Countries often perceive themselves as being in competition with each other for profitable international markets. In such a world export subsidies can appear as attractive policy tools because they improve the relative position of a domestic firm in noncooperative rivalries with foreign firms, enabling it to expand its market share and earn greater profits. In effect, subsidies change the initial conditions of the game that firms play. The terms of trade move against the subsidizing country, but its welfare can increase because, with imperfect competition, price exceeds the marginal cost of exports. International noncooperative equilibrium involves such subsidies by producing nations, even though they are jointly suboptimal.

1. Introduction

Considerable recent attention has been focused on the role of export subsidies in international trade policy. Effective subsidization of firms engaged in international rivalry has been a common practice in Western economies for some time, and there seems to be a growing belief that foreign subsidization of exports is 'unfair' and merits some sort of retaliation.

Such policies do not appear to make much sense from the standpoint of two-good competitive models of international trade. Even in markets where the domestic country can exercise some influence over world prices, the domestic interest is served by trade restriction, not by subsidization of trade.\(^1\) If foreigners wish to subsidize us to consume the goods they produce, so much the better for us.

\(^*\)We would like to thank Ron Jones for very helpful discussions in the early stages and we also thank an anonymous referee for inducing us to improve our exposition and for offering several suggestions leading to improvements in the substance of the paper.

\(^1\)One important extension of the standard optimum tariff argument, considered by Graaf (1949–50) and developed by Feenstra (1983) is that in an n-good world \((n \geq 3)\) the optimal tariff structure may involve export subsidies on some goods because of what Feenstra calls 'export linkages'.

How then are we to understand arguments in favor of export subsidization and in favor of retaliation against foreign subsidization. Rather obviously, domestic producers who stand to gain from protection or subsidization would be strong proponents of such arguments. Still, the alleged success of Japanese policies, for example, suggests that there may be more to the issue than just this.

In this paper we present an analysis based on imperfect competition to explain why export subsidies might be attractive policies from a domestic point of view. The central idea is that it is to the advantage of a country to capture a large share of the production of profit-earning imperfectly competitive industries.\(^2\) Export subsidies can be used to carry out such 'profit-shifting' policies. Such a motive for subsidization requires the presence of (at least) two exporting countries. We also assume a third country which imports the imperfectly competitive good.

Our results also depend (of course) on the equilibrium concept we use. The industry in question is modelled as a simple Cournot (or Nash quantity) duopoly: firms take as given subsidy levels set by governments and output levels set by their rivals. Governments are able to act first, and set subsidy levels before output levels, using their understanding of how subsidies influence the output equilibrium. We first consider a single government, then examine a Nash equilibrium in government subsidy levels. As expressed by the referee 'firms play Nash against all other players, and governments play Stackelberg against firms and Nash against other governments'. A slightly different expression of the same structure is that the equilibrium is subgame perfect\(^3\) in a two-stage model in which governments (simultaneously) choose subsidy levels in the first stage and firms (simultaneously) choose output levels in the second stage.\(^4\)

In relating our analysis to the literature of international trade theory there are several themes that should be mentioned. Recent papers by Spencer and Brander\(^5\) (1983) and Krugman (1984) explicitly use the idea that national governments may wish to help domestic firms expand market shares in

\(^2\)We model the rents as accruing to residual profit-takers in the firm. One could easily imagine more complex and more realistic structures in which benefits also accrue to workers in the form of wages above opportunity cost, or in expanded employment at a (high) fixed wage.

\(^3\)The subgame perfection concept is well exposted in Shubik (1982). The basic idea is that each player anticipates that other players will act in their own best interests when they choose the levels of their strategy variables: these are the only 'credible' choices.

\(^4\)Why the governments are able to act first is something we do not model explicitly. In essence the government subsidy announcements are simply assumed to be credible. This government credibility may arise because a government has some reputation which it has an incentive to maintain.

\(^5\)Spencer and Brander (1983) is a companion paper to this which shares the idea the governments may wish to help domestic firms capture large shares of profit-earning industries. In that paper governments subsidize R&D activities while firms themselves use R&D for strategic purposes. This gives rise to a farily complicated structure in which the simple general equilibrium aspects are surpressed.
profitable areas. This is tangentially related to Basevi (1970), Frenkel (1971),
and Pursell and Snape (1973), where a domestic monopolist can benefit by
exporting, and to Brander and Spencer (1981), where a government may
promote entry of a domestic firm which can earn rent from foreign sales.
One can also connect our arguments to the 'distortions' literature associated
with, among others, Bhagwati (1971). Specifically, the possibility of a second-
best policy, like subsidies, being in the national interest depends on the
existence of some distortion, which in our case is imperfect competition. The
focus here, however, is really quite different, since it is the effect of
government policies on the strategic interaction between domestic and
foreign firms which is of the essence.

