1. **Simple Ricardian model:**

Assume two countries (US, Mexico) and two goods (C, F). Labor productivities are given by:

<table>
<thead>
<tr>
<th></th>
<th>Cloth</th>
<th>Food</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Mexico</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

The labor force for the US and Mexico are $L, \bar{L}$ respectively.

a) Find the autarky equilibrium prices.

b) Find the pattern of trade (which good each country exports). Explain why demand is not needed to answer this question.

c) Suppose US productivity increases by the same proportion in each sector (to $4\lambda, 8\lambda; \lambda > 1$). Explain how this would affect autarky prices and real wages in the US.

d) How does the productivity increase in part (c) affect the post-trade equilibrium pattern of trade and equilibrium prices. How is Mexico affected by the US productivity growth? How is the US affected by this productivity growth?

2. **Multi-country Ricardian model.**

Consider a two good, 4 country Ricardian model with the following labor productivities in each country.

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Mexico</th>
<th>Japan</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloth</td>
<td>8</td>
<td>2</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Food</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Find autarky prices and real wages in each country. Can you state which country has the highest real wages?

b) Assuming free trade, what can you predict about the pattern of trade? Be specific.

c) Suppose labor productivity in the US increases by 25% in both sectors. How will this affect post-trade prices, the pattern of trade and welfare in each country. Be as specific as possible.

d) Given Cobb-Douglas demands, with equal shares for each good, and assuming the same size labor force in each country, use the productivity figures from the table above to find the post-trade equilibrium.

e) Repeat (d) if US productivity is $8\lambda, 16\lambda; \lambda > 1$. Show how the equilibrium changes as $\lambda$ increases.

3. **Gains from trade and market failures.**

Consider a “small country” Ricardian model. Suppose Costa Rica can produce two goods, food (F) and cloth (C). The production function for each good is given by:

$$Q_f = 4L_f, \quad Q_c = 2L_c$$
where $Q_i$ is output of good $i$ and $L_i$ is the amount of labor allocated to production of good $i$. There are $L$ workers, each endowed with one unit of labor which is supplied inelastically to produce output. The production of cloth generates pollution, $Z$, the aggregate output of which is proportional to cloth output:

$$Z = \alpha Q_c$$

All workers have identical preferences given by:

$$U = \left( D_i D_f \right)^{1/2} - \mu Z; \quad \mu > 0$$

where $D_i$ is consumption of good $i$.

a) Find autarky (no trade) consumption, production and utility. Is it (pareto) efficient? If not, how should resource allocation change in order to improve efficiency? (a qualitative answer suffices).

For the remainder of the question, assume that because it is small - Costa Rica can trade at the exogenously given world prices: $\rho^w \equiv \left( P^w_c / P^w_f \right)$, where the “w” stands for world prices.

b) Will Costa Rica necessarily gain from trade? Relate your answer to the magnitude of $\rho^w$ - that is, to the pattern of trade.

c) Assume $\rho^w < 2$. Will free trade necessarily benefit Costa Rica and will it be optimal? If not, are import tariffs/subsidies the optimal policy and, if not, what is?

d) Assume $\rho^w > 2$. Will free trade necessarily benefit Costa Rica and will it be optimal? If not, are import tariffs/subsidies the optimal policy and, if not, what is?

4. Consider the Dornbusch-Fischer-Samuelson model of Ricardian production. Assume there is a continuum of goods that can be produced in each country; let $z \in [0,1]$ index each good. Further, let $a(z)$ denote the output productivity of labor in the home country for good $z$, and $a^*(z)$ the labor productivity in the foreign country. Define: $A(z) = \frac{a(z)}{a^*(z)}$ and assume $A'(z) < 0$. Let $W$ denote the home wage rate, $W^*$ the foreign wage rate and define $w = \frac{W}{W^*}$. Similarly, let $L$ denote the home labor force and $L^*$ the foreign labor force.

Finally, assume demands are Cobb-Douglas, are identical in the two countries and are given by:
\[ C(z) = b(z) \frac{Y}{P(z)} \] where \( Y \) denotes income, \( P(z) \) denotes price of good \( z \), and \( b(z) \) denotes the share spent on good \( z \) such that: \[ \int_{z=0}^{1} b(z) dz = 1. \] Note that the resource constraints are given by:

\[
\int_{z=0}^{1} \left( \frac{Q(z)}{a(z)} \right) dz \leq L; \quad \int_{z=0}^{1} \left( \frac{Q^*(z)}{a^*(z)} \right) dz \leq L^*
\]

where \( Q(z), Q^*(z) \) denote the output of good \( z \) in the home and foreign country, respectively.

a. Graphically show how the pattern of trade is determined and how an increase in \( L \) affects trade and real wages in each country.

b. (Transfer problem) Suppose the home country is required to make a real transfer of \( T \) to the foreign country (foreign aid, reparations from a war, etc.). How will that affect equilibrium prices? How would your answer change if preferences were different, such that \( b^*(z) \) represented foreign preferences, and \( \int_{0}^{\phi} b^*(z) dz > \int_{0}^{\phi} b(z) dz \) for all \( 0 < \phi < 1 \), with \( \int_{0}^{1} b^*(z) dz = 1 \).

c. Transportation costs – iceberg model. Suppose that a constant fraction of each good shipped between countries “melts” so that if \( X \) units are exported, only \( (1-t)X \) units actually arrive. Show that, for this case, there are numbers \( \phi_1, \phi_2 \) such that: \( 0 < \phi_1 < \phi_2 < 1 \) so that all goods \( z < \phi_1 \) are exported by the home country, goods \( z > \phi_2 \) are exported by the foreign country and for \( z \in [\phi_1, \phi_2] \) the goods are not traded. Write down the equilibrium conditions for determining \( z \).

d. Finally, assume: \( a(z) = 2\lambda; \quad a^*(z) = (1+z); \quad b(z) = b^*(z) = 1 \). Assuming transportation costs (as in c), solve for the equilibrium relative wage, and \( \phi_1, \phi_2 \) as defined in part c. Show how the solution is affected by:
   (i) a reduction in transportation costs.
   (ii) an increase in \( \lambda \).