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Stephen J. Turnovsky


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PRODUCTION FLEXIBILITY, PRICE UNCERTAINTY AND
THE BEHAVIOR OF THE COMPETITIVE FIRM*

BY STEPHEN J. TURNOVSKY

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1. INTRODUCTION

THE PAST FEW YEARS HAVE WITNESSED important advances in the theory of
the firm under price uncertainty, with increasing attention being devoted to the
question of how a firm's decisions are affected by its attitude towards risk tak-
ing. Specifically, recent work by McCall [7] and Sandmo [15] has established
that a competitive firm's output under uncertainty will be smaller, larger, or
the same as it would be under certainty, depending upon whether it is risk
averse, risk attracted or risk neutral, respectively. Leland [6] has shown that,
with the addition of a mild restriction on the stochastic demand function, these
results obtained under quite general conditions by Sandmo, extend to the case
of a quantity setting monopolist as well. He also analyzes the same question
for the price setting monopolist and indicates some additional complications
that arise in that case.

Practically the entire literature on the subject makes the assumption (either
implicitly or explicitly) that the firm is required to make all its production
decisions for a given period before the selling price for that period is known and
that once these decisions are made, they are irrevocable. Demand and sales
are then determined after the market price has been established. In other
words, the firm has no flexibility in its production decisions and this is a rather
restrictive assumption. The purpose of this paper is to develop a one period
model of the competitive firm relaxing this assumption and to determine how
a firm having production flexibilities responds to uncertainty. The firm being
considered here makes its production decisions on the basis of stochastic informa-
tion about the selling price of its product, but possesses the ability to modify
these plans—at additional cost—after it learns the true selling price. Hence the

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1 This paper was begun while the author was an associate of the Institute for the Quantitative
Analysis of Social and Economic Policy, University of Toronto. I would like to thank Ted
Sieper and the referee for helpful comments on an earlier version.
2 The recent interest in this subject originates with the work by Mills [9, 10] and some of the
essays contained in Arrow, Karlin and Scarf [2, 3]. For a survey of many of the subsequent
contributions see McCall [8].
3 McCall [7] first established this result for the case where the firm has constant absolute risk
aversion (see Arrow [1] and Pratt [12]). Sandmo has proved that any risk averse firm will pro-
duce less than if it were risk neutral. He did not compare the output for a risk attracted firm,
but the proposition asserted can be easily verified for his more general model.
4 The assumption introduced by Leland is the intuitive one, which he names the principle of
increasing uncertainty and which asserts that as the total revenue increases so does its disper-
sion.
5 Clearly this comment does not apply to the price setting monopolist of Leland and one
other exception for quantity setting firms is contained in Tisdell. [17, (Chapter 7)].
firm's production plans are flexible in the sense introduced by Hart [4], three
decades ago. This would seem to characterize the situation confronting the
typical producer. For example, most firms are able to increase their output at
short notice in response to an unexpectedly high price by hiring additional
(more costly) overtime labor. Likewise, short run cuts in production resulting
from unanticipated adverse product market conditions are also common.

In order to analyze the problem it becomes necessary to distinguish between
those decisions made before, and those made after, the actual price is known.
These shall be referred to as ex ante decisions and ex post adjustments respec-
tively and as we shall see they are interdependent. The ex post adjustment will
depend upon the initially chosen plans; furthermore the crucial problem facing
the firm is to choose these initial plans optimally (i.e., its decisions under un-
certainty) taking into account its ex post adjustment possibilities. Hence this
formulation introduces some important dynamic elements into an essentially
static framework.6

There are two alternative cases one can consider. The first is where the initial
decisions are merely production plans which are not completely put into effect
before the price is known. In this case these decisions can be reduced as well
as increased (in either case at additional cost) after the true price becomes
known. The second alternative is the situation where the initial quantities are
actually produced before the true selling price is known. If this is so, then with
a one period horizon it will never pay the firm to reduce its original decisions
after it learns the selling price.7 In this case the only ex post adjustment is
upwards, where the firm responds to an unexpectedly high price by producing
additional output, introducing an asymmetry into the firm's adjustment. Since
the behavioral implications of the two alternatives are very similar, we shall
restrict ourselves to the former. The asymmetric case can be analyzed in the
same way and some of the differences which do arise are indicated in footnote
18 below.

The remainder of this paper proceeds as follows. Section 2 discusses the
firm's cost function, deriving several properties that are important in subsequent
analysis. In order to focus on the adjustment issues—which is the prime purpose
of this paper—Section 3 analyzes the behavior of the risk neutral firm. However,
the amount of risk encountered by the firm and its ex post adjustment possibil-
ities are intimately related. Thus in Section 4 we extend the analysis to deal
with the case where the firm has a non-neutral attitude towards risk taking.
The concluding section summarizes the main results of this paper.

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6 Available empirical evidence indicating a very high elasticity in the use of overtime labor
with respect to short run fluctuations in output, as well as other evidence describing the short
run variations in capacity utilization, suggest that the kinds of short run adjustments we are
considering are in fact important. For some examples of this evidence see [16, 5] respectively.
7 Since the firm is competitive and cannot affect the price it receives by reducing its supply,
it will obviously be better off by selling its total ex ante output.
2. PROPERTIES OF THE COST FUNCTION

The firm produces a single output, and possesses a cost function of the form $C(y, z)$ where

$y =$ initial production plan made by the firm before the stochastic selling price is known,

$z =$ additional output produced by the firm after it learns the price.

