THEORY OF EFFECTIVE PROTECTION, RESOURCE ALLOCATION AND THE STOLPER-SAMUELSON THEOREM

The many-industry case

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In this paper the following principal results will be established:

1. In the general equilibrium model with many industries, many primary factors, many imported inputs, and nonseparable production functions, necessary and sufficient conditions for which the ERP theory works in the sense defined by Bhagwati and Srinivasan under their (B-S) restriction on tariff changes are presented.

2. Necessary and sufficient conditions for the strong Stolper-Samuelson theorem to hold are presented.

3. Under certain special conditions the weak Stolper-Samuelson theorem holds.

1. Introduction

The theory of effective protection (ERP) has been developed in an attempt to seek a concept of protection which, in the presence of imports of intermediates, will perform the same role in predicting the effects of a change in tariff structure on domestic resource allocation as nominal tariffs do in their absence. The problem of generalization of the theory from the assumption of fixed production coefficients in respect of intermediates towards allowing their substitution in production in a general equilibrium model has received the attention of several economists.¹ The studies of

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Bruno (1973) and Bhagwati and Srinivasan (1973) on the effect of tariff change in a general equilibrium model with \( n \) industries, \( d \) primary factors and \( m \) imported inputs are particularly significant in this connection.

Bruno defines 'real' value added as the difference between the value at free trade prices of post-tariff outputs and nonfactor inputs, i.e. 
\[
F = \{X - e'M\},
\]
where \( X \) is output, \( M \) a column vector of nonfactor tradeable inputs in the post-tariff situation, and \( e' \) the row vector \((1, \ldots, 1)\) where the world prices are normalized at unity. On the other hand, nominal value added is
\[
G = \{(1 + T)X - (e + t)'M\},
\]
where \( T \) is the ad valorem tariff on output and \( t' \) the row vector \((t_1, \ldots, t_m)\) of ad valorem tariff on tradeable inputs. He defines the effective rate of protection (ERP) as
\[
B = \{(G/F) - 1\}.
\]
However, Bruno deals mainly with the issue of the conditions under which a tariff structure will increase (decrease) the use of all primary factors in an industry under a general equilibrium framework in which the factor endowment in the economy is exogenously fixed; the ERP defined in the above is discussed only in the case of functional separability.

The B-S analysis, on the other hand, is directly focused on ERP theory. They observe that the proportionate change of nominal value added \( V_i (i = 1, \ldots, n) \) (Bruno's \( G \)) induced by a change in tariff structure can be expressed as the sum of two terms: a Divisia index (I) of proportionate changes in all the prices of output and imported inputs faced by the \( i \)th industry, and a Divisia index (II) of proportionate changes in the quantities of the factors used by the \( i \)th industry. Then they define the ERP problem as one of investigating whether the ERP (Divisia) index (I) of price changes and the Divisia index (II) of quantity changes will move in the same direction for an activity. They propose two approaches to this problem.

The first approach is to look for restrictions on production functions that will be sufficient for indexes I and II to move in the same direction. This led Bhagwati and Srinivasan to the condition of functional separability.

The second approach, on the other hand, is to look for restrictions on tariff changes that will equally suffice to make indexes I and II move in the same directions. Since no special restriction on production functions can be used in this alternative approach, one can expect that the restriction required on tariff changes would have to be rather strong. Essentially Bhagwati and Srinivasan looked for tariff changes that would result in a positive flow of at least one domestic resource (and no withdrawal of any other resource) to the activity whose (price) index (I) was relatively higher than in other activities thanks to the tariff change, thus sufficing to make the (quantity) index (II) positive as well. This led them to the following restriction on tariff changes:

\[
\bar{P}_i^0 > (\leq) \bar{P}_i^0 = \bar{P}_k^M, \quad i = 2, \ldots, n; \quad k = 1, \ldots, m,
\]

where \( P_i^0 \) and \( P_k^M \) denote the domestic prices of the \( i \)th good and the \( k \)th
imported input, respectively, and \( \ddot{z} \equiv \dot{dz}/z \) for any variable \( z \). Furthermore, this sufficiency condition was established only for the two-industry case. For the many-industry case \((n > 2)\), they managed to show that condition (B–S) above is sufficient for index (I) and the change in gross outputs of the first activity to have the same sign, but they were unable to prove anything in general about the sign of index (II).

The B–S sufficiency restrictions on tariff changes were then generalized by Sendo (1974) for the two-industry case as follows

\[
\hat{p}_1^{0} \geq \hat{p}_k^{M} \geq \hat{p}_2^{0}, \quad k = 1, \ldots, m,
\]

with at least one strict inequality holding. In this case, semipositive resource flows to industry 1 will take place and index (II) will be positive (along with index (I), absolutely and relatively to industry 2) in industry 1. This condition was anticipated by Bruno (1973) for the two-industry case (though in Bruno's model the tariffs on imported intermediates can vary from industry to industry whereas they are identical in all industries in the Bhagwati–Srinivasan model).

Bruno (1973) also attempted to investigate the many-industry case, assuming the case where one industry's output and/or input tariffs have been changed leaving all tariffs in the rest of the economy unchanged. Then by placing the following additional restrictions on production functions, he concluded that all factors move to or away from the first industry. These conditions are

(i) The production functions are strictly decreasing returns to scale.
(ii) \( H \equiv (A^2)^{-1} + \ldots + (A^n)^{-1} \) has the Metzler property, i.e. the diagonal elements of \( H^{-1} \) are negative and the off-diagonal ones are non-negative,
where \( A^i \) is the Hessian of the \( i \)th nominal value added function \( G \) with respect to its primary inputs.

Bruno's theorem, however, is not appealing. First, it will not hold if the production functions are constant returns to scale. Secondly, condition (ii) is in terms of the sum of \((n - 1)\) Hessians of value-added functions (except for the first sector) and therefore its economic interpretation is not clear.

In this paper, we consider the case of equal number of domestic goods and factors. Our principal results are the following.

(1) It is shown by a counter example that the ERP theory will not necessarily work in the sense defined by Bhagwati and Srinivasan under (B–S) restriction on tariff changes for the many-industry case.

(2) A sufficient and necessary condition for the ERP theory to hold under (B–S) restriction on tariff change is established.

(3) The strong and the weak Stolper–Samuelson criteria under (B–S) restriction on tariff change will be defined in section 5.
(4) A necessary and sufficient condition for which the validity of the strong Stolper–Samuelson theorem will be guaranteed is established. At the same time it is shown that the ERP theory will work in the sense defined by Bhagwati and Srinivasan.

