Econ 655 Fall 2002 (given in Spring 2003!)
Lapan

Final Exam

Answer any four questions. Answer All Parts to Each Question.

1) Answer all parts.

a) Consider a standard Ricardian model with 2 goods (C, F) and 2 countries (US, Mexico). Let labor productivities in the US be given by: \(a_L^C, a_L^F\) and labor productivities in Mexico be given by: \(\bar{a}_L^C, \bar{a}_L^F\). Suppose that: \(a_L > \bar{a}_L, \ i = C, F\) so that the US has an absolute advantage in both goods. Let aggregate labor stocks in each country be given by \(L, \bar{L}\).

i. What determines the pattern of trade between the two countries, and how does trade affect the real wage (as compared to autarky)?

ii. Derive the world production possibility frontier, and show that aggregate autarky output levels will lie inside this frontier.

b) Consider the Dornbusch-Fischer-Samuelson model with a continuum of goods indexed by \(z \in [0,1]\). Let \(a(z), \bar{a}(z)\) denote labor productivity in the US and Europe, respectively, and, without loss of generality, assume \(A(z) = \left[ a(z)/\bar{a}(z) \right]\) is a decreasing function of \(z\). Finally, let \(L, \bar{L}\) denote the aggregate labor stocks in the US and Europe, and assume identical Cobb-Douglas preferences across countries.

i. Derive the free trade equilibrium trade pattern, and real wages in each country.

ii. Show how a uniform increase in productivity in the US will affect the pattern of trade, prices, and real wages in each country. Are the results you find concerning the impact of productivity increases on US and European real income general (i.e., will they necessarily hold for other demand structures)?

c) Discuss the difficulties that arise in attempting to predict trade patterns when there are more than two countries (and a continuum of goods).

i. Consider the special case of 3 countries (US, Europe, China) and a continuum of goods indexed by \(z \in [0,1]\). Let the labor productivities be as follows:

US: \(a^{US}(z) = z\); Europe: \(a^{E}(z) = (1-z)\); China: \(a^{C}(z) = (u/2), \ u > 0\)

Assuming all consumers have identical Cobb-Douglas preferences (with the same share spent on each good) derive the equilibrium equations that determine the trade pattern (you do not have to solve). For this special case, what can you conclude about the trade pattern? (i.e., which goods will the US export, which goods will Europe export and which goods will China export). Be as specific as possible.
2) Consider a general equilibrium trade model with N goods and M factors.

Assume there are J competitive firms, each with its own convex production set \( \Upsilon^j \), that allows the firm to produce the N final goods using the M primary factors of production. Let \( \bar{y}^j \in \mathbb{R}_+^N \) denote the output vector of the firm, and \( \bar{v}^j \in \mathbb{R}_+^m \) denote the input vector of the firm, and suppose each firm’s production set is described by the set of \( \{ \bar{y}^j, \bar{v}^j \} \) such that:

\[
\frac{\partial g^j}{\partial y^j_i} < 0, \quad i = 1, \ldots, n \quad \text{and} \quad \frac{\partial g^j}{\partial v^j_k} > 0, \quad k = 1, \ldots, m.
\]

Each firm behaves as a competitive profit-maximizer, where \( (\bar{p}^j, \bar{w}^j) \) denotes the output, input price vector facing the firm. We assume there are no production externalities.

Let \( \bar{V}^T \) denote the aggregate input vector, which we take as exogenously given.

