1. \textit{(First price auction)} An object is to be assigned to the highest among $N$ bidders, indexed by $i$. Players have individual values of $v_1 > v_2 > \ldots v_N > 0$ for the object which are common knowledge. A winner pays his/her bid. If multiple bidders have submitted the highest bid, the object will be assigned to the bidder with the lowest index and the bidders know this rule.

   \textbf{a.} Model this situation as a strategic game and find a PSNE.

   \textbf{b.} Characterize all PSNEs. As in second price auctions, are there equilibria that are behaviorally "not compelling"?

2. \textit{(Median voter theorem - version 1)} Three candidates are fighting an election on a specific issue. Voters are uniformly distributed according to their ideologies along $[0, 1]$ with 0 indicating "extreme left" and 1, "extreme right". A voter will select the candidate closest to his/her ideological position. If multiple candidates choose the same position, the votes are equally shared by them. The candidate with the most votes wins the election. Candidates do not have an ideology - they are only interested in winning. Each candidate needs to decide whether to "stay out" of the race or to choose a position on $[0, 1]$ to fight. That is, each candidate’s strategy set is an element of $[0, 1] \cup \{\text{Stay out}\}$. Candidates preferences are given by the following order on the outcomes: $\text{Win} \succ \text{Tie} \succ \text{Stay out} \succ \text{Lose}$.

   \textbf{a.} Does the game have a PSNE? Are there multiple PSNEs?

   \textbf{b.} Now assume that there are two candidates and everything else remains the same. Does the game have a PSNE? Does the game have multiple PSNEs?

3. \textit{(Median voter theorem - version 2)} Three candidates are fighting an election on a specific issue. Voters are uniformly distributed according to their ideologies along $[0, 1]$ with 0 indicating "extreme left" and 1, "extreme right”. A voter will select the candidate closest to his/her ideological position. If multiple candidates choose the same position, the votes are equally shared by them. The candidate with the most votes wins the election. Candidates do not have an ideology - they are
only interested in winning. The difference with the previous scenario is that candidates do not have an option to "Stay out". Each candidate needs to choose a position on \([0, 1]\) to fight. That is, each candidate’s strategy set is an element of \([0, 1]\). Candidates preferences are given by the following order on the outcomes: \(\text{Win} \succ \text{Tie} \succ \text{Lose}\).

a. Does the game have a PSNE? Are there multiple PSNEs?

b. Now assume that there are two candidates and everything else remains the same. Does the game have a PSNE? Does the game have multiple PSNEs?

4. (Median voter theorem - version 3) Three candidates are fighting an election on a specific issue. Voters are uniformly distributed according to their ideologies along \([0, 1]\) with 0 indicating "extreme left" and 1, "extreme right". A voter will select the candidate closest to his/her ideological position. If multiple candidates choose the same position, the votes are equally shared by them. The candidate with the most votes wins the election. Candidates do not have an ideology as before. The difference with the scenario of Problem 3, is that candidates want to maximize their votes (Obviously, a "Win" gives the maximum payoff, in that sense). Each candidate needs to choose a position on \([0, 1]\).

a. Does the game have a PSNE? Are there multiple PSNEs?

b. Now assume that there are two candidates and everything else remains the same. Does the game have a PSNE? Does the game have multiple PSNEs?

c. Compare your answers to Problems 3-5 and comment on them.

5. (Teams with substitutable effort) Three co-authors are writing a paper. Each of them has the ability to complete the paper on his/her own and in this sense their efforts are substitutable (also implying they can free ride on each other). But of course the probability of success increases if they work with each other. Each is choosing an effort level \(x_i \in [0, 1]\) where without loss of generality, \(x_i\) also represents the probability of successfully completing the paper on their own. The cost of effort is \(c x_i^2\). If the paper is completed, each player receives a reward of \(r > 0\) unit. Otherwise each receives 0.

a. Frame this as a strategic game. Assume \(r = c\) and find a PSNE for the game. Does the game have multiple PSNEs?
b. Under appropriate restrictions on \( r \) and \( c \), find a PSNE under which all co-authors choose the same level of effort.

6. \((Tournaments)\) Three players are trying to complete a task by himself or herself to get a reward. The reward \( r > 0 \) is shared by the players if multiple players succeed, in the following way. If one person succeeds, he gets \( r \) for sure, if two players succeed each gets \( r \) with probability one-half and if all succeeds, each gets \( r \) with probability one-third. Each player chooses an effort level \( x_i \in [0, 1] \) to succeed, where without loss of generality, \( x_i \) is the probability of success. The cost of effort is \( cx_i^2 \). Assume \( r = 2c \) for convenience.

   a. Frame this as a strategic game and find a PSNE for this game.
   b. Does the game have multiple PSNEs?

7. \((Symmetric games)\) Consider a two person strategic game that satisfies all the conditions stated in the PSNE existence theorem discussed in class. Assume that the game is symmetric. A symmetric is symmetric if (1) \( A_1 = A_2 \) and (2) \((a_1, a_2) \succeq_1 (b_1, b_2) \) iff \((a_2, a_1) \succeq_2 (b_2, b_1)\).

   a. Use Kakutani’s theorem to show that there is a PSNE of the form \((a_i^*, a_i^*)\).
   b. From the set of all the binary action games discussed in class, point out the ones which are symmetric. Are there any symmetric games with only asymmetric PSNEs? How do you reconcile your answer with the answer to 7a. above?