(5) A sufficient condition for which the validity of the weak Stolper–Samuelson theorem will be guaranteed is established.

2. Mathematics

Before proceeding, it is convenient to prove three basic theorems which are indispensable for our analysis. We are concerned with a matrix $A^i(i=1,\ldots,n)$ of order $d$ which has the following property:

(H) 

(i) $A^i$ is indecomposable, symmetric and singular.

(ii) $A^i$ is negative semidefinite with negative diagonal and non-negative off-diagonal elements.

The following notation is needed:

$D$ — the set \{1, \ldots, d\},

$J$ — a subset of $D$,

$\bar{J}$ — a complement of $J$ relative to $D$,

$A_{JI}$ — a submatrix of a matrix $A$ which consists of the $i$th row and the $j$th column of $A$ for $i \in J$ and $j \in \bar{J}$,

$x_j$ — a subvector of a vector $x$ which consists of the $j$th component of $x$,

$c$ — the sum vector $(1, \ldots, 1)'$,

$\emptyset$ — the empty set.

First, we shall prove the following theorem on a matrix with property (H).

**Theorem 1.** If a matrix $A$ has property (H), then the following statements hold:

(i) the equation

$$Ax = 0 \quad \text{and} \quad c'x = 1$$

has a unique positive solution $x$.

(ii) the inequality

$$Ax \leq 0 \text{ (or} \geq 0)$$

has no solution.

\[By \ x > 0 \ \text{we mean:} \ x_i > 0 \ \text{for all} \ i. \ By \ v \geq 0 \ \text{we mean:} \ x_i \geq 0 \ \text{for all} \ i \ \text{and} \ x_i > 0 \ \text{for at least one} \ i. \ By \ v \geq 0 \ \text{we mean:} \ x_i \geq 0 \ \text{for all} \ i.\]
(iii) any principal submatrix of $A$ (except $A$ itself) is negative definite.

**Proof.** (i) Since $A$ is singular, the equation

$$Ax = 0$$

has a nonzero solution $x$. Let $J = \{j : x_j > 0\}$ and $\bar{J} = \{j : x_j \leq 0\}$. We can assume that $J \neq \emptyset$, since eq. (3) is homogeneous. We get from (3)

$$A_{JJ}x_J = -A_{J\bar{J}}x_{\bar{J}} \geq 0.$$

(4)

$$A_{J\bar{J}}x_J = -A_{J\bar{J}}x_{\bar{J}} \leq 0.$$  

(5)

where the semipositivity of (5) results from the indecomposability of $A$.

Suppose $J \neq \emptyset$. Then we get from (5) that

$$(x_J)'A_{JJ}x_J \geq 0.$$  

(6)

But since $A_{JJ}$ is negative semidefinite, we see from (6) that $(x_J)'A_{JJ}x_J = 0$ and therefore $A_{JJ}x_J = 0$, a contradiction.\(^3\) Hence, eq. (3) has a positive solution $x$.

Suppose eq. (1) has two distinct positive solutions, $x^0$ and $\hat{x}$ ($x^0 \neq \hat{x}$). Then $\hat{x} = x^0 - \bar{x}$ is also a solution to eq. (3). But since $e'x^0 = e'\bar{x} = 1$, $\hat{x}$ has at least one negative component and at least one positive one, a contradiction with the above discussion. Hence, eq. (1) has a unique positive solution $x$.

(ii) By duality,\(^4\) (i) implies that the inequality (2) has no solution.

(iii) Since eq. (3) has a positive solution $x$, we have for any nonempty proper subset $J$ of $D$

$$A_{JJ}x_J \geq 0.$$  

(7)

\(^3\)Since $A_{JJ}$ is negative semidefinite, there exists a matrix $R$ such that $A_{JJ} = -RR$. Hence, since

$$(x_J)'A_{JJ}x_J = -(Bx_J)'(Bx_J) = 0,$$

we see that $Bx_J = A_{JJ}x_J = 0$.

\(^4\)Exactly one of the following alternatives holds. Either the equation

$$xA = 0$$

(1)

has a positive solution, or the inequality

$$A_{\bar{J}} \geq 0$$

(2)

has a solution [see Gale (1960, theorem 2.9, corollary 2, p. 49)].
We see from (7) that any principal submatrix of $A$ (except $A$ itself) has a q.d.d.\(^5\) and therefore, is negative definite. Q.E.D.

**Theorem 2.** Let $A$ have property (H) such that $z'A=0$ for some positive vector $z$, and $v$ be a nonzero vector such that $z'v=0$. Then the equation

$$Ax=v \quad (8)$$

has a semipositive solution $\bar{x}$ with at least one zero component. And the solution $\bar{x}$ is unique.

**Proof.** Suppose that eq. (8) does not have a non-negative solution. Then from Farkas's theorem of the alternative, the inequalities

$$y'A \geq 0 \quad \text{and} \quad y'v < 0 \quad (9)$$

have a solution $y$.\(^6\) Since $A$ satisfies the hypothesis of Theorem 1, we must have $y=tx$ for any nonzero real number $t$. But $0 > y'v = tz'v = 0$, a contradiction. Hence, eq. (8) has a non-negative solution $x^0$.

Suppose $x^0 > 0$. Since $Az=0$, $x^0 - tz$ for any real number $t$ is the general solution to eq. (8). Let $\bar{t} = \min_i x_i^0 / z_i$. Then, $\bar{x} = x^0 - \bar{t} z$ is a semipositive solution with at least one zero component to eq. (8).

Suppose $\bar{x}$ is also a semipositive solution with at least one zero component to eq. (8), where $\bar{x} \neq t\bar{x}$ for any real number $t$. Then we have

$$Ax=0, \quad \bar{x} \geq 0, \quad x \geq 0. \quad (10)$$

\(^5\)Professor McKenzie has revised the definition of a matrix with a quasidominant diagonal (or q.d.d.) made in his paper (1960) as follows. Let $A$ be an $n \times n$ matrix and $A_{J}$ be the principal submatrix with indices in $J$. $A$ is said to have a q.d.d. if there exist $d_j > 0, j = 1, \ldots, n$ such that for any principal submatrix $A_{J}$,

$$d_{|a_{ij}|} \geq \sum_{i \in J} d_{|a_{ij}|}$$

with strict inequality for some $j \in J$.

