If you took the midterm: Answer three questions; answer at most one question from Part I (you do not have to answer any question from Part I).

If you did not take the midterm: Answer one question from Part I and two questions from Part II.

Part I

1. Answer all parts

a) Consider the standard two good (M, F), two factor (K, L) Heckscher-Ohlin model. There are a large number \(2N\) countries of “similar” size. Endowments are given by \(\left(K^n, L^n\right), (n = 1, \ldots, 2N)\), and the countries are ordered such that: \(k^1 > k^2 > \ldots > k^{2N}\), where \(k^n \equiv \left(K^n/L^n\right)\). Assume good \(M\) is capital-intensive for all factor prices, and that technology is standard (CRS) and identical across countries. Also, preferences are identical and homothetic. Finally, assume that initially there is no trade between countries.

i. What, if anything, can be concluded when comparing autarky factor prices and goods prices across countries?

Next, assume the countries organize themselves into two trade blocs; by coincidence, countries 1 to \(N\) form trade bloc A, and countries \((N+1)\) to \(2N\) form bloc B. Free trade occurs within the bloc, but no trade occurs between blocs.

ii. Within each bloc, what – if anything – can be concluded about: (1) the trade pattern between any two countries; (2) the net trade pattern of each country (i.e., which good it exports and which it imports); and (3) factor prices across countries within the bloc? {Be precise in your answer but do not necessarily restrict yourself to the case in which all countries within a bloc are incompletely specialized}.

iii. In comparing the two blocs, what can you conclude about relative goods prices in the two blocs? Suppose countries \(N\) (in bloc A) and \(N+1\) (in bloc B) are quite similar in the endowments (though, of course, \(N\) is slightly more capital-abundant). What, if anything, can you say in comparing the returns to labor and capital between the countries?

iv. Finally, suppose \(A\) and \(B\) are discussing forming a single free trade bloc that includes all countries. (1) Will this alliance lead to a more efficient allocation (i.e., is the new allocation potentially pareto superior to the allocation with two blocs)? (2) In terms of per capita income, will all countries benefit from this super-free trade bloc? Clearly indicate which countries are likely to gain and which, if any, may lose. (3) Within a country, will everybody gain from trade? Be as specific as possible.
b) Consider a four good, two country Ricardian model with the following unit labor requirements:

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<tr>
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<th>Cloth</th>
<th>Planes</th>
<th>Autos</th>
<th>Steel</th>
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<tbody>
<tr>
<td>US</td>
<td>0.5</td>
<td>10</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>50</td>
<td>20</td>
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i. Let $W$ be the US real wage, and $\bar{W}$ the Chinese real wage. Find the production pattern in each country as a function of these real wages.

ii. Assume an initial equilibrium in which each country produced two goods. If Chinese labor productivity doubles, how will this productivity increase affect: (i) production patterns in each country; (ii) the real wage in each country; (iii) the relative prices of goods; and (iv) the trade pattern? Must both countries benefit from this productivity increase? Be as specific as possible.

iii. Suppose preferences in each country are given by: $U^h = (c^h \cdot p^h \cdot a^h \cdot s^h)$ where $C^h, P^h, S^h, A^h$ represent consumption of cloth, planes, steel and autos, respectively. Assuming China’s labor supply is three times that of the US, find the initial equilibrium wages in each country and then show how doubling China’s labor productivity alters real wages in each country {you may take as true that each country will produce two goods in the original equilibrium}.

2. Answer all parts.

a) Consider a Heckscher-Ohlin world with three goods ($M, F, C$) and two factors ($K, L$), where good $M$ is the most capital-intensive and good $C$ is the least capital-intensive. Within this context, we will consider the “world economy” in which, by definition, all three goods must be produced and a “small economy,” $B$, within this world, which has the same technology as the world economy. By definition, in equilibrium the “world economy” – with aggregate endowment vector: $K^w, L^w$ is producing all three goods, whereas $B$’s production pattern will depend on its own endowment vector.

i. Discuss, and show graphically, how production and factor prices within $B$ evolve as the capital-abundance in that country increases. Also, show how the country’s pattern of trade evolves {consider all cases: when $B$ is very labor abundant, and when it is very capital abundant, as compared to the world economy}.

ii. Suppose initially $B$’s endowments are such that the economy is incompletely specialized. How will a slight (Hicks neutral) increase in productivity in sector $F$ affect factor prices in $B$, and what will its resulting trade pattern be? Explain carefully {HINT – there are several cases}.

b) Consider a diversified small country ($A$) that produces footwear, steel and electronics. Its resources consist of capital and two types of labor, skilled and unskilled. Footwear is produced with unskilled workers only, steel with unskilled workers and capital, and electronics with skilled workers and capital, all under constant returns to scale. Country $A$ imports footwear and steel, but its steel imports are very small.

i. If you were hired to study country $A$’s factor content of trade flows, would you expect the country to be a net importer of unskilled labor services? of skilled labor services? of capital
services? What would you infer from this study about the country’s factor abundance?

ii. The government of country $A$ is considering protecting its import-competing sectors. How will a uniform rate of protection against all imports affect the real returns to each factor?

iii. Given world prices, how would a small increase in productivity in sector F affect factor prices and output? Be as specific as possible.