Section 2 sets out the basic model and shows the unilateral incentive of a
government to subsidize exports. In section 3 the argument is presented in a
simple general equilibrium framework, incorporating domestic consumption
of the imperfectly competitive good. The effects of the subsidy are interpreted
using terms of trade and output effects, and the relationship between our
results and the standard optimum tariff argument is discussed. Section 4
extends the analysis to the case in which governments in both exporting
countries may subsidize exports, leading to a Nash equilibrium in subsidies.
Section 5 extends the analysis further to include the optimal response of the
government in the importing nation, and section 6 contains concluding
remarks.

2. The model

We use the simplest possible structure capable of bringing out the main
points. As mentioned, firm behaviour is modelled as a simple Nash quantity
(or Cournot) duopoly, with one domestic firm and one foreign firm, who
produce identical products. We assume (for this section) that both firms
produce only for third markets: there is no consumption in the producing
countries. An important assumption is that the government understands the
structure of the industry and is able to set a credible subsidy on exports in
advance of the quantity decision by firms.

The domestic firm produces quantity $x$ and the foreign firm produces $y$.
The domestic firm maximizes variable profit $\pi$:

$$\pi(x, y; s) = xp(x + y) - c(x) + sx,$$  \hspace{1cm} (1)

where $c$ is variable cost, $s$ is a per unit subsidy, and $p(x + y)$ is the (inverse)

These assumptions could be relaxed to some extent without affecting the central principles
under consideration. However, adding more domestic firms weakens the incentive for domestic
subsidies [see Dixit (1984) and Salant (1984)], and the introduction of more imperfectly
competitive industries may weaken the subsidy case for any one industry [see Dixit and
Grossman (1984)]. In addition, changing the nature of the oligopolistic rivalry between firms
may also affect the results, as shown in Eaton and Grossman (1983).
world demand (or price) for the good. There may be some additional sunk
cost which explains the existence of imperfect competition in this industry. It
is omitted since it plays no role in our analysis. The first-order condition for
profit maximization is then:

$$\pi_x = xp' + p - c_x + s = 0,$$

(2)

with second-order condition:

$$\pi_{xx} = 2p' + xp'' - c_{xx} < 0,$$

(3)

where derivatives are denoted by subscripts except for $p'$, the derivative of
inverse demand.

Similarly, the variable profit of the foreign firm, $\pi^*$, is given by:

$$\pi^* = yp(x + y) - c^*(y),$$

(1*)

leading to first- and second-order conditions:

$$\pi_y^* = yp' + p - c^*_y = 0,$$

(2*)

$$\pi_{yy}^* = 2p' + yp'' - c_{yy}^* < 0.$$ 

(3*)

We also use the following conditions:

$$\pi_{xy} \equiv p' + xp'' < 0; \quad \pi_{yx}^* \equiv p' + yp'' < 0,$$

(4)

$$\pi_{xx} < \pi_{xy}; \quad \pi_{yy}^* < \pi_{yx}^*.$$ 

(5)

Condition (4) means that own marginal revenue declines with an increase in
the output of the other firm. This is equivalent, given satisfaction of the
second-order conditions, to reaction functions being downward sloping. This
is a fairly standard regularity condition in noncooperative models, but it can
be violated by feasible demand structures, in particular if demand is very
convex. From second order conditions (3) and (3*) and from (4), condition
(5) always holds if marginal cost is nondecreasing. Only if marginal cost falls
more steeply than demand can it be violated. Condition (5) means that own
effects of output on marginal profit dominate cross effects.

Conditions (4) and (5) imply:

$$D \equiv \pi_{xx}\pi_{yy}^* - \pi_{xy}\pi_{yx}^* > 0.$$ 

(6)

If conditions (3), (3*) and (6) hold globally, they imply global uniqueness of
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the equilibrium [see Nikaido (1968, ch. 7)]. Condition (6) is also the Routh-Hurwitz condition for reaction function stability.

Rather obviously, comparative static properties of the model will depend on these conditions. If one wishes to consider cases in which the conditions are violated, 'perverse' comparative static properties and policy implications can be obtained. This is of some interest, but we focus on structures which satisfy (4) and (5), since they include most economically relevant cases.

First-order conditions (2) and (2*) are reaction functions for the two firms in implicit form. Each shows the best response of the firm to any particular output chosen by its rival. The simultaneous solution to (2) and (2*) is the noncooperative solution.