\(^6\)Exactly one of the following alternatives holds. Either the equation

$$xA = b \quad (i)$$

has a non-negative solution or the inequalities

$$Ay \geq 0 \quad \text{and} \quad by < 0 \quad (ii)$$

have a solution. [see Gale (1960, theorem 2.6, p. 84)].
Subtracting (11) from (10) we get

$$A(x - \bar{x}) = 0.$$  \hfill (12)

We see from Theorem 1 that $\bar{x} - x = tz > 0$ for some positive number $t$ without loss of generality. But $\bar{x}$ has at least one zero component, say, $\bar{x}_k = 0$. Hence $\bar{x}_k - \bar{x}_k \leq 0$, a contradiction. Hence $\bar{x}$ is unique. Q.E.D.

Let

$$A = \begin{bmatrix}
A_{22} & \cdots & A_{2n} \\
\vdots & \ddots & \vdots \\
A_{n2} & \cdots & A_{nn}
\end{bmatrix}
$$

and the square matrix $A_{ij}$ of order $d$ for $i, j = 2, \ldots, n$ be given by

$$A_{ij} = A^i + A^1, \quad \text{if } i = j,$$

$$= A^1, \quad \text{if } i \neq j.$$

Furthermore, let $A^i D^i = 0$ ($i = 1, \ldots, n$) for some positive vector $D^i$. Then we have the following theorem.

**Theorem 3.** Suppose $A^i$ ($i = 1, \ldots, n$) has property (H). Then, $A$ is negative definite if and only if the rank of the matrix $[D^1, \ldots, D^n]$ is $n$.

**Proof.** (sufficiency). It is clear that $A$ is negative semidefinite. Suppose $A$ is singular. Then there exists a nonzero vector $x' = ((x_2)', \ldots, (x_n)')$ such that

$$x'Ax = ((x_2)', \ldots, (x_n)')$$

$$\begin{bmatrix}
A^2 + A^1 & A^1 & \cdots & A^1 \\
A^1 & A^3 + A^1 & \cdots & A^1 \\
\vdots & \vdots & \ddots & \vdots \\
A^1 & A^1 & \cdots & A^n + A^1
\end{bmatrix} \begin{bmatrix}
x_2 \\
x_3 \\
\vdots \\
x_n
\end{bmatrix} = 0,$$

$$\sum_{i=2}^{n} (x_i)' A^i x_i + \left( \sum_{i=2}^{n} (x_i)' \right) A^1 \left( \sum_{i=2}^{n} x_i \right) = 0,$$  \hfill (13)

where $x_i$ ($i = 1, \ldots, n$) is a column vector of order $d$. Since $A^i$ ($i = 1, \ldots, n$) is negative semidefinite, eq. (13) implies that

$$(x_i)' A^i x_i = 0, \quad \text{for } i = 2, \ldots, n,$$  \hfill (14)
and
\[
\left( \sum_{i=2}^{n} (x^i)' \right) A^1 \left( \sum_{i=2}^{n} x^i \right) = 0.
\] (15)

Furthermore, we see from Theorem 1 that (14) and (15) imply
\[
x^i = t_i D^i \quad (i = 2, \ldots, n), \text{ for some real number } t_i,
\] (16)
\[
t_1 D^1 + \ldots + t_n D^n = 0,
\] (17)

where \((t_1, \ldots, t_n) \neq 0\). But (17) implies that the rank of \([D^1, \ldots, D^n]\) is less than \(n\), a contradiction.

(necessity). Suppose that the rank of \([D^1, \ldots, D^n]\) is less than \(n\). Then there exists a nonzero vector \((t_1, \ldots, t_n)\) such that
\[
t_1 D^1 + \ldots + t_n D^n = 0.
\] (18)

Let \(x^i = t_i D^i\) and \(x = ((x^2)', \ldots, (x^n)')'\). Then we see
\[
x'Ax = \left( \sum_{i=2}^{n} t_i (D^i)' \right) A^1 \left( \sum_{i=2}^{n} t_i D^i \right) = (t_1)^2 (D^1)' A^1 D^1 = 0.
\] (19)

Eq. (19) implies that \(A\) is singular, a contradiction. Q.E.D.

3. The model

The B–S model (1973) is the following. Let the production function for the \(i\)th good be \(F'(D^i, M^i)\), \((i = 1, \ldots, n)\), where \(D^i = (D_1^i, \ldots, D_j^i)'\) is the column vector of domestic inputs and \(M^i = (M_1^i, \ldots, M_m^i)'\) is the column vector of imported inputs. In this paper we deal with the case where the number of goods is equal to the number of factors, i.e. \(n = d\).

It is assumed that:

(A.1) All inputs enter into the production of each commodity.
(A2) Each production function is homogeneous of degree one, concave and its Hessian matrix is indecomposable with negative diagonal and non-negative off-diagonal elements.\(^7\)

(A.3) Production takes place under perfect competition, given the domestic price vector, \(P^0 = (P_1^0, \ldots, P_n^0)'\) and \(P^M = (P_1^M, \ldots, P_m^M)'\), respectively, of the outputs and imported inputs.

\(^7\)Bhagwati and Srinivasan assume that the Hessian of each production function has negative diagonal and positive off-diagonal elements.
(A.4) Every commodity is produced.
(A.5) The rank of the primary input coefficient matrix, \([D_1, \ldots, D^n]\) is \(n\).

In this section we need the following notation due to B-S:

\[
F_D^i = \left(\frac{\partial F^i}{\partial D_1}, \ldots, \frac{\partial F^i}{\partial D_n}\right), \quad i = 1, \ldots, n, \tag{20}
\]

\[
F_M^i = \left(\frac{\partial F^i}{\partial M_1}, \ldots, \frac{\partial F^i}{\partial M_n}\right), \quad i = 1, \ldots, n, \tag{21}
\]

\[
F_{DD}^i = \left(\frac{\partial^2 F^i}{\partial D_j \partial D_k}\right), \quad i, j, k = 1, \ldots, n, \tag{22}
\]

\[
F_{DM}^i = \left(\frac{\partial^2 F^i}{\partial M_j \partial D_k}\right), \quad i, j = 1, \ldots, n; \quad k = 1, \ldots, m, \tag{23}
\]

\[
F_{MM}^i = \left(\frac{\partial^2 F^i}{\partial M_j \partial M_k}\right), \quad i = 1, \ldots, n; \quad j, k = 1, \ldots, m. \tag{25}
\]

The competitive equilibrium conditions are:

\[
P_i F_D^i = w, \quad i = 1, \ldots, n. \tag{26}
\]

\[
P_i^0 F_M^i = P_M, \quad i = 1, \ldots, n, \tag{27}
\]

\[
\sum_{i=1}^{n} D^i = \bar{D}, \tag{28}
\]

where \(\bar{D} = (\bar{D}_1, \ldots, \bar{D}_n)\) is the endowment vector of the primary factors in the economy and \(w\) a primary factor prices vector.