Part II

3. Consider a small economy that produces shirts and food. Production of shirts creates a negative externality, which harms individuals. For simplicity, assume all consumers are alike (and, for simplicity, normalized to one) and have quasi-linear preferences. Production is carried on by competitive profit-maximizing firms, and hence supply curves can be found from the societal production possibility frontier (ppf). Preferences and the ppf are:

$$U^h = c^h_j + \left(\frac{200c^h_s - (c^h_s)^2}{2\alpha}\right) - \mu z^2; \quad z = Q_s; \quad Q_f + \frac{(Q_f)^2}{2\sigma} \leq 1000;$$

$$\alpha + \sigma = 10, \alpha > 0, \sigma > 0$$

where $Q_s$ is output, $z$ is pollution, and $c^h_j$ is consumption. \{Note the parametric restriction on $\alpha, \sigma$.\}

a) Find the autarky equilibrium.

b) Assume the country can trade at a given world price, $P^w$. Assuming the government does not implement any domestic policy, will free trade raise welfare? Be as specific as possible.

c) Assume the world price $P^w = 20$. What are the first best policies for the country? Does it include free trade?

d) If trade policy is the only feasible policy instrument, what is the optimal (second best) export tariff (or subsidy)? \{NOTE: for parts d-f you may assume the parametric values are such that it is always optimal to export some of good f\}

e) Is there an export quota that would have the same impact as the export tariff you determined in part (d)?

f) Finally, assume the world price is a random variable, with mean $P^w = 20$ (you can assume, for example, the variation in world price is not large enough to change the pattern of trade).

i. Assuming the trade policy instrument must be set before the world price is known, will tariffs and quotas be equivalent? Explain.

ii. Find the optimal specific tariff.

iii. Find the optimal export quota.

iv. Which policy is better? Explain.
4. Consider a Heckscher-Ohlin-Helpman type trade model with production differentiation. There are two countries (US, Japan) and two industries (F, M). The output of industry F is a homogeneous good, whereas the output of industry M is a heterogeneous good. What makes M an industry - rather than a number of different industries - is that all (potential) M producers have the same technology, and the substitutability in consumption among M goods is much higher than that between M goods and F goods.

In particular, all individuals have the following “love of variety” preferences:

\[ U_f = f^h \cdot \beta\left(\bar{m}\right); \quad \beta\left(\bar{m}\right) \equiv \left(\sum_i \left(m_i^h\right)^{\gamma/4}\right)^{4/3} \]

where \( f^h \) is the individual’s consumption of \( F \), and \( m_i^h \) is that person’s consumption of variety \( i \) of good \( M \). Turning to technology, which is internationally identical, the dual cost curves are given by:

Industry F:

\[ TC\left(Q_f, W, R\right) = Q_f W^{2/3} R^{1/3}, \]

Industry M:

\[ TC\left(Q_{M,i}, W, R\right) = \eta\left(Q_{M,i}\right) W^{1/3} R^{2/3}; \quad \eta\left(Q_{M,i}\right) = \left(\frac{3}{2}\right)\left(Q_{M,i}\right)^{1/2} + \left(\frac{Q_{M,i}}{12}\right)^{3/2} \]

where, of course, \( Q_{M,i} \) is the output of the \( i \)th variety of good \( M \).

Finally, let the world factor endowments be given by: \( K^T = L^T \).

a) Find the individual’s demand curves.
b) Assume M producers all have the same technology, and that they behave as Bertrand competitors (i.e., maximizing profits, taking the price of all other firms as given). Assuming free entry into sector M, find the equilibrium size of each firm, and the price charged (as a function of factor prices).
c) Find the integrated equilibrium.
d) Given individual country’s endowments \( \left(K^{us}, L^{us}\right), \left(K^J, L^J\right) \) such that \( K^{us} + K^J = K^T = L^T = L^{us} + L^J \):

i. Find the set of allocations for which free trade reproduces the integrated equilibrium.
ii. Assuming trade reproduces the integrated equilibrium, characterize the production pattern in each country (output of F, number of M producers) and the pattern of trade between the countries. Show that trade will occur between countries that are identical except for scale (i.e., \( \left(K^{us}/L^{us}\right) = \left(K^J/L^J\right) \)) and that this trade is mutually beneficial.