What is the effect of the subsidy, s? It is fairly easy to calculate the comparative static effects dx/ds and dy/ds. Total differentiation of first-order conditions (2) and (2*) yields:

$$\pi_{xx} dx + \pi_{xy} dy + \pi_{xs} ds = 0,$$

(7)

$$\pi_{yx} dx + \pi_{yy} dy + \pi_{ys} ds = 0.$$ 

(7*)

Since $\pi_{xs} = 1$ and $\pi_{ys} = 0$, these equations can be put in matrix form and solved, using Cramer's rule, to yield:

$$x_s = \frac{dx/ds}{D} = -\frac{\pi_{yx}}{D} > 0,$$

(8)

$$y_s = \frac{dy/ds}{D} = \frac{\pi_{yx}}{D} < 0,$$

(8*)

where $D$ is defined in expression (6).

Naturally enough, an increase in the export subsidy, s, increases domestic exports, as expressed in (8) [using (3) and (6)]. Similarly, from (4) and (6), a domestic subsidy reduces the output of the foreign firm as shown in (8*).

Diagrammatically, the subsidy shifts out the reaction function of the domestic firm, increasing its exports and reducing foreign exports, as shown in fig. 1, where the subsidy shifts the equilibrium from N to S.

The subsidy lowers marginal cost to the domestic firm, which commits it to a higher reaction function. Proposition 1 summarizes the (comparative static) effects of the subsidy on prices and profits.

**Proposition 1.** An increase in the domestic subsidy

(i) lowers the world price of the good;

(ii) increases domestic profit; and

(iii) reduces foreign profit.

**Proof.**

(i) The change in price is given by the slope of inverse demand times the
change in total quantity:

\[ p_s = \frac{dp}{ds} = p'(x_s + y_s) \]

\[ = p'(\pi^*_x - \pi^*_y)/D, \quad \text{by (8) and (8*)} \]

\[ < 0, \quad \text{by (3*), (5) and (6).} \]

(ii) From total differentiation of \( \pi \) with respect to \( s \),

\[ \pi_s = \frac{d\pi}{ds} = \pi_x x_s + \pi_y y_s + \frac{\partial \pi}{\partial s}. \]

Since \( \pi_x = 0 \) by (2), \( \partial \pi/\partial s = x \) by (1), \( \pi_y = xp' \) and \( y_s < 0 \) by (8*),

\[ \pi_s = xp'y_s + x > 0. \]  \hspace{1cm} (9)

(iii) Similarly, from (1*), (2*), (8) and \( \pi^*_x = yp' \):

\[ \pi^*_s = \frac{d\pi^*}{ds} = \pi^*_x x_s + \pi^*_y y_s \]

\[ = yp'x_s < 0. \]  \hspace{1cm} (10)

A domestic subsidy, not surprisingly, lowers price, increases domestic profit and lowers foreign profit. What is perhaps more surprising is that the subsidy actually increases domestic welfare net of the subsidy. In our simple
case with all production for export, domestic surplus, $G$, net of the subsidy, is the profit of the domestic firm (earned from exports) minus the cost of the subsidy:

$$G(s) = \pi(x, y; s) - sx \quad (11)$$

Taking the derivative of (11) and using (9) to substitute for $\pi_s$, yields:

$$G_s = \pi_s - x - sx_s$$
$$= xp'y_s - sx_s. \quad (12)$$

At $s=0$, $G_s$ is clearly positive [by(8*)] indicating that a marginal increase in the subsidy will increase welfare. Alternatively, setting $G_s=0$ to obtain the optimal subsidy yields:

$$s = xp'y_s/x_s > 0, \quad (13)$$

leading to the following proposition.

**Proposition 2.** The domestic country has a unilateral incentive to offer an export subsidy to the domestic firm.

This is a simple and familiar model with a striking result: export subsidies which enable the domestic firm to capture a larger share of profitable international markets can appear to be attractive policies from a domestic perspective. This occurs despite the fact that the contribution of the subsidy to profit exactly offsets the cost of the subsidy to the government, leading to a domestic benefit function which is the same as the profit function of the domestic firm with no subsidy. In essence, the government’s prior action in setting a subsidy changes the domestic firm’s set of credible actions (i.e. its reaction function) in the output rivalry with its rival. The noncooperative equilibrium in the inter-firm rivalry is altered in favour of the domestic firm.

The formal structure is similar to Dixit (1980), where a firm may influence later output rivalry by an earlier irreversible capital stock decision, or to Prescott and Visscher (1976), where an irreversible location decision determines later output reaction functions. Instead of a prior capital or location decision, however, we have a prior government subsidy. In acting first, the government can actually move the domestic firm to the Stackelberg leader position in output space.