The final output of the firm is $(y + z)$ and the cost function describes the total cost of producing this quantity when an amount $y$ is planned in advance and $z$ is produced ex post. We shall refer to $C(y, 0)$ as the planned costs, and the difference between this function and $C(y, z)$ as the adjustment costs. Note that since $y$ denotes the initial plan it is constrained to be non-negative; however, as long as symmetric adjustment is possible, this plan can be reduced so that $z$ may be negative. Since final output cannot be negative, we must necessarily have $y + z \geq 0$. The quantity $C(y, -y)$, the costs of producing zero output after an initial planned output of $y$, can be interpreted as the fixed or sunk costs corresponding to this plan. The difference between this amount and $C(y, z)$ can accordingly be viewed as the associated variable costs.

The following assumptions are made about the cost function:

**Assumption 1.** $C(0, 0) = 0$.

**Assumption 2.**

$C_1(y, z) > 0$, for all $y$ and all $z$ such that $y + z > 0$.\(^9\)

$C_2(y, z) > 0$, for all $y$ and all $z$ such that $y + z > 0$.

**Assumption 3.** The matrix \[
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\] is positive definite.

**Assumption 4.** $C(y, z) > C(y + z, 0)$ for all $y$, and for all $z \neq 0$.

The first assumption asserts that the total cost of producing nothing, when this was the original plan is zero. The first part of Assumption 2 asserts that the marginal cost of increasing plans is positive, the second part asserts that the marginal cost of any ex post adjustment is positive. Interpreting $C(y, -y)$ as measuring fixed costs, it follows that $C_2(y, -y)$ equals the marginal cost of producing an infinitesimally small final output, having planned for $y$. This must clearly be non-negative and the positive definiteness of $(C_i)$ ensures that $C_2(y, z)$ is positive for all $y$ and $z$ such that $y + z > 0$, as asserted. The positive definiteness of the matrix in Assumption 3 implies that the cost function has the usual convexity properties. Finally, Assumption 4 formalizes the crucial assumption that ex post production is more costly than ex ante production. That is, it is cheaper to produce an amount $(y + z)$ when this amount is exactly the quantity that was planned in advance, than when a portion $z$ is produced at short notice after the price is known.

These assumptions—particularly Assumption 3 and Assumption 4—enable us to derive a number of properties of the cost function $C(y, z)$. From the second

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\(^8\) Note in this terminology planned costs are of course not the same as fixed costs in that the former include the variable costs incurred in producing the planned output.

\(^9\) Following the usual convention, subscripts applied to functions denote partial derivatives and primes denote derivatives.
mean value theorem, it follows that

\[(1a) \quad C(y, z) = C(y, 0) + C_z(y, 0)z + \frac{1}{2}C_{z^2}(y, \xi)z^2\]

\[(1b) \quad C(y + z, 0) = C(y, 0) + C_1(y, 0)z + \frac{1}{2}C_{11}(y + \eta, 0)z^2\]

where both \(\xi, \eta\) lie between 0 and \(z\) and using Assumption 4 we obtain

\[(2) \quad C_2(y, 0)z + \frac{1}{2}C_{22}(y, \xi)z^2 > C_1(y, 0)z + \frac{1}{2}C_{11}(y + \eta, 0)z^2.\]

Since this inequality is true for all \(z \neq 0\), when \(z > 0\), we derive

\[C_3(y, 0)z + \frac{1}{2}C_{22}(y, \xi)z^2 > C_1(y, 0) + \frac{1}{2}C_{11}(y + \eta, 0)z\]

and letting \(z \to 0\) through positive values yields

\[C_2(y, 0) \geq C_1(y, 0).\]

Repeating these steps in the case where \(z < 0\), we similarly deduce

\[C_2(y, 0) \leq C_1(y, 0)\]

and from these two inequalities we obtain the first property,

\[(3) \quad C_1(y, 0) = C_2(y, 0).\]

Substituting this result back into equation (2) and letting \(z \to 0\) (which implies that both \(\xi, \eta \to 0\)) we deduce a second result that

\[(4) \quad C_{11}(y, 0) \leq C_{22}(y, 0).\]

Hence the marginal cost of plans equals the marginal cost of \textit{ex post} adjustment at the point where \(z = 0\), although the latter increases at a faster rate at that point.

By differentiating (1a) with respect to \(y\), using result (3), differentiating again with respect to \(y\), and finally letting \(z \to 0\), we obtain a third result, use of which is made subsequently\(^{10}\)

\[(5) \quad C_{12}(y, 0) = C_{11}(y, 0).\]

This result, together with Assumption 3 implies that (4) must in fact hold with strict inequality, yielding

\[(4') \quad C_{11}(y, 0) < C_{22}(y, 0).\]

\(^{10}\) This procedure assumes that at least the first four derivatives of \(C\) exist and are finite. Actually the equality in (5) can be established by applying the first mean value theorem to \(C\), in which case only the first three derivatives need exist. Implicitly our entire analysis rests on the existence of the first three derivatives.
Further results can be obtained by expanding $C(y + z, 0)$ about the point $(y, z)$. Thus using the second mean value theorem we obtain

$$C(y + z, 0) = C(y, z) + [C_1(y, z) - C_2(y, z)]z + \frac{1}{2}[C_{11} - 2C_{12} + C_{22}]z^2$$

where the partial derivatives $C_{ij}$ are evaluated at a point $(y + \eta', \xi')$ such that both $\eta', \xi'$ lie between 0 and $z$. Hence Assumption 4 implies

$$(6) \quad [C_1(y, z) - C_2(y, z)]z + \frac{1}{2}[C_{11} - 2C_{12} + C_{22}]z^2 \leq 0,$$

with the equality holding for $z = 0$. Since the matrix $(C_{ij})$ is positive definite, it follows further that the second term on the left hand side of (6) is positive, implying that:

(7a) \quad $C_2(y, z) > C_1(y, z)$ \quad for $z > 0$

(7b) \quad $C_2(y, z) < C_1(y, z)$ \quad for $z < 0$.