Differentiating eqs. (26)–(28) totally, we get

\[
P_i^0 [F_{DD}^i \bar{D}^i + F_{DM}^i \bar{M}^i] + P_i^0 w = \hat{w}, \quad i = 1, \ldots, n, \tag{29}
\]

\[
P_i^0 [F_{MD}^i \bar{D}^i + F_{MM}^i \bar{M}^i] + \bar{P}_i^0 P_M = [\bar{P}_M] \bar{P}_{MM}, \quad i = 1, \ldots, n, \tag{30}
\]

\[
\sum_{i=1}^{n} \hat{D}^i = 0, \tag{31}
\]

where, for any vector \(z = (z_k)\), \(\dot{z} \equiv (dz_k)\) and \(\ddot{z} \equiv (dz_k/z_k)\), and \([[\dot{z}]]\) is the diagonal matrix whose \((i, i)\)th element is \(z_i\) \((i = 1, \ldots, n)\).

Eliminating \(\bar{M}^i\) \((i = 1, \ldots, n)\), we get

\[
A_i \dot{D}^i = -F_{DM}^i (F_{MM}^i)^{-1} [\bar{P}_M] (\bar{P}_M - P_i^0 e) - P_i^0 w + \hat{w}, \quad i = 1, \ldots, n, \tag{32}
\]

where \(A_i \equiv P_i^0 [F_{DD}^i - F_{DM}^i (F_{MM}^i)^{-1} F_{MD}^i]\) and \(F_{MM}^i\) is negative definite by (A.2) (see theorem 1) and therefore \((F_{MM}^i)^{-1} < 0\).
4. Tariff change and resource allocation

Suppose that protection is now conferred only on the \( i^0 \)th industry by the following change in the tariff structure according to Bhagwati and Srinivasan (1973):

\[
\hat{\theta}_i \rightarrow \hat{\theta}_i^M = \hat{\theta}_j^0 = \sigma \quad (k = 1, \ldots, m; j = 1, \ldots, n, j \neq i^0).
\]

In what follows we shall assume \( i^0 = 1 \) without loss of generality. Then we see from (32) that

\[
A^i \hat{D}^i = (\hat{\theta}_1^0 - \sigma) \{ - e - [\hat{\mathbf{w}}]^{-1} F_{DM}^1 (F_{MM}^1)^{-1} P \} + [\hat{\mathbf{w}}] (\hat{\mathbf{w}} - \sigma e),
\]

\[
A^i \hat{D}^i = [\hat{\mathbf{w}}] (\hat{\mathbf{w}} - \sigma e), \quad i = 2, \ldots, n.
\]

Subtracting (34) from (33), we get

\[
A^i \hat{D}^i = A^i \hat{D}^i - b, \quad i = 2, \ldots, n,
\]

where

\[
b = (\hat{\theta}_1^0 - \sigma) \{ - e - [\hat{\mathbf{w}}]^{-1} F_{DM}^1 (F_{MM}^1)^{-1} P \} > 0.
\]

Substituting \( \hat{D}^i = - \sum_{i=2}^{n} \hat{D}^i \) in (35), we get

\[
A \cdot ((\hat{D}^2)\gamma, \ldots, (\hat{D}^n)\gamma) = (b^\prime, \ldots, b^\prime)\gamma,
\]

where

\[
A \equiv \begin{bmatrix}
\Delta_{22} & \cdots & \Delta_{2n} \\
\vdots & \ddots & \vdots \\
\Delta_{n2} & \cdots & \Delta_{nn}
\end{bmatrix},
\]

and the square matrix \( A_{ij} \) of order \( n \) for \( i, j = 2, \ldots, n \) is given by

\[
A_{ij} = A^i + A^j, \quad \text{if} \quad i = j,
\]

\[
= A^1, \quad \text{if} \quad i \neq j.
\]

By condition (A.5), \( A \) is negative definite by Theorem 3. Therefore eq. (36) has a unique nonzero solution \( ((\hat{D}^2)\gamma, \ldots, (\hat{D}^n)\gamma) \). Hence, \( \hat{D}^1 \) \( (\neq - \sum_{i=2}^{n} \hat{D}^i) \) is also uniquely determined.

Let \( v_{ij} \) and \( u_{kj} \) be the amounts of the \( i \)th factor and the \( k \)th imported input needed in the production of one unit of the \( j \)th good. Let \( v^\prime \) be an \( n \times n \) matrix whose \((i,j)\)th element is \( v_{ij} \) and \( U \) an \( m \times n \) matrix whose \((k,j)\)th element is \( u_{kj} \).
Then we have

\[ V'w + U'P^M = P^0 \quad \text{or} \quad V'[\tilde{w}]e + U'[\tilde{i}^M]e = [P^0]e. \quad (37) \]

Differentiating (37) totally, we have

\[ V'[\tilde{w}]\dot{w} + U'[\tilde{P}^M]\sigma e = [\tilde{P}^e]P^0, \quad (38) \]

where \( \tilde{P}^M = \sigma e \) and \( \tilde{P}^0 = (\tilde{P}_1^0, \ldots, \tilde{P}_n^0)' \) by (B–S) condition. Multiply (37) by \( \sigma \) and subtract it from (38). Then we have that

\[ V'[\tilde{w}](\dot{w} - \sigma e) = (\tilde{P}_1^0 - \sigma)[P^0]e_1, \quad (39) \]

where \( e_j \) is the \( j \)th column vector of the \( n \times n \) identity matrix. We see from (39) that

\[ \dot{w} - \sigma e = (\tilde{P}_1^0 - \sigma)S^{-1}e_1, \quad (40) \]

where \( S \equiv [P^0]^{-1}V'[\tilde{w}] \equiv (s_{ij}) \). \( S \) is the share matrix of order \( n \times n \) with respect to primary factors. Hence we see from (33), (34) and (40) that

\[ A^1 \hat{D}^1 = (\tilde{P}_1^0 - \sigma)[\tilde{w}]e + [\tilde{w}]^{-1}F_{hM}(F_{MM})^{-1}P^M + S^{-1}e_1 \]

\[ \equiv -b + h \quad (41) \]

and

\[ A^i \hat{D}^i = h, \quad (42) \]

where \( h = (\tilde{P}_1^0 - \sigma)[\tilde{w}]S^{-1}e_1. \)