e) Finally, suppose the US decides to subsidize its producers of manufactured goods (the subsidy is a constant percent of total costs, regardless of where output is sold). Assuming firms still behave as Bertrand competitors, discuss how this subsidy is likely to affect firm size and the number of M producers in the US. You do not need to solve explicitly but indicate what steps in your derivation of equilibrium for parts (a-d) are modified.
Consider a two good (M, C), specific factor model of a small country. Technology is given by:

\[
Q_c = 10\left(V + L_c\right) - \left(1/2\right)\left(V - L_c\right)^2; \quad Q_m = 10\left(K + L_m\right) - \left(1/2\right)\left(K - L_m\right)^2
\]

\(K\) and \(V\) are specific factors, whereas labor (\(L\)) is potentially mobile. The total endowments of each factor are: \(\bar{V} = \bar{K} = 10\), \(\bar{T} = 20\). When preferences are needed, assume all individuals have identical preferences given by: \(U = c^{3/4}m^{1/4}\).

a) Given world prices \(P_c = P_m = 1\), find the equilibrium allocation, assuming labor is (costlessly) mobile between sectors and the country pursues free trade.

b) Using the labor allocation found in part (a), assume no (further) labor mobility. Assume the world price of good \(c\) falls to \(P_c = 0.5\) (\(M\) is the numeraire). Also, assume the wage paid to workers in sector C, as measured in terms of the numeraire, is downward rigid (it cannot fall below the level determined in part a).

i. Given the labor immobility, absent government policy how does the price decline affect employment in each sector? Given this wage rigidity, what is the optimal policy for the government to pursue?

ii. If the government adopts the policy in part (i) and labor is potentially mobile over time, but only at a cost, how does the policy affect the incentives of labor to move between sectors?

iii. Again, assume no labor mobility and assume the only policy the government can pursue is trade policy. Find the (magnitude of the) optimal trade policy.

Next, complicate the model by assuming labor movement between sectors is feasible, but costly (time lost from work plus retraining costs). To analyze this case we need to introduce a simple two period dynamic structure with the following sequence of actions:

1) The initial allocation of potential workers to each sector is given by: \(N_c = N_m = 10\).

2) The world price of \(C\) is revealed (good \(M\) is the numeraire); it will be the same for both periods.

3) At the beginning of period 1, a decision is made as to how many workers will be trained to move from one sector to the other; \(T_c\) denotes the number of sectors trained to move from \(C\) to \(M\) and \(T_m\) is its counterpart; \(L_{c1}, L_{mt}\), which denotes the number of people working in each sector in each time period, are:

\[
L_{c1} = N_c - T_c = 10 - T_c; \quad L_{m1} = N_m - T_m = 10 - T_m; \quad T_c \geq 0, \quad T_m \geq 0
\]

\[
L_{c2} = N_c + T_m - T_c = 10 + T_m - T_c; \quad L_{m2} = N_m + T_c - T_m = 10 + T_c - T_m;
\]

Workers who retrain in period one cannot be employed in that period.

4) Once period one “retraining” decisions are made, the remaining workers \((L_{c1}, L_{m1})\) produce output in period one.

5) For simplicity, period two is the “last” period, so all labor in each sector is used to produce output. Production technology for each period is given by:
Period 1: $Q_{c1} = 10(V + L_{c1}) - (1/2)(V - L_{c1})^2$; $Q_{m1} = 10(K + L_{m1}) - (1/2)(K - L_{m1})^2$

Period 2: $Q_{c2} = 9\left[10(V + L_{c2}) - (1/2)(V - L_{c2})^2\right]; Q_{m2} = 9\left[10(K + L_{m2}) - (1/2)(K - L_{m2})^2\right]

(the rationale for the multiplier of “9” is to represent the stream of income from all future periods).