These two inequalities can be given the following interpretation. The marginal cost of producing an additional unit of output is greater \textit{ex post}, than it is \textit{ex ante}. On the other hand, the reduction in cost resulting from cutting back on output is less when this reduction is made \textit{ex post} than when it is made in advance.

The inequality (6) immediately implies the following simple lemma.

**Lemma.** \textit{Under certainty, the firm will always choose $z = 0$.}

**Proof.** Under certainty the firm will choose its initial production plan, as well as its short run adjustment, at the point where their respective marginal costs equal the known, non-stochastic price $p$. That is,

$$C_1(y, z) = p$$

$$C_2(y, z) = p$$

implying $C_1(y, z) = C_2(y, z)$. Thus substituting into (6) it immediately follows from the positive definiteness of $(C_{ij})$ that $z = 0$, as was asserted. The significance of this result is that it enables us to interpret the planned cost curve $C(y, 0)$ to be the firm's cost curve under conditions of certainty.

At this point it may be useful to illustrate the cost function $C(y, z)$ graphically and this is done in Figure 1. Suppose that the entrepreneur plans to produce the quantity $OA$. The cost of this plan is denoted by the point $X$ on the planned cost curve $C(y, 0)$. The actual total costs incurred, including those resulting from changing these plans after the price becomes known, is illustrated by the curve $C(y, z)$ and there will be one such curve corresponding to each point on the planned cost curve $C(y, 0)$. Note that when $z = 0$, its slope is equal to that of $C(y, 0)$ and hence the two curves are tangents at $X$. This, of course is the meaning of equation (3). Note also, that since $C(y, z) > C(y + z, 0)$ the total cost curve lies above $C(y, 0)$ everywhere, except at the tangency point $X$. 


Suppose now that after learning the price, the firm wishes to produce an additional quantity \( AB \). In this case the total cost of doing so, having initially decided to produce \( OA \), will be \( BZ \). On the other hand, had the firm initially chosen to produce \( OB \), and therefore made the appropriate preparations at the outset, the total cost would have been only \( BY \). The difference \( YZ \) represents the additional costs brought about by the \textit{ex post} adjustment. Similarly we see that downward adjustments also involve the firm in additional costs.\(^{11}\)

Finally, it is useful to identify the two limiting cases of totally inflexible and perfectly flexible plans respectively. The former, which is the conventional assumption in stochastic models, is characterized by the condition

\[
C_2(y, z) \to \infty \quad \text{for } z \neq 0.
\]

If complete flexibility exists then Assumption 4 is replaced by an equality

\[
C(y + z, 0) = C(y, z) \quad \text{for all } y, z.
\]

so that total cost of producing a quantity \( y + z \) is the same irrespective of how much was planned in advance, in which case the total cost curve coincides with the planned curve. Thus it immediately follows that inequality (6) holds with equality for all \( z \), implying in particular that

\(^{11}\) Note that in the asymmetric case, where the originally planned production is actually carried out before the price is known, downward adjustment is not economically sensible and only that portion of the \( C(y, z) \) curve which corresponds to \( z > 0 \) would apply.
\[ C_1(y, z) = C_2(y, z) \quad \text{for all } y, z. \]

3. BEHAVIOR OF RISK NEUTRAL FIRM

Suppose that the initial production plan \( y \) has been decided. Once the price is known, the firm is back in a world of certainty and having made its initial plan \( y \), the optimal additional output \( z \) that it will wish to produce is determined according to the usual marginal condition, namely

\[ C_2(y, z) = p \]

where the selling price \( p \geq 0 \). There are, however, two complications that must be considered. First, since the firm cannot reduce its output by more than it had originally planned to produce, this equation can hold only provided \( y + z \geq 0 \). If the solution to (11) yields a value of \( z < -y \), then it follows from the fact that for given \( y \), profit is a concave function of \( z \), that the firm will set \( z = -y \). Furthermore, the convexity of \( C \) implies that \( z \geq -y \), if and only if \( p \geq C_2(y, -y) \).

Secondly, we must also take account of the fact that \( \text{ex post} \) (under certainty) the firm will produce only if it can cover its variable costs. As discussed in Section 2, these equal \( C(y, z) - C(y, -y) \) so that we also require

\[ p \geq \min_z \left[ \frac{C(y, z) - C(y, -y)}{y + z} \right]. \]

From the convexity of \( C(y, z) \) it can be easily shown that marginal costs (with respect to \( z \)) always exceed average variable costs. Hence this constraint is always satisfied and does not impose any further restrictions on the adjustment.

Defining\(^{12}\)

\[ p_n = C_2(y, -y) \geq 0 \]

the optimal \( \text{ex post} \) adjustment in \( z \) is described by

\begin{align*}
(13a) \quad & C_2(y, z) = p \quad \text{if } p \geq p_n \\
(13b) \quad & z = -y \quad \text{if } p < p_n
\end{align*}

and solving these equations, yields the following functions for \( z \)

\begin{align*}
(14a) \quad & z = z(y, p) \quad \text{if } p \geq p_n \\
(14b) \quad & z = -y \equiv z(y, p_n) \quad \text{if } p < p_n.
\end{align*}

Thus equations (13) or (14) determine the optimal adjustment that the firm will make to its original production plan, once the price is known.

