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On the Theory of the Competitive Firm
Under Price Uncertainty: Note

By YASUNORI ISHII*

Agnar Sandmo, in his excellent paper, attempted to analyze the behavior of the
competitive firm under price uncertainty.
He presumed that: (a) the utility function of
the firm is a concave, continuous and differen-
tiable function of profits, so that

\[ U'(\pi) > 0, \quad U''(\pi) < 0 \]

and the absolute risk aversion \( R_A(\pi) \)
\( = -U''(\pi)/U'(\pi) \) is a decreasing function
of \( \pi \), that is,

\[ \partial R_A(\pi)/\partial \pi < 0 \]

(b) The firm’s profit function can be de-

\[ \pi(x) = Px - c(x) - B \]

where \( P \) is the price of output, assumed to be a (subjectively)
random variable with density function \( f(P) \) and expected value
\( E[P] = \mu \), \( x \) is output, \( c(x) \) is the variable
cost function with \( c'(x) > 0 \) and \( c(0) = 0 \),
and \( B \) is fixed cost.

(c) The expected utility of profits can be
written as

\[ E[U(Px - c(x) - B)] \]

where \( E[\cdot] \) is the expectations operator.
The competitive firm decides the output so
as to maximize the expected utility of
profits.

I

Sandmo analyzed the effects of change in
the expected value and uncertainty of the
price on the optimal output of the com-
petitive firm, which were written as \( \partial x/\partial \theta \)
and \( \partial x/\partial \gamma \) in his article, under the assump-
tions mentioned above. As a result of his
analysis, he asserted that while the sign of
\( \partial x/\partial \theta \) can be clearly judged to be positive,
the sign of \( \partial x/\partial \gamma \) is ambiguous in general.

The purpose of this note is to show that
investigating the sign of \( \partial x/\partial \gamma \) under the
same assumptions and analytic apparatus
as Sandmo’s, we can decide the sign of
\( \partial x/\partial \gamma \), manifestly in opposition to his
assertions.

II

Necessary and sufficient conditions for a
maximum are written as

\[ E[U'(\pi)(P - c'(x))] = 0 \]

\[ D = E[U''(\pi)(P - c'(x))^2 \]

\[ - U'(\pi)c''(x)] < 0 \]

If \( c''(x) \geq 0, D < 0 \) is always satisfied. And
there is a possibility of \( D < 0 \) even in the
case \( c''(x) < 0 \). It is interesting to examine
the conditions of existence and uniqueness
of interior optimal solution. We, however,
assume for the simplification of analysis
that (3) and (4) determine a nonzero, finite,
and unique solution \( x^* \) to the maximiza-
tion problem.

Following the introduction of lemmas,
we shall investigate the sign of \( \partial x^*/\partial \gamma \).

LEMMA 1:

\[ \mu - c'(x^*) > 0 \]

LEMMA 2:

\[ E[(P - c'(x^*))U''(\pi^*)] \geq 0 \]

where\(^1\) \( \pi^* = Px^* - c(x^*) - B \)

The proofs of these lemmas are omitted
here for they are shown in Sandmo’s paper.

\(^1\)We assume that \( R_A(\pi) \) is a nonincreasing function
of \( \pi \).

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III

Now that all of the necessaries have been presented, we shall proceed to prove our proposition.

**PROPOSITION:** Nonincreasing absolute risk aversion is a sufficient condition for $\frac{\partial x^*}{\partial \gamma}$ to be negative.

**PROOF:**

By adopting the same procedures that Sandmo used, we can obtain

$$\frac{\partial x^*}{\partial \gamma} = -\frac{1}{D} \left[ E[U'(\pi^*)(P - \mu)] + x^* E[(P - c'(x^*))U''(\pi^*)(P - \mu)] \right]$$

which is equal to his equation (13). Since $D < 0$, the sign of $\frac{\partial x^*}{\partial \gamma}$ is equivalent to that of $E[U'(\pi^*)(P - \mu)] + x^* E[(P - c'(x^*))U''(\pi^*)(P - \mu)]$. Consequently we will investigate the sign of this.

First, modifying $E[U'(\pi^*)(P - \mu)]$, we obtain

$$E[U'(\pi^*)(P - \mu)] = E[(U'(\pi^*) - E[U'(\pi^*)])(P - \mu)] = \text{cov} (P, U'(\pi^*))$$

where $\text{cov} (P, U'(\pi^*))$ is the covariance between $P$ and $U'(\pi^*)$. Differentiating $U'(\pi)$ with respect to $P$ at $x = x^*$ and taking into consideration $U''(\pi) < 0$, we can obtain

$$\frac{\partial U'(\pi)}{\partial P} = x^* U''(\pi^*) < 0$$

which means

$$(7) \quad \text{cov} (P, U'(\pi^*)) < 0$$

in the case when the competitive firm faces price uncertainty. We therefore have, substituting (7) for (6),

$$(8) \quad E[U'(\pi^*)(P - \mu)] < 0$$

Next, from $E[(P - c'(x^*))U''(\pi^*)(P - \mu)]$, we can get

$$E[(P - c'(x^*))U''(\pi^*)(P - \mu)]$$

$$= E[(P - c'(x^*)(P - c'(x^*) + c'(x^*) - \mu)U''(\pi^*)]$$

$$= E[(P - c'(x^*))^2 \cdot U''(\pi^*)]$$

$$- (\mu - c'(x^*)E[(P - c'(x^*))U''(\pi^*)]\]

It is clear immediately that the first term of the right-hand side of (9) is negative and that the second term of the right-hand side of (9) is nonpositive in consideration of Lemma 1 and Lemma 2. We therefore can obtain

$$x^* E[(P - c'(x^*))U''(\pi^*)] \cdot (P - \mu) < 0$$

It follows that substituting (8) and (10) in (5) and considering $D < 0$ from (4), we have $\frac{\partial x^*}{\partial \gamma} < 0$.

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2We can show another proof of (8):

$$E[U'(\pi^*)(P - \mu)] = E[U'(\pi^*)(P - c'(x^*) + c'(x^*) - \mu)] = E[U'(\pi^*)(P - c'(x^*)]$$

$$- (\mu - c'(x^*)E[U'(\pi^*)] = E[U'(\pi^*)] < 0$$

**REFERENCES**