There are H consumers, who possess an exogenous endowment vector of the primary inputs, denoted \( (\bar{x}^h) \), and partial ownership in each firm \( (\theta^j) \), and who consume the N final goods, with convex preferences represented by \( U^h (\bar{c}^h) \), where \( \bar{c}^h \) denotes the household’s consumption vector. Let \( \{ \bar{p}^c, \bar{w}^c \} \) denote the output and input price vectors, respectively, facing households, and let \( T^h \) denote the net government transfer to households. Each household maximizes utility, subject its budget constraint:

\[
\sum_i p_i^c c_i^h \leq \left[ \sum_i w_i^c x_i^h + \sum_j \theta^j \Pi^j + T^h \right]; \quad \sum_j \theta^j = 1 \quad \forall j.
\]

where \( \Pi^j \) denotes the maximized profits of firm \( j \). The government budget constraint requires that net tax revenue be zero. In a pure laissez-faire equilibrium there are neither lump sum taxes nor commodity or factor taxes \( (T^h = 0, \quad \bar{p}^f = \bar{p}^c, \quad \bar{w}^f = \bar{w}^c) \).

Finally, the resource constraints facing the economy are:

\[
\sum_j \bar{y}^j \leq \bar{V}^T = \sum_h \bar{x}^h \quad \text{if there is no trade in inputs: and:}
\]

\[
\sum_h \bar{x}^h \leq \sum_j \bar{y}^j \quad \text{if there is no trade in goods (autarky)}.
\]

a) Under what conditions will individual profit maximization lead to GNP maximization, so that we can utilize the GNP function \( R(\bar{p}^f, \bar{V}^T) \)? Prove your answer.

b) Assuming no distortion and no domestic taxes, prove that the movement from autarky to trade leads to a potentially Pareto superior allocation, provided net tariff revenues are non-negative (and assuming lump sum transfers are feasible). Why do you need the assumption that lump
sum transfers are feasible?

c) Modify preferences so that household utility depends on consumption of primary inputs (denoted $z^h$) as well as consumption of final goods, so that the household optimization problem becomes: 

$$\max \left[ U(z^h, z^f) \right] \text{ such that:}$$

$$\sum_i p_i c_i^h \leq \left( \sum_i w_i \left( x_i^h - s_i^h \right) + \sum_j \theta_j^h \Pi^j + T^h \right).$$

Further, assume lump-sum taxes are not feasible ($T^h = 0$), that firm’s production sets exhibit constant returns to scale, and that world prices are exogenous.

ii. Assuming no government intervention, will the movement from autarky to trade benefit everyone? Prove that there exists government policies so that this movement is (potentially) welfare-improving for all people.

iii. Assume the government’s objective is to maximize social welfare, $S$ (where $S = \phi(U^1, \ldots, U^H)$, $(\partial S/\partial U^h) > 0$). Derive the optimal policy to achieve this objective; are trade restrictions part of the optimal policy? Explain. If only trade policy were feasible, would free trade maximize social welfare? {Note: if you cannot formally derive the optimal policy, discuss its characteristics}.

3) Answer all parts.

a) Consider the standard 2 good (C, M), 2 factor (K, L), 2 country (US, UK) Heckscher-Ohlin model. Technology in each country is given by:

$$Q^c_i = \lambda^i_c F^c_i \left( K^i_c, L^i_c \right); \quad Q^m_i = \lambda^i_m F^m_i \left( K^i_m, L^i_m \right); \quad i = \text{US, UK}$$

where the basic production functions $(F^c, F^m)$ exhibit constant returns to scale, and the specification indicates that the only possible difference in technology across countries is due to Hicks-neutral technological progress.

i. Assuming internationally identical technology $(\lambda^US_j = \lambda^UK_j, \quad j = C, M)$, and identical and homothetic preferences, state the main theorems of the H-O model concerning the pattern of trade and factor prices.

ii. Suppose $\lambda^US_c$ increases (so $\lambda^US_c > \lambda^UK_c$, $\lambda^US_m = \lambda^UK_m$); given output prices, show how this affects supply and factor prices in the US (relate to factor intensity). Will free trade in goods be able to replicate the integrated equilibrium? If not, how can this equilibrium be achieved? Explain.