Therefore, we see that both \( -b + h \) and \( h \) have at least one positive and at least one negative element, since from the definitions of \( b \) and \( h \)

\[ (D^1)'(-b + h) = 0 \quad \text{and} \quad (D^i)'h = 0 \quad (i = 2, \ldots, n). \quad (43) \]

Consider the equation

\[ A^1x^1 = -b + h. \quad (44) \]

We see from Theorem 2 that eq. (44) has a semipositive solution \( \bar{x}^1 \) with at least one zero element and \( \bar{x}^1 \) is unique. Similarly, the equation

\[ A^ix^i = h, \quad i = 2, \ldots, n, \quad (45) \]

\(^8\)In order that eqs. (41) and (42) hold, we see by Theorem 1 that \( -h + h \) and \( h \) must have at least one positive and at least one negative element, respectively.
has a unique semipositive solution $\bar{x}^i$ with at least one zero component. Note that $(D^1)^{i}h=(D^1)^{i}b>0$ from (43), since $D^1>0$ and $b>0$.

Since $A^iD^i=0$ ($i=1,\ldots,n$), we see that the general solutions to eqs. (44) and (45) are $\bar{x}^i+tD^i$ ($i=1,\ldots,n$) for any real number $t$.

Now we shall prove the following theorem.

**Theorem 4.** The sufficient and necessary condition for $D^1$ to be positive is that

$$h\left(\sum_{i=1}^{n} \bar{x}^i\right)<0.$$  

**Proof.** (sufficiency). Let $\bar{D}^i=\bar{x}^i+t_iD^i$ ($i=1,\ldots,n$) for some real number $t_i$. Suppose $h'(\Sigma_{i=1}^{n} \bar{x}^i)<0$. Since $\Sigma_{i=1}^{n} \bar{D}^i=0$, we have

$$\sum_{i=1}^{n} \bar{x}^i = -\sum_{i=1}^{n} t_iD^i. \quad (46)$$

The $\bar{x}^i$ values ($i=1,\ldots,n$) are uniquely determined from eqs. (44) and (45), and the $D^i$ values ($i=1,\ldots,n$) are linearly independent. Hence we see that the $t_i$ values ($i=1,\ldots,n$) are uniquely determined from eq. (46). Furthermore, we see from (43) and (46) that

$$h\left(\sum_{i=1}^{n} \bar{x}^i\right)=-t_1 h'D^1, \quad (47)$$

since $h'D^1=0$ ($i=2,\ldots,n$). But since the left-hand side of (47) is negative and $h'D^1>0$, we have $t_1>0$. Hence, $\bar{D}^1=\bar{x}^1+t_1D^1>0$.

(necessity). Suppose that $\bar{D}^1=\bar{x}^1+t_1D^1>0$. Then $t_1>0$, since $\bar{x}^1$ is semipositive with at least one zero component and $D^1>0$. From the proof of the sufficiency we see that eq. (47) holds. Since $t_1>0$ and $h'D^1>0$, we have $h'(\Sigma_{i=1}^{n} \bar{x}^i)<0$. Q.E.D.

The following counterexample shows that the hypothesis of Theorem 4 is not satisfied and therefore $\bar{D}^1$ is not positive.
**Counterexample.** Let

\[
A^1 = \begin{bmatrix} -21 & 6 & 3 \\ 6 & -12 & 6 \\ 3 & 6 & -5 \end{bmatrix}, \quad A^2 = \begin{bmatrix} -4 & 12 & 10 \\ 12 & -52 & 10 \\ 10 & 10 & -125 \end{bmatrix}
\]

\[
A^3 = \begin{bmatrix} -3 & 8 & 5 \\ 8 & -26 & 10 \\ 5 & 10 & -125 \end{bmatrix}
\]

\[
D^1 = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}, \quad D^2 = \begin{bmatrix} 20 \\ 5 \\ 2 \end{bmatrix}, \quad D^3 = \begin{bmatrix} 15 \\ 5 \\ 1 \end{bmatrix}
\]

\[
b = \begin{bmatrix} 1 \\ 7 \\ 1 \end{bmatrix}, \quad h = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}
\]

Then we see that \( h'D^1 = 90 > 0 \), \( h'D^2 = 0 \), for \( i = 2, 3 \) and \( \bar{x}^1 = (1/4, 13/24, 0)' \), \( \bar{x}^2 = (7/16, 1/16, 0)' \) and \( \bar{x}^3 = (5/7, 1/7, 0)' \). Therefore, \( \Sigma_{i=1}^{3} \bar{x}^i = (471/336, 251/336, 0)' \). Hence the hypothesis of Theorem 4 is not satisfied, since \( h' (\Sigma_{i=1}^{3} \bar{x}^i) = 31/336 > 0 \).

A solution to the equation

\[
A \cdot ((D^2)', (D^3)')' = (b', b')'
\]

is \( \bar{D}^2 = (3.6922, 0.8762, 0.3255)' \) and \( \bar{D}^3 = (-3.9371, -1.4076, -0.3101)' \) where

\[
A = \begin{bmatrix} -25 & 18 & 13 & -21 & 6 & 3 \\ 18 & -64 & 16 & 6 & -12 & 6 \\ 13 & 16 & -130 & 3 & 6 & -5 \\ -21 & 6 & 3 & -24 & 14 & 8 \\ 6 & -12 & 6 & 14 & -38 & 16 \\ 3 & 6 & -5 & 8 & 16 & -130 \end{bmatrix}
\]

Thus we see that \( \bar{D}^1 = -\bar{D}^2 - \bar{D}^3 = (0.2449, 0.5314, -0.0154)' \).

The hypothesis of Theorem 4 depends on the share matrix, \( S \), of primary factors and the Hessian matrices, \( F_{jk}^i \), \( i = 1, \ldots, n; \ j, k = D, M \), of production functions. Especially, \( h \equiv [\bar{w}] (\bar{\psi} - \sigma \bar{e}) \) is determined from the share matrix \( S \) of primary factors. Note that Theorem 4 is valid for the case where the number
of primary factors exceeds that of goods, i.e. \( d > n \). However, \( h \) depends on the factor endowment vector \( \bar{D} \) as well as the share matrix \( S \) for the case. The economic meaning of the hypothesis in Theorem 4 is not clear.

