNEs (two of them pure and one totally mixed) and we have demonstrated the existence of at least two correlated equilibrium. Any convex combination of these five payoffs at least, can be derived as a correlated equilibrium corresponding to some device. Further note that convex combinations of convex combinations are also convex combinations. Thus the set of correlated equilibrium payoffs achievable, *assuming that we can think of no further device* beyond the two laid out in Examples 1 and 2, is shown in Fig 1.

### 4 The Universal device

There are an infinite number of possible ways to draw up a device \( \Omega \) and examples 1 and 2 show that it is possible to obtain different expected payoffs using different devices. A natural question
therefore is - can any level of expected payoffs be obtained by an appropriate choice of a device. The following theorem implies that the answer to this question is "no". The theorem shows that any expected payoff obtained under any correlated equilibrium using any given device \( \Omega \) can be obtained using the device \( \Omega' \) whose set of outcomes is the set of joint action profiles \( A = \times_{i=NA_i} \). Hence \( \Omega' \) is sometimes known as the Universal device. Hence if a finite strategic game has finite payoffs, any arbitrary level of expected payoffs cannot be obtained under a correlated equilibrium using an appropriate device.

**Theorem 3** Let \( G \) be a finite strategic game. Every probability distribution over outcomes that can be obtained in a correlated equilibrium of \( G \) can be obtained in a correlated equilibrium in which the set of states is \( A \) and player \( i \)’s information partition consists of all sets of the form \( \{a \in A | a_i = b_i\} \), for some action \( b_i \in A_i \).

Without going through the proof, we check why this result is true by obtaining the expected payoff of the correlated equilibrium of Example 2, with the use of the Universal device.

The random device, the prior probabilities of the states and the information partitions of Example 2 are as follows:

\[
\Omega = \{A, B, C\} \quad \text{with} \quad p(A) = p(B) = p(C) = 1/3. \quad H_1 = \{h_1^1, h_1^2\} = \{(A), (B,C)\} \quad \text{and} \quad H_2 = \{h_2^1, h_2^2\} = \{(A,B), (C)\}.
\]

Using this device and these information partitions, under the correlated equilibrium strategies described earlier, the agents earn an expected payoff of 3.33 each. Under the correlated equilibrium strategies, the joint action profiles could occur with the following probabilities:

\[
p(U, L) = p(D, L) = p(D, R) = 1/3 \quad \text{and} \quad p(U, R) = 0.
\]

To use, Theorem 3, we define the new random device as \( \Omega' = \{(U,L), (U,R), (D,L), (D,R)\} \). The information partition of player 1 is \( H_1 = \{h_1^1, h_1^2\} \) where \( h_1^1 = \{(U,L), (U,R)\} \) and \( h_1^2 = \{(D,L), (D,R)\} \). The information partition of player 2 is \( H_2 = \{h_2^1, h_2^2\} \) where \( h_2^1 = \{(U,L), (D,L)\} \) and \( h_2^2 = \{(U,R), (D,R)\} \). Theorem 3 specifies what the device is and what the information partitions are. It does not specify any rule to assign the prior probabilities for the states. Theorem 3 can be shown to be true, precisely because we have this control in our hands. We shall the assign
the priors on \( \Omega' \) according to the probabilities of the outcomes under the equilibrium we want to replicate. Thus we assign the priors as \( p(U, L) = p(D, L) = p(D, R) = 1/3 \) and \( p(U, R) = 0 \).

A correlated equilibrium is a PSNE of the above extended game. In general, there can be multiple such PSNEs. To show that Theorem 3 is correct, we have to show the existence of at least one that provides an expected payoff of 3.33. The following is the one: Player 1 plays \( s_1^*(h_1^1) = U, s_1^*(h_2^2) = D \), Player 2 plays \( s_2^*(h_1^1) = L, s_2^*(h_2^2) = R \). We show that these two are best responses to each other.

If \( h_1^1 \) occurs, player 1’s conditional probabilities for the joint outcomes are: \( P((U, L)|h_1^1) = \frac{P((U, L))}{P(h_1^1)} = \frac{1/3}{1/3} = 1 \) and \( P((U, R)|h_1^1) = \frac{P((U, R))}{P(h_1^1)} = \frac{0}{1/3} = 0 \). Hence according to \( s_2^* \), player 1 expects player 2 to play \( L \) with prob 1. Thus \( U \) is player 1’s best response.

If \( h_2^2 \) occurs, player 1’s conditional probabilities for the joint outcomes are: \( P((D, L)|h_2^2) = \frac{P((D, L))}{P(h_2^2)} = \frac{1/3}{2/3} = 1/2 \) and \( P((D, R)|h_2^2) = \frac{P((D, R))}{P(h_2^2)} = \frac{1/3}{2/3} = 1/2 \). Hence according to \( s_2^* \), player 1 expects player 2 to play \( L \) with prob 1/2 and \( R \) with prob 1/2. For player 1, expected payoff from \( U \) equals \( (1/2.5 + 1/2.0 = 2.5) \) and expected payoff from \( D \) equals \( (1/2.4 + 1/2.1 = 2.5) \). Thus \( D \) is player 1’s best response.

Hence \( s_1^* \) is a best response to \( s_2^* \).

If \( h_1^1 \) occurs, player 2’s conditional probabilities for the joint outcomes are: \( P((U, L)|h_1^1) = \frac{P((U, L))}{P(h_1^1)} = \frac{1/3}{2/3} = 1/2 \) and \( P((D, L)|h_1^1) = \frac{P((D, L))}{P(h_1^1)} = \frac{1/3}{2/3} = 1/2 \). Hence according to \( s_1^* \), player 2 expects player 1 to play \( U \) with prob 1/2 and \( D \) with prob 1/2. For player 2, expected payoff from \( L \) equals \( (1/2.1 + 1/2.4 = 2.5) \) and expected payoff from \( R \) equals \( (1/2.5 + 1/2.0 = 2.5) \). Thus \( L \) is player 2’s best response.

If \( h_2^2 \) occurs, player 2’s conditional probabilities for the joint outcomes are: \( P((U, R)|h_2^2) = \frac{P((U, R))}{P(h_2^2)} = \frac{0}{1/3} = 0 \) and \( P((D, R)|h_2^2) = \frac{P((D, R))}{P(h_2^2)} = \frac{1/3}{1/3} = 1 \). Hence according to \( s_1^* \), player 2 expects player 1 to play \( D \) with prob 1. Thus \( R \) is player 2’s best response.

Hence \( s_2^* \) is a best response to \( s_1^* \).

From a computational point of view, the following are the main lessons of Theorem 3.

(1) If we want to find a correlated equilibrium, the natural device to start with is the Universal device \( \Omega' \) along with the information partitions as specified in Theorem 3. We then find a PSNE
for this extended game.

(2) If we want to find a correlated equilibrium that provides higher expected payoffs than under a MSNE of the original strategic game (which is after all, the motivation for such a solution concept), we choose the priors on the states of $\Omega'$ appropriately. We reduce the probabilities of the less desirable outcomes under the MSNE and increase the probabilities of the more desirable outcomes.
Fig 1: Examples 1 and 2 game.