b) Consider the extension of the Heckscher-Ohlin model to the case of $N$ goods and $M$ factors, where $N \geq M > 2$. Make all the usual assumptions, including identical and homothetic preferences, internationally identical technology which exhibits all the standard properties: constant returns to scale, non-joint production, etc., so that:
\[ Q_i = F_i (X_{i1}, \ldots, X_{iM}) ; \quad i = 1, \ldots, N \]

i. **Discuss what can be deduced about the pattern of trade for this model. Be specific and consider the case where \( N = M \) and the case where \( N > M \).**

ii. **How, if at all, can the basic Stolper-Samuelson theorem and the magnification theorem be extended (assume \( N = M \)). Be as specific as possible.**

iii. **Assume \( N > M \). Is factor price equalization possible? If so, is it more or less likely to occur in this case than when \( N = M \). Explain your answer.**

c) **Finally, consider the case of joint production. Suppose there are \( N \) constant returns to scale production processes (sectors) which use \( N \) inputs, but – unlike the basic H-O model – these production processes entail joint production:**

\[ H_i (y_{i1}, \ldots, y_{in}) \leq G_i (x_{i1}, \ldots, x_{in}) ; \quad i = 1, \ldots, n \]

where \( y_{ij} \) is output of good \( j \) in process (sector) \( i \), and \( x_{ik} \) is the use of input \( k \) in sector \( i \).

Each of the functions: \( H_i \) and \( G_i \) is homogeneous of degree one and:

\( \left( \frac{\partial H_i}{\partial y_{ij}} \right) > 0 ; \quad \left( \frac{\partial G_i}{\partial x_{ik}} \right) > 0 \quad \forall i, j, k \)

i. **Assuming internationally identical technology, will free trade in goods lead to factor price equalization? Will the Stolper-Samuelson theorem hold? Explain.**

ii. **Assuming identical technology and identical, homothetic preferences what can you conclude about the trade pattern? Discuss for the special case of \( N = 2 \).**

4) **Answer all parts.**

a) **Using a partial equilibrium model, demonstrate the equivalence between tariffs and quotas in a competitive setting.**

i. **Assume there is a single domestic firm, and that world prices are exogenous. Show how a non-zero quota affects domestic price and production. Compare the effects of the quota to that of a tariff that has the same impact on domestic price. Which policy leads to greater domestic output?; to higher profits for the domestic firm?**

b) **Much of the modern literature on protection emphasizes political economy explanations for protection. Assume that, due to political power, the government places a higher weight on producer surplus than on consumer surplus (or tax revenue). Thus, using a simple partial equilibrium framework, suppose the government’s objective function is to maximize the sum:**

\[ \lambda PS + (CS + TR) ; \quad \lambda > 1 \]

where \( PS, CS, TR \) stand for producer surplus, consumer surplus and tax revenue, respectively. **Assuming the good in question is imported, and that the feasible instruments are tariffs and domestic production subsidies, find the optimal policy (assume domestic producers behave**
competitively). In answering, be sure to consider both the case in which the country is a price taker on world markets, and the case in which its imports affect world prices.

i. How does the optimal solution change as \( \lambda \) increases?

c) Consider a model with one large country and many identical smaller countries. There are two goods, \( X \) and \( Y \), and the large (home) country exports \( X \) under free trade. Using either a partial or a (simple) general equilibrium model, derive the optimal tariff for the large country (the small countries, because they act non-cooperatively, pursue free trade).

i. Next, consider actions are made in the following sequence: (a) the large country government sets its tariff and production policies; (b) production decisions are made; (c) the large country government revises its tariff to maximize welfare (since production decisions are made, there is no incentive to revise production policies); and (iv) trade decisions are made. Assuming only tariffs are used, show how the optimal tariff and the resulting equilibrium are determined and compare to the optimal tariff derived above (with commitment).

ii. Given the sequence of actions described in part i, and given the inability for the government to commit to its tariff, does the domestic government have an incentive to use production policies? If so, what should they be (you do not need to formally prove your answer, but motivate your answer).