The remaining problem—and indeed the basic problem—is to determine the

\(^{12}\) Since there is nothing in our previous assumptions to ensure that \( p \geq p_n \), explicit consideration must be given to the possibility that \( p < p_n \). Presently we shall show that by imposing a mild restriction on the probability distribution of \( p \) the case \( p < p_n \) can be ruled out.
optimal production plan itself, taking into account the \textit{ex post} adjustment. Using (14), it follows that profit $\Pi$ is given by
\begin{equation}
\Pi(y, p) = p[y + z(y, p)] - C[y, z(y, p)] \quad \text{if } p \geq p_n
\end{equation}
\begin{equation}
\Pi(y, p_n) = -C[y, -y] \quad \text{if } p < p_n
\end{equation}
and the expected profit is
\begin{equation}
E[\Pi] = -\int_0^{p_n} C(y, -y) dF(p) + \int_{p_n}^{\infty} \{p[y + z(y, p)] - C[y, z(y, p)]\} dF(p)
\end{equation}
where $E$ denotes expected value and $F(p)$ is the probability distribution function describing the random selling price $p$.\textsuperscript{13} Thus the risk neutral firm makes its decisions by maximizing (16) with respect to $y$, taking into account the adjustment described by (13). Omitting details of the calculation, we obtain the optimality condition\textsuperscript{14}
\begin{equation}
-\int_0^{p_n} [C_1(y, -y) - C_1(y, -y)] dF(p) + \int_{p_n}^{\infty} [p - C_1(y, z)] dF(p) = 0.
\end{equation}
Denoting expected price by $\bar{p}$ and letting
\begin{equation}
I = \int_0^{p_n} [C_2(y, -y) - p] dF(p) \geq 0
\end{equation}
the optimality condition can be written as
\begin{equation}
E[C_1(y, z)] = \bar{p} + I.
\end{equation}
Thus we see that the firm should select its initial production plan by equating expected marginal costs (taking into account \textit{ex post} adjustment) to a quantity \textit{exceeding} expected price, where this quantity depends upon the probability that $p < p_n$. If this probability is zero, then $I = 0$ and we obtain the condition that expected marginal costs equal expected price.

There are several questions that we wish to consider in connection with the optimal solution.

(i) How does planned production under uncertainty ($y_n$) compare to output under certainty ($y_c$)?

(ii) Denoting the firm's actual output under uncertainty by $O_n$, is its expected output $E(O_n)$ greater or less than the output under certainty ($y_c$)?

(iii) What is the effect of uncertainty on the firm's expected profit?

In making such comparisons we must first decide what certainty output we wish to use. There is no unambiguous answer to this question and the proce-

\textsuperscript{13} The fact that $dF(p)$ is not conditional on $y$ or $y+z$ is a consequence of the assumption that we are dealing with a perfectly competitive firm.

\textsuperscript{14} The second order condition requires the second derivative of (16) with respect to $y$ to be positive. It can be readily verified that the convexity of $C(y, z)$, together with the \textit{ex post} adjustment (13), ensures it will be satisfied.
dure, which we follow here is the plausible one of assuming \( y_c \) to be the output that would result if \( p = p_n \) with certainty.

In order to proceed further it is convenient to write the first order condition (17) in the form

\[
\frac{\partial E(\Pi)}{\partial y} = \int_0^\infty \phi(p, y) dF(p) = 0
\]

where

\[
\phi(p, y) = -C_1(y, -y) + C_2(y, -y) \quad \text{if } p < p_n
\]
\[
= p - C_1[y, z(y, p)] \quad \text{if } p \geq p_n.
\]

The solution to the first question, which in turn affects the answer to (ii) and (iii), depends upon the shape of the function \( \phi(p) \) (for given \( y \) and thus considered as a function of \( p \)). In particular we make use of the following fundamental result, derived from Jensen's inequality.\(^{15}\)

(i) If \( \phi(p) \) is concave in \( p \) then \( y_n \leq y_c \), with strict inequality holding when \( \phi(p) \) is strictly concave. Moreover, further increases in uncertainty will lead to further reductions in planned output.

(ii) If \( \phi(p) \) is convex in \( p \) then \( y_n \geq y_c \), with strict inequality holding when \( \phi(p) \) is strictly convex. Moreover, further increases in uncertainty will lead to further increases in planned output.

(iii) If \( \phi(p) \) is neither concave nor convex then the response to uncertainty is ambiguous.

The following observations can be made about the function \( \phi(p) \).

(i) \( \phi(p) \) is continuous for all \( p \), including \( p = p_n \).

(ii)

\[
\begin{align*}
\phi'(p) &= 0 \quad \text{for } p < p_n, \\
\phi'(p_n^-) &= 1 - \frac{C_{12}(y, z)}{C_{22}(y, z)} \quad \text{for } p > p_n,
\end{align*}
\]

so that \( \phi'(p) \) is discontinuous at \( p = p_n \).\(^{16}\)

(iii)

\[
\begin{align*}
\phi''(p) &= 0 \quad \text{for } p < p_n, \\
&= \frac{C_{12}C_{22} - C_{12}C_{22}}{C_{22}^3} \quad \frac{\partial}{\partial z}\left[ \frac{C_{12}}{C_{22}} \right] \quad \text{for } p > p_n.
\end{align*}
\]

None of our previous assumptions imposes any restrictions on \( \phi''(p) \) and it is clear that either sign is possible. Consequently we consider the following cases in turn (see Figure 2).

\(^{15}\) For a discussion of the use Jensen's inequality in this connection see Rothschild and Stiglitz [13, 14].