Let us evaluate the sign of the change in gross output of the first industry, as well as the change in value added by it:

\[
\dot{F}^1 = (F_D')\dot{D}^1 + (F_M')\dot{M}^1
\]

\[
= (F_D')\dot{D}^1 + (F_M')\left[ \frac{1}{P_{10}} (F_{MM}^1)^{-1} \bar{P}^M \right] (\hat{P}^M - \hat{P}_{10}^0) - (F_{MM}^1)^{-1} F_{MD}^1 \dot{D}^1
\]

\[
= \left[ (F_D') - (F_M')(F_{MM}^1)^{-1} F_{MD}^1 \right] \dot{D}^1
\]

\[
+ (1/P_{10})(F_M')/(F_{MM}^1)^{-1} \bar{P}^M \right] (\hat{P}^M - \hat{P}_{10}^0).
\]  

(48)

Since nominal value added in industry 1 is expressed as \( V^1 = P_{10}^0 F^1 - (P^M)'M^1 \), we get

\[
\dot{V}^1 = \hat{P}_{1i}^0 P_{10}^0 \left[ (F_D'/D^1 + (F_M')M^1 \right] + P_{1i}^0 ((F_D')\dot{D}^1 + (F_M')\dot{M}^1)
\]

\[
- (\hat{P}^M)'[\bar{P}^M]M^1 - (PM)'M^1
\]

\[
= \hat{P}_{1i}^0 w'D^1 + [(\hat{P}_{1i}^0)' - (\hat{P}^M)'][\bar{P}^M]M^1 + w'D^1.
\]

(49)

Furthermore, according to Bhagwati and Srinivasan, the two Divisia indices, \( \hat{P}_{1i}^0 \) and \( \hat{Q}_{1i}^0 \) are defined as

\[
\hat{V}^i = \hat{P}_{1i}^0 + \hat{Q}_{1i}^0,
\]

where

\[
\hat{P}_{1i}^0 = \frac{\hat{P}_{1i}^0 - \sum_{k=1}^{m} \theta_{ik} M \hat{P}_{1k}^0}{1 - \sum_{k=1}^{m} \theta_{ik} M} - \hat{P}_{1i}^0 + \frac{\sum_{k=1}^{m} \theta_{ik} M (\hat{P}_{1i}^0 - \hat{P}_{1k}^0)}{1 - \sum_{k=1}^{m} \theta_{ik} M},
\]

(51)

\[
\hat{Q}_{1i}^0 = \frac{\sum_{j=1}^{n} \theta_{ij} D_j}{\sum_{j=1}^{n} \theta_{ij}},
\]

(52)

in which \( \theta_{ik} M = (P_{ik}^M M_k)/(P_{1i}^0 F^i) \) is the competitive share of the \( k \)th imported input in the \( i \)th output, and \( \theta_{ij} = (w_i D_j)/(P_{1j}^0 F^i) \) is the competitive share of the \( j \)th domestic primary input in the \( i \)th output.

If protection is conferred only on industry 1 (i.e. \( \hat{P}_{1i}^0 > \hat{P}_{1k}^0 = \hat{P}_{10}^0 \), \( k = 1, \ldots, m \); \( i = 2, \ldots, n \)), it can be seen from (51) that this structure results in \( \dot{P}_{1i}^0 > \hat{P}_{1i}^0 \), \( i = 2, \ldots, n \). Furthermore, we have already proved that \( \dot{D}^1 \) is positive under this tariff structure. Hence, we see from (48), (49) and (52) that

\[
\dot{r}^1 > 0, \dot{V}^1 > 0 \text{ and } \hat{Q}_{1i}^0 > 0.
\]

(53)
Hence, if protection is conferred only on industry 1 by the change in the
tariff structure, then there will be a positive resource inflow into this industry
if and only if the hypothesis of Theorem 4 is satisfied, and its gross output,
nominal value added and Q^\text{t}_l will go up. ERP theory will work, in the sense
defined by B S.

Note that the gross output will go up independently of the hypothesis in
Theorem 4, as Bhagwati and Srinivasan have already proved. This is seen
from the following: We have from (36)

\[(\mathbf{D}^2)^{(i)}, \ldots, (\mathbf{D}^p)^{(i)} A((\mathbf{D}^2)^{(i)}, \ldots, (\mathbf{D}^p)^{(i)})' = \sum_{i=2}^{i=n} (\mathbf{D}^i)^{(i)}' \mathbf{h} = - (\mathbf{D}^1)' \mathbf{h}\]

\[= (\mathbf{D}^0 - \sigma) \mathbf{P}_1^{(i)} (\mathbf{F}_1^{(i)})' (\mathbf{F}_1^{(i)})' \mathbf{F}_1^{(i)} \mathbf{D}^1 < 0,\]

since \( A \) is negative definite. Hence, \( (\mathbf{F}_1^{(i)})' (\mathbf{F}_1^{(i)})' \mathbf{F}_1^{(i)} \mathbf{D}^1 > 0 \) and therefore we see from (48) that \( \mathbf{F}^1 > 0 \).

5. The Stolper–Samuelson theorem

First, we define factor intensity in the sense that the \( i \)th factor \( (i=1, \ldots, n) \)
is intensive in the production of the \( i \)th good, as follows.\(^9\) Let \( J \subset N \) and
\( J \neq N \), and \( I \) be the diagonal matrix obtained from an identity matrix by
replacing each \( j \)th row \( e^j \) of the identity matrix by \( -e^j \) \( (j \in J) \), where \( N \) is the
set \( \{1, \ldots, n\} \) and \( \subset \) a proper inclusion.

**Condition [I].** For any nonempty proper subset \( J \) of \( N \) and any given
positive vector \( x_j > 0 \), the inequality

\[(x_j' \mathbf{x}_j)[I_j SI_j] > 0\]

has a solution \( x_J > 0 \).\(^{10}\)

Let \( \delta_j \) be the sum of the shares of primary factors in the \( j \)th sector, i.e. \( \delta_j = \sum_{i=1}^{i=n} s_{ij} = (P_0^j)^{-1} \sum_{i=1}^{i=n} v_{ij} w_j \). Let \( \gamma_j \) be the sum of the shares of imported inputs in the \( j \)th sector, i.e. \( \gamma_j = (P_0^j)^{-1} \sum_{i=1}^{i=n} u_{ij} p_i^M \). Let \( \delta = (\delta_1, \ldots, \delta_n)' \) and \( S' \)
be an \( n \times n \) matrix obtained from \( S \) by replacing the \( j \)th column vector of \( S \)
by \( \delta \).