Finally, assume training costs (TC), as measured by the numeraire, are given by:

$TC = (T_c + T_m)$

c) Assume there is a central planner and that his goal is to maximize welfare, which implies maximizing the present discounted value of net GNP, which is given by: (for simplicity, assume no discounting):

$Y = \{P_c(Q_{c1} + Q_{c2}) + (Q_{m1} + Q_{m2}) - TC\}

i. Find the optimal solution as a function of the world price of good C, where $P_c < 1$ (pay attention to corner solutions). Show how current (period 1) employment and income changes as $P_c$ falls. Assuming the country imports good C (for all $P_c \leq 1$), how does a decline in $P_c$ affect welfare?

ii. Next, consider the laissez-faire solution (no central planner). Each worker makes her own decision as to whether to remain in her current job or move to a new sector. Each worker can: (1) remain in her current sector for both periods or (2) retrain and move to the other sector in period 2. Total private costs of relocating are: (1) lost period one wages, as the worker is unemployed while retraining, and (2) actual training costs she must pay, which is one unit of good M. In period two the person will be in the other sector. Markets are competitive and wages are flexible, so all people who want to work can find employment and all employed workers will be paid their marginal value product in each period. Given this setup:

Show how workers decide whether to retrain or stay in their current job. Will these private decisions lead to efficiency (i.e., replicate the optimal solution found in part (ci))? If not, what government policy is needed to insure that private decisions are optimal? Explain carefully.

iii. Suppose legal constraints require “comparable worth” - i.e., since workers are identical in their skills, all workers must be paid the same wage (i.e., $W_c = W_m$). Under these conditions, will laissez faire decisions lead to efficiency? If not, what policy should the government implement and what complications arise in implementing this policy (you do not need to solve explicitly, just discuss).

d) Assume the same setup as in part (c) - with flexible wages - except that total training costs are modified to:

$TC = (T_c + T_m)^2$

Each worker pays the average cost of her training (so the cost per worker is $(T_c + T_m)$).

i. Assuming $P_c = (1/2)$, find the competitive equilibrium. Is it efficient? If not, do private decisions results in too little, or too much, (voluntary) unemployment and mobility?

ii. If government intervention is required to support the optimal decision, what is the appropriate policy/policies? Are trade policies part of the optimal policy? Explain (you need not solve).
6. Answer all parts

a) \{Infant industry\}. Consider a simple two period model of a small economy which can produce two goods, \( F \) or \( M \). Using a specific factor type model, assume there are a fixed number of firms \((N)\) in each sector, with the following technologies:

1. \( Q^n_{ci} = \left( L^n_{ci} \right)^{\frac{1}{2}} \); \( Q^n_{ci} = \theta \cdot \left( L^n_{ci} \right)^{\frac{1}{2}} \); \( n = 1, ..., N; \)

2. \( Q^k_{m1} = 2 \left( L^k_{m1} \right)^{\frac{1}{2}} \); \( Q^k_{m2} = 2 \cdot \phi \cdot \left( L^k_{m2} \right)^{\frac{1}{2}} \); \( k = 1, ..., N; \)

3. \( \theta = \left[ 1 + \alpha \left( \sum_n Q^n_{ci} \right)^{\frac{1}{2}} \right] \); \( \phi = \left[ 1 + \beta \left( \sum_k Q^k_{m1} \right)^{\frac{1}{2}} \right] \)

where \( Q^n_{ci} \) is output, by firm \( s (=n,k) \) in sector \( j (=c,m) \) in time period \( i (=1,2) \) and the terms \( \theta, \phi \) represent the “infant industry” argument. The resource constraints for the economy are:

4. \( \sum_n L^n_{ci} + \sum_k L^k_{m1} \leq L^i \); \( i = 1, 2 \)

Finally, preferences, if needed, are given by: \( U = c \cdot m \) and world prices by: \( P_c = P_m = 1 \)

i. Find the laissez-faire equilibrium outputs. What market failure, if any, occurs in this setting?

ii. Find the central planner’s optimal allocation (which maximizes the present discounted value of GNP; assume the interest rate is zero).

iii. What policy can support the efficient solution? If only trade policy is allowed, are import restrictions part of the second best policy? Explain carefully (you do not have to solve).

b) Consider a simplified 3 good model. The goods are \( M, S, \) and \( F \). Perfect competition prevails and production technology, resource constraints, preferences and world prices are given by:

\[ Q_m = 2 \text{Min} \left[ S_m, L_s^{\frac{1}{2}} \right]; \quad Q_s = 2L_s^{\frac{1}{2}}; \quad Q_f = L_f; \quad L_m + L_s + L_f \leq 100 \]

\[ U = c_f + 26c_m - \left( \frac{1}{2} \right) \left( c_m \right)^2; \quad P_f \equiv 1; \quad P_m = 5, \quad P_s = 1 \]

i. Find the free trade equilibrium production and consumption allocations (note that good \( S \) is a pure intermediate good).

ii. Suppose a tariff of 50% is imposed on imports of good \( M \). Find the resulting production allocation.

iii. Repeat part (ii), assuming a 50% tariff on all imports. Contrast the effect of the two tariff structures on production of good \( M \).