\(^{16}\) \( \phi'(p_n+) \) is the right hand derivative of \( \phi(p) \) at \( p_n \); i.e., the limit as \( p \) tends to \( p_n \) through positive values; \( \phi'(p_n-) \) is defined analogously.
Figure 2
Shape of $\phi(p)$
\[
\phi''(p) < 0 \text{ for } p > p_n.
\]

This implies that \( \frac{\partial}{\partial z} \left[ \frac{C_{12}}{C_{22}} \right] > 0 \) which together with (4') and (5) yields

\[
C_{12}(y, -y) - C_{22}(y, -y) = \frac{d}{dy} C_2(y, -y) < 0
\]

so that \( \phi'(p_n+) > 0 \). If follows further from (23) and (3) that for \( y > 0 \)

\[
C_2(y, -y) < C_2(0, 0) = C_1(0, 0).
\]

We now introduce an additional assumption

**Assumption 5.** \( p \geq C_1(0, 0) \) with probability one. This assumption asserts that price must always exceed the marginal cost of planned output at \( y = 0 \). This imposes a lower bound (in general strictly positive) on the range of \( p \), but in view of the convexity of \( C(y, 0) \), this is unlikely to be a severe restriction (see Figure 1). Assuming Assumption 5 we obtain the inequalities

\[
p \geq C_1(0, 0) > C_2(y, -y) = p_n \quad \text{for all } p
\]

so that the shape of the cost function, together with the restriction on prices rules out the possibility that \( p < p_n \). We therefore deduce that \( \phi(p) \) is strictly concave for all relevant \( p \) and we conclude that \( y_n < y_c \); production plans are reduced under uncertainty. Moreover, production plans decrease in response to further increases in uncertainty.

\[
\phi''(p) = 0 \text{ for } p > p_n.
\]

If Assumption 5 holds then we again deduce that \( p \geq p_n \) for all \( p \). Thus \( \phi''(p) = 0 \) for all \( p \), so that \( \phi(p) \) is linear in \( p \), over the relevant range implying that \( y_n = y_c \), so that plans are unaffected by uncertainty.

If Assumption 5 does not hold and there is a positive probability that \( p < p_n \), then the statements made in (i) and (ii) no longer hold. In that case, if \( \phi''(p) < 0 \text{ for } p > p_n \), the function \( \phi(p) \) is neither concave nor convex and the relationship of \( y_n \) to \( y_c \) becomes indeterminate. Moreover, if \( \phi''(p) = 0 \text{ for } p > p_n \), then the function \( \phi(p) \) has a kink at \( p = p_n \) and becomes convex (but not strictly, since it is now piece-wise linear) implying that \( y_n \geq y_c \).

\[
\phi''(p) > 0 \text{ for } p > p_n.
\]

The third case \( \phi''(p) > 0 \text{ for } p > p_n \) is even more ambiguous since we can now no longer ensure that

\[
C_{22}(y, -y) - C_{12}(y, -y) > 0.
\]

However, if this inequality does hold, then irrespective of Assumption 5 it follows that \( \phi(p) \) is convex implying that \( y_n > y_c \). On the other hand if the inequality is reversed then \( \phi'(p_n+) < 0 \text{ and } C_2(y, -y) > C_2(0, 0) = C_1(0, 0) \) so that even if Assumption 5 holds, \( p < p_n \) for some \( p \). In this case \( \phi(p) \) is neither
convex nor concave over the entire relevant range of \( p \) and hence, the response of \( y \) to uncertainty becomes ambiguous. While the reversal of inequality (26) seems unlikely, nevertheless it cannot be ruled out on an \textit{a priori} grounds.

While all these cases—and indeed others where \( \phi''(p) \) changes sign with \( p \)—are all possible, it is worthwhile pausing to consider which is most likely to occur. The crucial factor in determining the likely behavior of the firm is the sign of \( \partial / \partial z [C_{12}/C_{22}] \) and unfortunately this is hard to evaluate. Case (i) will arise, if for example, the cost function is quadratic in \( z \) and the rate at which the marginal costs of adjustment increase also increases with the original plans (i.e. \( C_{221} > 0 \)). Figure 1 would suggest that this is a most plausible case. The second case occurs if say \( C_{22} \) is constant, so that \( C_{221} = 0 = C_{222} \). Finally, the third case will arise if costs are a positive cubic function in \( z \) (i.e., \( C_{222} > 0 \)) and if \( C_{221} \) is zero or perhaps mildly positive. On balance, we probably have a slight preference for case (i), although it is clear that any one of these possibilities could quite easily arise. Thus, while our analysis leads to the conclusion that in general a risk neutral firm, possessing \textit{ex post} adjustment possibilities, \textit{will} alter its initial production plan in response to uncertainty, we cannot make any unambiguous statement as to the direction of this response.

Actual output under uncertainty is simply given by \( o_n = y_n + z \) so that expected output is \( E(o_n) = y_n + E(z) \) where the expectation takes into account the asymmetry when \( p < p_n \). Thus the comparison of expected output with output under certainty involves determining the sign of \([y_n - y_c + E(z)]\). With some manipulation we obtain the following expression\(^{17}\)

\[
E(o_n) - y_c = \frac{1/2C_{12}(y_n, 0)(y_n - y_c)^2 - 1/2(I_1 + I_2) + I}{C_{12}(y_n, 0)}
\]

where \( \eta \) lies between \( y_n \) and \( y_c \), \( I \) is defined in (18) and

\[
I_1 = \int_0^{p_n} C_{122}(y_n, \xi_1)\xi_1^2dF(p)
\]

\[
I_2 = \int_{p_n}^{\infty} C_{122}(y_n, \xi_2)\xi_2^2dF(p)
\]

with \( -y_n < \xi_1 < 0 \) and \( \xi_2 \) between \( 0 \) and \( z \). It implies that in general the expected output of a risk neutral firm will differ from that under certainty and that their relative magnitudes depend critically upon the shape of the cost function. For example, if \( C_{122} = 0 \), so that the rate at which the marginal adjustment costs increase is independent of the initial plans, and if \( C_{111} > 0 \), then expected output will exceed output under certainty. On the other hand, if the cost function is quadratic in plans and \( C_{122} > 0 \), and the asymmetric portions do not arise, the opposite will be true.