**Condition [II].**

(i) \( \delta_j \geq \gamma_j \) \( (j = 1, \ldots, n) \),


\(^{10}\)Condition [I] is equivalent to a non-negative matrix \( S \) that has the Minkowski property, i.e. it possesses an inverse with \( s_{ii} \geq 0 \) and \( s_{ij} \leq 0 \) for \( i \neq j \). Especially, since the share matrix \( S \) is positive, \( s_{ii} > 0 \), and any column and any row of \( S^{-1} \) have at least one negative element. See Uekawa, Kemp and Wegge (1973, theorem 3).
(ii) $S'_{ij}(j=1,\ldots,n)$ has a dominant diagonal (or d.d.).

Note that condition [II] implies that $S$ and $S'_{ij}(j=1,\ldots,n)$ are $P$-matrices.\(^{11}\)

Next, we shall define the Stolper–Samuelson (1941) criteria, extended to the present model involving imported intermediates, as follows. Suppose that a change in the tariff structure such as $(B-S)$, i.e. $\hat{p}^0_{ik} > \hat{p}^0_{kj} = \sigma$ ($k=1,\ldots,m$; $j=1,\ldots,n$, $j \neq i^0$) for any given $i^0$ ($-1,\ldots,n$) will take place.

The strong Stolper–Samuelson criterion. The price of the corresponding intensive factor (i.e. $i^0$th factor) will go up more than $\hat{p}^0_{i^0}$, while some other factor prices may increase too, their increase rate must not be larger than $\sigma$, i.e. $\hat{w}_{i^0} > \hat{p}^0_{i^0}$ and $\hat{w}_j \leq \sigma$ for any $j \neq i^0$.

The weak Stolper–Samuelson criterion. The price of the corresponding intensive factor (i.e. $i^0$th factor) will go up more than $\hat{p}^0_{i^0}$, while some other factor prices may increase too, their increase rate must not be larger than $\hat{p}^0_{i^0}$, i.e. $\hat{w}_{i^0} > \hat{p}^0_{i^0}$ and $\hat{w}_j \leq \hat{p}^0_{i^0}$ for any $j \neq i^0$.

**Theorem 5.** Condition [I] is necessary and sufficient for the strong Stolper–Samuelson theorem to hold.

**Proof (necessity).** We see from (40) that

$$\hat{w} - \sigma e = (\hat{p}^0_{i^0} - \sigma) S^{-1} e_{i^0},$$

for any given $i^0$ ($-1,\ldots,n$).

Since

$$\hat{w}_{i^0} > \hat{p}^0_{i^0} \quad \text{and} \quad \hat{w}_j \leq \sigma, \quad \text{for any} \ j \neq i^0,$$

and $S$ is a positive matrix, we see from (54) that $S^{-1} = (s^{ij})$ is indecomposable with positive diagonal and nonpositive off-diagonal elements, i.e.

$$s^{ii} > 0 \quad \text{and} \quad s^{ij} \leq 0, \quad \text{for} \ i \neq j.$$

By Uekawa–Kemp–Wegge's theorem [see Uekawa, Kemp and Wegge (1973, theorem 3)], (56) is equivalent to condition [I].

---

\(^{11}\)An $n \times n$ matrix $A = (a_{ij})$ is said to be a $P$-matrix if all its principal minors are positive. An $n \times n$ matrix $A$ is said to have a dominant diagonal (or d.d.) if there are $d_j > 0$, $j=1,\ldots,n$ such that

$$d_j a_{jj} > \sum_{i \neq j} d_i a_{ij} \quad (j=1,\ldots,n).$$

If a matrix $A$ with positive diagonal elements has a d.d., then $A$ is a $P$-matrix. See McKenzie (1960)
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(sufficiency). Since Condition [I] is equivalent to (56), (54) implies that

\[ \hat{w}_j \leq \sigma, \text{ for any } j \neq i^0. \]  

(57)

Furthermore, we have from (54) that

\[ \hat{w} - \hat{P}_i^0 e = \hat{P}_i^0 \delta_i^0 [S_1 e - e]. \]  

(58)

Let us show that

\[ \hat{w}_i^0 > \hat{P}_i^0, \]  

(59)

Since \( S\delta = \delta \), we have that

\[ S^{-1}\delta = e. \]  

(60)

The \( i^0 \)th component of (60) is

\[ s_i^{i^0} \delta_i^0 + \sum_{j \neq i^0} s_j^{i^0} \delta_j = 1 \]

and therefore, we see from (56) that

\[ s_i^{i^0} \delta_i^0 = 1 - \sum_{j \neq i^0} s_j^{i^0} \delta_j > 1, \]  

(61)

since \( S^{-1} \) is indecomposable. Thus, we have from (61) that

\[ s_i^{i^0} > 1/\delta_i^0 \geq 1, \]  

(62)

since \( 1 \geq \delta_i^0 > 0. \)

We see from (57), (58) and (62) that \( \hat{w}_i^0 > \hat{P}_i^0 \) and \( \hat{w}_j \leq \sigma \) for any \( j \neq i^0. \) Q.E.D.

Theorem 6. If Condition [I] is satisfied, then ERP theory under the B-S restriction on tariff change will work in the sense defined by Bhagwati and Srinivasan.

Proof. We assume \( i^0 = 1 \) without loss of generality. Consider eq. (44):

\[ A^1 x^1 = (\hat{P}_i^0 - \sigma)^{-1} w + F_{DM} (F_{MM}^{-1} P^M + [\hat{w}] S^{-1} e_1) \equiv -b + h \equiv q. \]

(44)
Then, we see from (56) that

$$q_1 > 0 \quad \text{and} \quad q_j < 0, \quad \text{for} \quad j = 2, \ldots, n, \quad (63)$$

since $$(D^1)'q = (P^0 - \sigma)(- (D^1)'w - (M^1)'P^M + P^0 P^1) = 0.$$ Furthermore, (63) implies that

$$\hat{x}_1^i = 0 \quad \text{and} \quad \hat{x}_j^i > 0, \quad \text{for} \quad j = 2, \ldots, n. \quad (64)$$

For suppose that $\hat{x}_i^1 = 0$ for some $i \neq 1$. Then we must have from (44) that

$$q_i = \sum_{j=1}^{n} a_{ij} \hat{x}_j^1 \geq 0, \quad \text{for} \quad i \neq 1,$$

a contradiction. Hence (64) is true.