5) Answer all parts.

a) Consider a model with \( N \) goods, and consider a small economy that can trade at given world prices. Assume that there are no domestic market failures, but that this economy has a number of tariffs in place.

i. Show how a tariff reduction on only one good affects welfare. Under what conditions can you be sure the tariff reduction is welfare enhancing?

ii. Consider the same situation except assume all trade barriers are quotas, instead of tariffs. Under what conditions can you be sure that liberalizing (increasing) one quota will be welfare-enhancing? Prove your answer. Might your answer change if world prices were endogenous?

b) Suppose two countries (US, Mexico) form a free trade area (FTA), eliminating internal tariffs but maintaining external tariffs. Will this FTA raise welfare for both countries? Provide some analysis to support your conclusion.

i. Assuming that transfers between the two countries are permitted, can you devise a trade policy that would ensure both countries gain by eliminating internal tariffs? Be specific.

c) Using the two good (\( C, M \)), two factor (\( K, L \)) Heckscher-Ohlin model, discuss how an import tariff affects the domestic shadow price of capital (in answering this question you need not assume internationally identical technology). How would a limited inflow of foreign capital, which earns the domestic rate of return, affect domestic welfare? Explain. Would your answer change if the economy used a quota instead of a tariff?
6) Answer all parts.

a) Consider a partial equilibrium model in which there are two firms, one US and one Japanese. Both firms sell all their output in a third market. The firms produce differentiated products at constant (and equal) marginal cost (c), and compete by setting prices simultaneously (Nash-Bertrand behavior). The demands in the third market are given by:

\[ D^{US} = 20 - 2p^{US} + p' \]
\[ D' = 20 - 2p' + p^{US} \]

where \( D^{US} \) and \( D' \) are the demands in the foreign market for the US and Japanese produced goods, respectively, and \( p^{US} , p' \) are the prices charged by the firms.

i. Assuming that only the US government is policy active, derive its optimal policy (a specific answer is expected).

ii. Assuming both countries are policy active, derive the equilibrium. Would they both be better off if they could commit, via an international agreement, to no policy intervention?

iii. Assuming that the goods were sold in the US (instead of in a third country), and assuming the only feasible policy is trade policy, what is the optimal policy for the US?

b) Consider a standard two factor, two sector (M, F) H-O type model with CRS technology:

\[ Q_m = H^m (K_m, N_m); \quad Q_f = H^f (K_f, N_f) \]

Consider a small, developing country facing given world prices: \( \{P_f^w, P_m^w\} \). Assume good \( M \) is capital-intensive, and that under free trade and no distortions the country would export good \( F \). Inputs within the economy are mobile, but there is a minimum wage \( \bar{W}^m \) that must be paid in sector \( M \) (the urban, manufacturing sector); there is no restriction on the (rural) wage in sector \( F \) and, assuming no other government policies, the return on capital is the same in the two sectors. Since the minimum wage is set above the equilibrium wage, workers would rather work in sector \( F \) if they could find employment. We assume that people respond to the expected wage, so labor mobility implies:

\[ W^f = (1-u)\bar{W}^m \quad \text{where:} \quad (1-u) = (N_m/L_m); \quad N_f = L_f \]

where \( u \) is the urban (sector M) unemployment rate, \( N_i \) denotes employment in sector \( i \), and \( L_i \) denotes the number seeking employment in that sector. The resource constraints are:

\[ L_m + L_f = \bar{L}; \quad K_m + K_f = \bar{K} \]

i. Show how an increase in the minimum wage affects production of each good, the return on capital and the rural wage, and the unemployment rate.
ii. Given the minimum wage, how does an import tariff (on good M) affect the economy (factor prices, output, unemployment rate).

iii. How would an inflow of foreign capital, through foreign direct investment, affect the economy (output levels, factor prices and the unemployment rate). Assuming the foreign investors earned the domestic return on capital, would this inflow benefit the developing country?