Despite the indeterminacies in the comparisons of \( y_n \) or \( E(o_n) \) with \( y_c \), we can deduce unequivocably that expected profit under uncertainty will exceed

\(^{17}\) Briefly, this is obtained by applying the second mean-value theorem to the optimality condition (19), recognizing that under certainty \( C(y_c, 0) = p \) and finally making use of the equality in (5).
profit received under certainty. Since \( y_n \) is the expected profit maximizing value of planned output, \( E[H(y_n, p)] \geq E[H(y_e, p)] \). From (15) we see that provided \( z \neq 0 \), \( H \) is convex in the random variable \( p \) so that Jensen's inequality implies \( E[H(y_e, p)] \geq H(y_e, p) \), and combining these two inequalities we immediately deduce \( E[H(y_n, p)] \geq H(y_e, p) \). This result generalizes the same conclusion reached earlier by Oi [11] which he derived in the case where the firm has perfect flexibility and can delay all its decisions until after the price is known.

Finally, it is of interest to consider the implications of the two limiting cases of zero and infinite flexibility. The former is characterized by \( C_2(y, z) \to \infty \) for \( z \neq 0 \) and since the \textit{ex post} adjustment is obtained by equating \( C_2 \) to the finite selling price \( p \), it follows that \( z = 0 \). We are thus back to the case considered by Sandmo and others. With perfect flexibility, on the other hand, it immediately follows from (10) and (13) that the first order condition (17) vanishes identically, implying that the initial plan is indeterminate. This is an expected result; if the firm can produce as cheaply after as it can before it learns the price and if it can adjust its initial decisions in either direction it clearly does not matter what it initially plans to do.\textsuperscript{18}

4. THE CASE OF RISK AVERSION

We now drop the assumption of risk neutrality and assume instead that the firm evaluates its profits in terms of a concave utility function \( U(H) \), possessing the properties:

\[
U'(H) > 0, \quad U''(H) < 0
\]

(28)

reflecting an attitude of risk aversion. The behavior of the risk attracted firm can be treated similarly and brief reference to the corresponding results for that case will be indicated in footnote 21 below.

The firm's objective is to choose its initial plan so as to maximize expected utility, taking into account its \textit{ex post} adjustment. Analogous to the previous section, the second stage optimization implies an adjustment up to the point where the marginal utility of \( z \) equals price (assuming \( z \geq -y \)). Since \( U'(H) > 0 \), this implies that the \textit{ex post} adjustment is given by the identical expression to that under risk neutrality,

\[
C_2(y, z) = p \quad \text{if} \quad p \geq p_a
\]

(13a')

\textsuperscript{18} Essentially the same implications are obtained for the asymmetric case where the initially selected \( y \) is actually produced before the selling price is known so that the firm will not wish to exercise any downward \textit{ex post} adjustment. There are, however, two differences worth noting.

The first occurs if \( \partial(C_12/C_0)/\partial z = 0 \), which under these circumstances can be shown to imply a decrease in the firm's \textit{ex ante} production under uncertainty. This contrasts with the result obtained previously for the symmetric case, where it was shown that with the same cost function, such a firm's plans remain unaffected by uncertainty if Assumption 5 was applicable and that they would increase with uncertainty if Assumption 5 did not hold. Secondly, with perfect flexibility upwards, the asymmetric model implies \( y = 0 \), whereas \( y \) is indeterminate with perfect flexibility in the symmetric case.
(13b')  
\[ z = -y \text{ if } p < p_a \]

where we now let  
(29)  
\[ p_a = C(y, -y). \]

Of course it is clear that since the plans chosen by a risk averse firm are unlikely to equal those made under risk neutrality, \( p_a \) as we have defined it in (29) need not equal \( p_a \) as defined previously and that is the reason for introducing a new symbol.

Thus profit is still given by the expression (15) so that expected utility is  
(30)  
\[ E[U(\Pi)] = \int_0^{p_a} U[\Pi(y, p_a)] dF(p) + \int_{p_a}^{\infty} U[\Pi(y, p)] dF(p) \]

where the subscript \( a \) is analogous to the subscript \( n \) used earlier for the risk neutral firm. Differentiating (30) with respect to \( y \), and taking into account the definition of \( z \), results in the following first order condition  
(31)  
\[ \theta(y) \equiv \int_0^{p_a} U'[\Pi(y, p_a)][C_1(y, -y) - C_1(y, -y)] dF(p) + \int_{p_a}^{\infty} U'[\Pi(y, p)][p - C_1(y, z)] dF(p) = 0 \]

which determines the initial production plan.

Our task is to compare the decisions implied by (31) with those investigated earlier for a risk neutral producer. In making the comparison we assume that the two firms are identical in all respects except for their attitude to risk. We append subscripts \( a, n \) to both \( y \) and \( z \) to identify the quantities with the risk averse and risk neutral firm respectively. Thus, the optimal production plan of the risk averse firm is obtained by solving the following equation for \( y_a \)

(31')  
\[ \theta(y_a) = 0. \]

Because of the second order condition, we know that \( y_n \gtrless y_a \) according as \( \theta(y_n) \lesssim \theta(y_a) \) and thus the comparison depends crucially upon evaluating \( \theta(y_n) \). The argument we use is based upon one first used by Leland. By the concavity of \( U \), we know that  
\[ U'[\Pi(y_n, p)] \gtrless U'[\Pi(y_n, p)] \]

and hence taking the expected value it follows that:

\[ \theta(y_n, \rho) \lesssim 0. \]