iv. An escalated tariff structure is one in which the tariff on final goods is higher than that on intermediate goods (or raw materials). Economists tend to argue that escalated tariffs provide even more protection for the final goods than the nominal tariff rate indicates. Use your answer to parts (ii) and (iii) to discuss that point.

v. Finally, assume that, for political reasons, a 50% tariff on good \( M \) is mandatory. Given that tariff, find the optimal (second best) tariff on good \( S \). (a specific answer is required).
7. {Optimal tariff}. Consider a world of two goods (M, F). There are two types of countries, which differ by their production technology. “Industry” production functions in each type economy are (i.e., you can act as though there is one price-taking profit-maximizing firm in each industry):

Country Type I: \( Q_m = (8L_m)^{1/2} \); \( Q_f = L_f \); \( L_f + L_m \leq 10^6 \)

Country Type II: \( \bar{Q}_m = \left( \frac{2\mu L_m}{N} \right)^{1/2} \); \( \bar{Q}_f = \bar{L}_f \); \( \bar{L}_m + \bar{L}_f \leq \left[ 2 \cdot 10^6 / N \right] \)

where \( L_i \) is the allocation of land to good \( i \). Assume there is a single, price-taking, utility maximizing representative individual in each country with the following preferences (and hence demands)

Country Type I: \( U^I = c_f + 10c_m - \left( c_m^2 / 2 \right) \rightarrow d^I_m = \left( 10 - P^I_m \right) \)

Country Type II: \( U^I = c_f + \left[ \frac{1}{10 - \mu} \right] \left[ 100c_m - N \left( c_m^2 / 2 \right) \right] \rightarrow d^II_m = \left[ \frac{1}{N} \right] \left( 100 - (10 - \mu) P^II_m \right) \)

Finally, there are 2 countries \( (A, B) \) of type I and \( N \) countries of type II.

**NOTE 1**: The \( \mu \) in the supply curve for Type II countries and the \( \mu \) that appears in the demand curve for these countries is the same; the import demand should be independent of \( \mu \). This distinction does not matter until part (e)).

**NOTE 2**: The role of \( N \) here is simply to make it plausible that type II countries pursue free trade. The equilibrium of the model should be independent of \( N \), given free trade so you could set \( N = 1 \).

**NOTE 3**: The total labor size makes no difference - it is chosen to be large to assure interior solutions prevail. You may assume that all solutions are interior).

a) Find the free trade equilibrium prices, production, consumption and trade (let \( P_f \equiv 1 \)).

b) Suppose countries of type II continue to pursue free trade, but the two type I countries decide to use tariffs to improve their terms of trade. Each country, non-cooperatively and simultaneously, chooses its own (specific) export tariff, \( t^A \) and \( t^B \) in order to maximize its own welfare.

i. Find country \( A \)’s optimal export tariff, \( \hat{t}^A \), as a function of \( t^B \).

ii. Use the symmetry of the problem to find the Nash equilibrium tariffs, the resulting world price of good \( M \), and welfare for the type I countries.

iii. Do type I countries benefit from this trade policy? What happens to the welfare of type II countries and overall “efficiency”?

c) Suppose countries A and B form a customs union, meaning they have no tariffs on trade between the two countries but they agree on a common external tariff. Will this common external tariff be higher or lower then the equilibrium you calculated in part (b)? Find the optimal common tariff.

d) For simplicity, suppose A and B merge into one country (or, equivalently, choose policy cooperatively). Next, suppose there is a time lag between when production and consumption decisions are made and there is no way that the (unified) country can commit, before any production decisions are made, never to revise tariffs. Hence, decisions are made in the following sequence: (1) competitive, profit-maximizing producers in all countries make output decisions based upon their forecast of the equilibrium price; (2) after production decisions are irrevocably made, the combined
country A-B sets its export tariff; (3) given production decisions and the tariff, trade takes place. Assuming the tariff in stage (2) is set to maximize A-B’s welfare:

i. Explain why it matters that production decisions are predetermined when the tariff is set.
ii. Find the optimal tariff, given production decisions.
iii. Find the time consistent equilibrium, using the assumption that producers’ price expectations are accurate. Note that the equilibrium here depends on $\mu$ (the elasticity of supply in type II countries), but it does not in the pre-commitment equilibrium. Explain why and compare this tariff to the optimal tariff found in part (c).

e) Given that the policy active country (A-B) cannot commit to its tariff at the beginning of the game (i.e., the equilibrium is determined as in part d), does that country have any incentive to tax or subsidize output of its exportable (a decision which is irrevocably made at the first stage)? Explain your answer and indicate which policy, if either, is desirable. (You do not have to explicitly solve for this part).