Answer any two questions; answer all parts to each question.

1. Answer all parts

a) Consider the specific factor model with two goods (C, M), one mobile factor (L), and one factor specific to each sector \( (K_c, K_m) \). Each good is produced under constant returns to scale, using the two factors \( Q_i = F^i(K_i, L_i); \quad i = C, M \). Do the magnification and Stolper-Samuelson theorems hold for this model? Does trade bring factor price equalization? Explain.

b) Consider a two country Ricardian model with a continuum of goods. Call the countries US, Europe (E); index the goods by \( z \in \{0, 1\} \); let \( A(z) \) denote the labor requirement per unit output in each country, and order the goods so that: \( A^U(z) < A^E(z) \), \( \forall z \). Assume preferences are Cobb-Douglas and internationally identical so that (flow) demand for good \( z \) in country \( i \) is given by: \( D^i(z) = \alpha(z)Y^i / P^i(z) \), where \( Y^i \) denotes income, and \( \int_0^1 \alpha(z)dz = 1 \).

i. Given the labor stocks for each country, find the trading equilibrium and compare to the autarky equilibrium for each country. If the US is uniformly more productive than Europe (i.e., \( A^U(z) < A^E(z) \) \( \forall z \)) how can European producers compete with U.S. producers?

ii. Show how a uniform increase in U.S. productivity (i.e., the new US labor input requirement, \( A^U(z) = \left( A^U(z) / \theta \right), \theta > 1 \)) affects: (1) the pattern of trade (which goods the U.S. exports); (2) real wages in each country; and (3) relative prices of goods. How is welfare in each country affected by this productivity increase?

c) Consider the N good, M country extension of the H-O model. Under the usual assumptions (identical and homothetic tastes, constant returns to scale), can you determine the pattern of trade in goods if \( N > M \)? Explain. Assuming factor price equalization holds, what can you deduce about the factor content of trade? Prove your answer.
2. Consider a general equilibrium trade model with N goods and M factors. Assume there are J competitive firms, each with its own convex production set, that allows the firm to produce the N final goods using the M primary factors of production. Each firm behaves as a competitive profit-maximizer, where \((\bar{p}^f, \bar{w}^f)\) denote the output, and input, price vector facing the firm. Let \(V\) denote the aggregate input vector, and assume (for now) there are no production externalities, so that \(R(\bar{p}^f, V)\) denotes the GNP function for this economy.

Also, there are \(H\) consumers, with identical and homothetic preferences, and each has an exogenous endowment vector of primary inputs \((\bar{x}^h)\). Let \(\bar{c}^h\), \(U(\bar{c}^h)\) denote the household’s consumption vector, and preferences, respectively, \(\{\bar{p}^c, \bar{w}^c\}\) the output, input price vector facing households, and \(T^h\) the net government transfer to households. The household maximizes utility, subject to the budget constraint: 
\[
\sum p_i^c c_i^h \leq \sum w_i^c x_i^h + T^h.
\]

The government budget constraint requires that net tax revenue be zero. In a laissez-faire equilibrium there are neither lump sum taxes nor commodity or factor taxes \((T^h = 0, \quad \bar{p}^f = \bar{p}^c, \quad \bar{w}^f = \bar{w}^c)\).

a) Assuming no distortions and no domestic taxes, prove that the movement from autarky to trade leads to a potentially Pareto superior allocation, provided net tariff revenues are non-negative (and assuming lump sum transfers are feasible). If there are no lump sum transfers or taxes of any kind, must everyone gain from trade? Explain carefully.

b) Modify preferences so that household utility depends on consumption of primary inputs \((\bar{s}^h)\) as well as consumption of final goods, so that the household optimization problem becomes: 
\[
\max_{\{\bar{c}^h, \bar{s}^h\}} U(\bar{c}^h, \bar{s}^h) \text{ such that: } \sum p_i^c c_i^h \leq \sum w_i^c x_i^h - s_i^h + T^h.
\]
Assume lump-sum taxes are not feasible \((T^h = 0)\), and that world prices are exogenous. Assume the government’s objective is to maximize social welfare, \(S\) (where \(S = \phi(U^1, ..., U^H)\)), \(\partial S / \partial U^h > 0\). Derive the optimal policy to achieve this objective; are trade restrictions part of the optimal policy? Explain. If only trade policy were feasible, would free trade maximize social welfare? {Note: if you cannot formally derive the optimal policy, discuss its characteristics}.

c) For part (c), assume utility does not depend on consumption of primary inputs and assume all consumers have the same endowment vector (income distribution issues can be ignored). Further, assume that production of good one generates a negative externality that hurts consumers (the externality depends only on total output of good one, not on which firms produced the good). Thus, consumer preferences are modified to: 
\[
U^h = U(\bar{c}^h) \cdot \beta(Q_1);
\]
\([d\beta/dQ_1] < 0\). For this case, will the movement from autarky to trade be beneficial? Explain carefully and prove your answer. (you may assume there are only two goods, if you want). Can trade policy can be used to improve welfare if no other policy is feasible? Explain.
3. Answer all parts

a) Consider the standard two country (US, E), two factor (K, L), two good (C, M) H-O model. Using the usual assumptions (constant returns to scale, no joint production, identical & homothetic preferences,...) briefly discuss the main conclusions of the model concerning trade pattern and factor prices. Also, discuss the conditions under which factor price equalization occurs; can factor price equalization hold even if the technology allows for factor intensity reversals? (relate your answer to the integrated equilibrium).

i. Which of the basic propositions of the model (Stolper-Samuelson, magnification, factor price equalization,...) would still hold in the face of joint production? Explain.

b) Consider a variant of the standard two country (US, E), two final good (C,M) H-O model in which there are 2 primary factors (K, L) and one produced intermediate good (S). The intermediate good is used to produce two final goods (C,M), but is not consumed. All production functions (given below) exhibit constant returns to scale, are internationally identical, and preferences are identical and homothetic.

\[ Q_c = F^c(K_c, L_c, S_c); \quad Q_m = F^m(K_m, L_m, S_m); \quad Q_s = F^s(K_s, L_s) \]

If the intermediate good is not traded, then - in addition to the resource constraints on primary inputs - the constraint: \( S_c + S_m \leq Q_c \) must hold.

i. What determines the pattern of trade in this model and will factor price equalization occur? Prove your answer (be as specific as possible).

ii. How, if at all, would your predictions concerning the pattern of trade and FPE be altered if all three goods could be traded?

c) Use the model of part (b) but assume that, while technologies for producing C and M are internationally identical, the technology for producing S may vary across countries. For simplicity, consider a small country (Denmark) facing given world prices: \( (P_c = P_m = P_s = 1) \). Assume the dual cost curves are given by:

\[ TC^d(Q_c, W, R) = Q_c \left[ W^{2/3} R^{1/3} \right]; \quad TC^d(Q_m, W, R, P_s) = Q_s \left[ W^{1/6} R^{1/6} P_s^{2/3} \right]; \quad TC^d(Q_s, W, R) = (Q_s / \delta) \left[ W^{1/4} R^{3/4} \right] \]

Preferences are internationally identical and given by: \( U = CM \). Factor endowments in Denmark are given by: \( K^d = L^d \).

i. Assume \( \delta = 1 \). Given world prices, and assuming good S is not tradable, find factor prices and the price of good S in Denmark. If good S were also tradable, would this alter the equilibrium?

ii. Next, assume \( \delta = 2 \); treating world prices as fixed, find the equilibrium factor prices and production pattern in Denmark assuming S is not tradable. Repeat when good S is tradable. Which equilibrium will result in a more efficient allocation of resources? Explain.