The second order condition requires \( \theta(y_n, \rho) < 0 \). The concavity of \( U(\Pi) \), together with the convexity of \( C(y, z) \) ensures that \( \theta(y) < 0 \) for all \( y \) (and in particular \( \theta(y, \rho) < 0 \)), and hence the statement made in the text follows. It is worth noting that provided the firm is sufficiently risk averse, a unique equilibrium can be reached even where the cost function is concave, so that the firm has decreasing marginal costs. Moreover, with constant marginal costs equilibrium will be unique for any degree of risk aversion. On the other hand, for any given convex cost function, there is a limit to the degree of risk attraction the firm may have, which is consistent with the second order condition.
(32) \[ E[U'(\Pi(y_n, p))(p - \bar{p})] = \text{cov}(U', p) \leq 0. \]

As we indicated in Section 3, the optimality condition for a risk neutral firm may be written in the form

(33) \[ E[C_1(y_n, z_n)] = \bar{p} + I \]

where \( I \geq 0 \) is defined in equation (18). Substituting from (33) into (32) we deduce that

(34) \[ E[U'(\Pi(y_n, p))(p - E(C_1) - I)] \leq 0 \]

from which it follows that

(35) \[ \theta(y_n) + \text{cov}(U', C_1(y_n, z_n)) - IE[U'] \leq 0. \]

Note that in the absence of \textit{ex post} flexibility, \( C_1 \) is non-stochastic, so that \( \text{cov}(U', C_1) = 0 \). Likewise \( I = 0 \), so that (35) implies \( \theta(y_n) \leq 0 \) and therefore that \( y_n \geq y_a \), confirming the Sandmo-Leland result. However, in general when the firm has production flexibility this conclusion need not hold. Because of the concavity of \( U \), as \( p \) increases, \( U' \) decreases; at the same time an increase in \( p \) increases \( z_n \) and provided \( C_{12} > 0 \), as it almost certainly is, it will lead to an increase in \( C_1 \). Thus \( \text{cov}(U', C_1) \) is almost certainly negative and we shall assume this to be the case. Furthermore, since \( I > 0 \), \( U' > 0 \), the third term on the left hand side of (35) is also positive and thus all we can deduce from this equation is that \( \theta(y_n) \leq \varepsilon \) where \( \varepsilon \) is some positive number.

Thus we reach the conclusion that in the presence of production flexibility, an attitude of risk aversion on the part of the firm may lead it to either decrease or increase its production plan over what it would select if it were neutral to risk. Furthermore, the magnitude of \( \varepsilon \) and therefore the possibility that \( y_n > y_a \) increases with

(i) the magnitude of \( C_{12} \)

(ii) the probability that \( p < p_n \) and consequently the magnitude of \( I \). Note that if \( C_{12} \leq 0 \) and \( I = 0 \), then of course the Sandmo-Leland result would still apply, but there is no reason to expect this to be so, and indeed upon further reflection our result seems perfectly plausible.

Suppose \( C_{12} > 0 \). Then the marginal cost of making adjustments of a given magnitude \( z \) increases with the size of the original plans \( y \). The risk encountered by the firm is contracted as soon as it commits itself to a production plan before the price is known and with \textit{ex post} adjustment possibilities it is by no means clear that a reduction in planned activity will lead to a reduction in risk. By increasing its original production plan, the firm reduces the likelihood that it will eventually wish to increase its output over and above what was originally planned. At the same time, of course, it increases the chances that it will wish to make an \textit{ex post} downward adjustment. However, because of the convexity

\(^{20}\) As \( C_{12} \) increases, the term \( \text{cov}(U', C_1(y_n, z_n)) \) becomes more negative. Since the left hand side of (35) equals \( \text{cov}(U', p) \) which is independent of \( C_{12} \), it follows that as \( \text{cov}(U', C_1) \) becomes more negative, \( \theta(y_n) \) becomes more positive (or at least less negative).
of the cost function, it follows that
\[(36) \quad C(y, z) - C(y, 0) > C(y, 0) - C(y, -z)\]
that is, the marginal cost of increasing plans by a given amount exceeds the marginal savings obtained if these plans are reduced by the same amount. Thus the risk averse firm will wish to minimize the risks of having to make relatively costly upward adjustments and will therefore tend to increase its plans at the outset. Furthermore, the possibility that \( p < p_n \) provides an added incentive to increase initial plans. The amount of downward flexibility is limited by the original plans, thus by increasing these initial decisions the firm is able to increase its overall flexibility, thereby reducing its risk.

In summary then, both these effects combine to raise the possibility that a risk averse firm may actually plan to produce more than if it were risk neutral.\(^{21}\)

To determine how the expected output of a risk averse firm compares with what it would be if it were risk neutral, we compare \( E(o_a) \) with \( E(o_n) \). These are given by
\[(37a) \quad E(o_a) = \int_{p_a}^{\infty} (y_a + z_a)dF(p)\]
\[(37b) \quad E(o_n) = \int_{p_n}^{\infty} (y_n + z_n)dF(p)\]
where \( z_n, z_a \) are the solutions to (13a) and (13a') respectively. Subtracting, equating (13a) and (13a') and applying the first mean value theorem yields
\[(38) \quad E(o_a) - E(o_n) = (y_a - y_n) \int_{p_n}^{\infty} [1 - C_{12}/C_{22}]dF(p)\]
\[\quad + \int_{p_a}^{\infty} (y_a + z_a)dF(p)\]
where \( C_{12}, C_{22} \) are evaluated at a point between \( y_a \) and \( y_n \). To consider (38) further let us introduce the assumption
**Assumption 6.** \( C_{22} \geq C_{12} \) for all \( y, z \).