Since $h = (P^0 - \sigma)[\tilde{w}]S^{-1}e_1$, (56) implies that

$$h_1 > 0 \quad \text{and} \quad h_j \leq 0, \quad (65)$$

with at least one strict inequality for $j = 2, \ldots, n$. Thus, we have from (64) and (65) that

$$h'\hat{x}^1 = \sum_{j=2}^{n} h_j \hat{x}_j^1 < 0. \quad (66)$$

Since $A^1$ is negative semidefinite and $\hat{x}^i$ is not proportional to positive $D^i$, we see from (45) and Theorem 1 that

$$(\hat{x}^i)'h = h'\hat{x}^i < 0, \quad \text{for} \quad i = 2, \ldots, n. \quad (67)$$

Eqs. (66) and (67) imply that $h'(\Sigma_{i=1}^{n} \hat{x}^i) < 0$. Hence, we have from Theorem 4 that $\hat{D}^1 > 0$. Q.E.D.

**Theorem 7.** If condition [II] is satisfied, then the weak Stelper–Samuelson theorem holds.

**Proof.** We assume that $i^0 = 1$ without loss of generality. Let $S_{ij}$ be the cofactor of $s_{ij}$. Then, we get for any $j \neq 1,$
\[
\begin{vmatrix}
S_{11} & S_{12} & \ldots & S_{1n} \\
S_{21} & S_{22} & \ldots & S_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
S_{n1} & S_{n2} & \ldots & S_{nn}
\end{vmatrix}
- \begin{vmatrix}
S_{11} & \cdots & S_{1,j-1} & 1 & S_{1,j+1} & \cdots & S_{1n} \\
S_{21} & \cdots & S_{2,j-1} & 0 & S_{2,j+1} & \cdots & S_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{n,j-1} & 0 & S_{n,j+1} & \cdots & S_{nn}
\end{vmatrix}
= \begin{vmatrix}
S_{11} & \cdots & S_{1,j-1} & S_{1j} - 1 & S_{1,j+1} & \cdots & S_{1n} \\
S_{21} & \cdots & S_{2,j-1} & S_{2j} & S_{2,j+1} & \cdots & S_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{n,j-1} & S_{nj} & S_{n,j+1} & \cdots & S_{nn}
\end{vmatrix}
= \begin{vmatrix}
S_{11} & \cdots & S_{1,j-1} & \delta_1 - 1 & S_{1,j+1} & \cdots & S_{1n} \\
S_{21} & \cdots & S_{2,j-1} & \delta_2 & S_{2,j+1} & \cdots & S_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
S_{n1} & \cdots & S_{n,j-1} & \delta_n & S_{n,j+1} & \cdots & S_{nn}
\end{vmatrix}
\equiv \det E^j,
\]

where the third equality results from adding the sum of the \( n - 1 \) columns, except the \( j \)th one, to the \( j \)th column.

First we shall show that the matrix \( E^j \) \( j = 2, \ldots, n \) is a P-matrix. Since \( S^j \) has a d.d. by condition [II-(ii)], we see that there exist \( d_k > 0, k = 1, \ldots, n \) such that

\[
\tilde{s}_k s_k > \sum_{i=1}^{n} d_i s_{ik}, \quad \text{for } k = 1, \ldots, n \quad \text{and} \quad k \neq j,
\]

and

\[
d_j \delta_j > \sum_{i=1}^{n} d_i \delta_i.
\]

By condition [II-(ii)], (70) implies that

\[
d_j \delta_j > \sum_{i=1}^{n} d_i \delta_i \geq d_1 \gamma_1 + \sum_{i=j}^{n} d_i \delta_i = d_1 |\delta_1 - 1| + \sum_{i=j}^{n} d_i \delta_i,
\]

since \( \delta_1 \geq \gamma_1 = 1 - \delta_1 = |\delta_1 - 1| \). Thus, we see from (69) and (71) that \( E^j \) has a d.d. and therefore, is a P-matrix. Hence, we have from (68) that

\[
\det S > S_{1j}, \quad \text{for } j = 2, \ldots, n.
\]
On the other hand, we have

\[
\det S - S_{11} = \begin{vmatrix}
    s_{11} & s_{12} & \cdots & s_{1n} \\
    s_{21} & s_{22} & \cdots & s_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{n1} & s_{n2} & \cdots & s_{nn}
\end{vmatrix} - \begin{vmatrix}
    1 & s_{12} & \cdots & s_{1n} \\
    0 & s_{22} & \cdots & s_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & s_{n2} & \cdots & s_{nn}
\end{vmatrix}
\]

\[
= \begin{vmatrix}
    \delta_1 - 1 & s_{12} & \cdots & s_{1n} \\
    \delta_2 & s_{22} & \cdots & s_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \delta_n & s_{n2} & \cdots & s_{nn}
\end{vmatrix}
\]

\[
= -\gamma_1 \left| S\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right| - \sum_{j=2}^{n} s_{1j} \left| S^{i}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right|,
\]

where \(A(i)\) is a matrix obtained from \(A\) by deleting the first row and the first column of \(A\). Since both \(S\) and \(S'\) are \(P\)-matrices, \(S(i)\) and \(S'(i)(j=2,\ldots,n)\) are also \(P\)-matrices. Thus, we see from (73) that

\[S_{11} > \det S > 0.\]

Let \(S^{-1} = (s^{ij})\). Then we get from (72) and (74) that

\[s^{11} > 1 \quad \text{and} \quad s^{ij} < 1, \quad \text{for} \quad j=2,\ldots,n.\]

Thus, we see from (21) that

\[\hat{\omega}_1 > \hat{\rho}_1^0 \quad \text{and} \quad \hat{\omega}_j < \hat{\rho}_0^j, \quad \text{for} \quad j=2,\ldots,n.\]

We see that the weak Stolper–Samuelson theorem holds. Q.E.D.

It is an open question whether under condition [II] the ERP theory as defined by Bhagwati and Srinivasan holds or not.

References