From equation \((4')\) this inequality holds strictly when \( z = 0 \) and it does not seem too unreasonable that it should hold elsewhere as well. It certainly will hold for small \( z \)—i.e., for small risks. Note that if both Assumption 5 and Assumption 6 hold, then equalities (23)—(25) are satisfied, thereby excluding

\(^{21}\) In the case of risk preference (i.e., a convex utility function) (35) becomes,
\[\theta(\gamma_a) + \text{cov}(U', C_t) - I E[U'] \geq 0.\]
In the absence of flexibility, this inequality becomes \( \theta(y_a) \geq 0 \), and provided the second order condition holds, this implies \( y_a \leq y_p \), the output under risk preference. If \( C_{12} > 0 \), then \( \text{cov}(U', C_t) > 0 \) so that the bound on \( \theta(y_a) \) becomes indeterminate. Consequently the comparison between \( y_n \) and \( y_p \) tends to be more ambiguous than the corresponding comparison between \( y_n \) and \( y_a \).
the possibility that \( p < p_n \) or \( p_a \), in which case the second integral on the right hand side of (38) vanishes. Assuming Assumption 6 (but not necessarily Assumption 5) then (23) immediately follows so that from the definitions of \( p_n \) and \( p_a \), we infer that \( \text{sgn}(y_a - y_n) = \text{sgn}(p_n - p_a) \). Thus if \( y_a > y_n \), then
\[
\int_{p_a}^{p_n} (y_a + z_a)dF(p) > 0
\]
implying that \( E(o_a) > E(o_n) \); signs are reversed when \( y_a < y_n \), implying that
\[E(o_a) \geq E(o_n)\]
according as \( y_a \approx y_n \).

Hence, provided Assumption 6 holds, we conclude that if risk aversion induces a firm to increase (decrease) its planned output over what it would choose if it were risk neutral, then its expected output will increase (decrease) as well.

The final question we wish to consider is the comparison of the planned and expected outputs of a risk averse firm with the output which would be produced under certainty. We have already mentioned the well known result that in the case of no flexibility \( y_n = y_c \), so that comparisons made between \( y_a \) and \( y_n \) in that case apply between \( y_a \) and \( y_c \) as well. However, one of the main results of earlier sections is that in general \( y_n \neq y_c \) so that the comparison becomes more complicated. In fact we can show that the relative sizes of \( y_a, y_n, \) and \( y_c \) as well as \( E(o_a), E(o_n) \) and \( y_c \) are in general ambiguous.

For example, consider the case where both Assumption 5 and Assumption 6 hold. Suppose further that the firm's cost function possesses the property that \( C_{111} > 0, C_{222} > 0 \)—the marginal costs of both plans and adjustment increase at an accelerated rate—and also that \( C_{122} = 0 \). In that case it follows from (22) that \( \phi(p) \) is convex, implying that \( y_n > y_c \), while (27) implies \( E(o_n) > y_c \). Furthermore, provided \( C_{12} \) is sufficiently large, then it follows from (35) that it is possible for \( y_a > y_n \), in which case it would also be true that \( E(o_a) > E(o_n) \). Thus combining these results, this set of assumptions imply

\[
\begin{align*}
\{ & y_a > y_n > y_c \\
& E(o_a) > E(o_n) > y_c 
\}
\end{align*}
\]
so that a risk averse producer would increase both his plans and expected output under uncertainty.

On the other hand, if \( C_{111} = 0 = C_{222} \) but \( C_{122} > 0 \), then \( y_n < y_c \) and \( E(o_n) < y_c \). If further \( C_{12} \) is only small so that \( y_a < y_n \), then \( E(o_a) < E(o_n) \) and the pair of inequalities in (39) will be reversed. Similarly, other cases can give rise to partial changes in the ordering as well as indeterminacies between various pairs and many such examples can be readily constructed.

5. CONCLUSIONS

This paper has extended the theory of the perfectly competitive firm under price uncertainty to deal with the case where the firm has production flexibility, being able to modify its initial decisions, at extra cost, after it learns the true
value of the selling price. The presence of such *ex post* adjustment possibilities is shown to lead to significantly different implications from those derived in the usual case where, once made, the firm’s initial production decisions (made under uncertainty) are irrevocable.

The main conclusions of this paper are as follows:

(i) Unlike the conventional case, a risk neutral firm possessing *ex post* adjustment possibilities *will* in general alter its production plan under uncertainty. No unambiguous statement can be given as to the direction of this response; that will depend upon the exact nature of the firm’s cost function and under almost equally plausible sets of assumptions it is possible for its planned output level to be larger, smaller or remain the same as it would be under certainty.

(ii) The expected output of a risk neutral firm (i.e., its initial planned output plan plus the expected *ex post* adjustment) in general does not equal its output under certainty, and again the comparative magnitudes are ambiguous, depending upon the shape of the cost function.

(iii) An aversion to risk does not necessarily lead the firm to reduce its productions plan below what it would choose if it were risk neutral and the opposite response may occur. Indeed it is argued that with production flexibilities it is quite plausible to expect a firm to *increase* its planned output if it has an aversion to risk, since with the ability to modify initial decisions it is by no means clear that risk is reduced by decreasing initial commitments.

(iv) The same comments apply to expected output and under plausible conditions it is shown that the expected output of a risk averse firm is greater (less) than that under risk neutrality according to whether this change in attitude to risk induces it to increase (decrease) its planned output.

(v) Combining the results of (i)—(iv) we show that both the planned and expected outputs of a risk averse firm may be either greater or less than its output under certainty, depending upon the precise properties of its cost function and in many cases no comparative statement may be possible.

*Australian National University, Australia*

REFERENCES