**Proposition 3.** The optimal export subsidy, $s$, moves the industry equilibrium to what would, in the absence of a subsidy, be the Stackelberg leader-follower position in output space with the domestic firm as leader.
Proof. Suppose the domestic firm were a Stackelberg leader without a subsidy. From differentiation of (1), the first-order condition for a profit maximum is then:

$$\pi_x(x, y; 0) + \pi_y(x, y; 0)(dy/dx) = 0,$$

where $dy/dx$ is the slope of foreign firm's reaction function. Total differentiation of ($2^*$) [as shown in ($7^*$)] implies that $dy/dx = -\pi_{xx}^{*}/\pi_{xy}^{*}$, which, using ($8$) and ($8^*$), is equal to $y_s/x_s$. Also substituting for $\pi_x$ and $\pi_y$, (14) becomes:

$$xp' + p - c_x + (xp'y_s/x_s) = 0.$$  \hspace{1cm} (15)

If we then compare condition (15) with first-order condition (2) [substituting in the optimal subsidy given by (13) for $s$] we find that the conditions are identical, which proves the result.

That this case for subsidies is dependent upon the existence of at least one foreign competitor producing for the world market should now be clear. To take the simplest case, if the domestic firm had an international monopoly in the imperfectly competitive good, then by the envelope theorem $\pi_x = x$ and from (12), $G_s = -sx_s$, which is always negative if $s$ is positive.

3. Terms of trade effects

How is it that a country can perceive an incentive to subsidize an exported good when such an action will lead to a worsening of the country's terms of trade. This would never be optimal in a two-good perfectly competitive neoclassical world. If the country were large enough to influence the price of the exported good, then an export tax would be appropriate so as to improve the terms of trade.

However, expanded output sold at a price above marginal cost can lead to a net increase in profit, more than offsetting the adverse terms of trade offset.

In this section we see how the terms of trade effect is incorporated by examining the argument for export subsidies in a simple general equilibrium framework familiar to trade theorists. This allows illustration of the argument using a production possibility curve diagram. We also extend the analysis to consider the possibility of domestic consumption.

The inclusion of domestic consumption is particularly simple if firms can price discriminate between the home and foreign (third country) market and if each enjoys a monopoly in its home market. If marginal cost is constant, an export subsidy then does not affect the profit-maximizing level of sales in the domestic market. The existence of domestic consumption therefore does not affect the noncooperative export subsidy levels. However, if marginal
costs were decreasing, the export subsidy would reduce the cost of production for the domestic market as well as for the export market so that the optimal subsidy would be higher in the presence of domestic consumption. Conversely, the optimal noncooperative export subsidy would be lower if marginal cost were increasing.

On the other hand, if a subsidy must be applied to all production, whether it is exported or sold domestically, then the presence of domestic consumption tends to increase the optimal subsidy. This is because with imperfect competition the level of domestic sales is below the Pareto efficient level. We model this case in this section under the additional assumption that markets are unified so that the price of \( x \) is the same in the home and in the export market. Also, although the distribution of each firm's sales between markets is indeterminate with the zero transport costs that we are implicitly assuming, it is convenient to assume that each producing country supplies its own consumers (as well as exporting to the third country).

Consider a utility function \( u(z, m) \), where \( z \) is domestic consumption of the imperfectly competitive good and \( m \) is consumption of a numeraire competitive good. The change in utility associated with marginal changes in consumption levels is given by

\[
du = u_z dz + u_m dm. \tag{16}
\]

Dividing through by \( u_m \) yields:

\[
dl = 
\frac{du}{u_m} = \frac{dz + dm}{p} = \frac{dz + dm}{p}, \tag{17}
\]

where \( p = \frac{u_z}{u_m} \) is the relative price of the imperfectly competitive export good. \( dl \) is the change in 'real income' and must have the same sign as the change in utility.

Domestic production levels are \( x \) and \( m_p \). Assuming all profits from sales of \( x \) accrue to domestic residents, balanced trade requires that the value of production equal the value of consumption.

\[
px + m_p = pz + m. \tag{18}
\]

Totally differentiating (18) and substituting into (17) yields:

\[
dl = (x - z) dp + p dx + dm_p. \tag{19}
\]

\(^7\text{We are assuming an economy with identical consumers who receive the same income based on identical endowments and an equal share of the profits of the imperfectly competitive domestic firm. This is the usual assumption one makes so as to abstract from the problem that the national distribution of income affects demand and welfare.}\)
This is the standard breakdown of a change in real income into a 'terms of trade' effect, \((x-z)dp\), and a 'value of output' effect, \(pdx+dmP\).

A frequent reference for this kind of analysis is Caves and Jones (1981).] The marginal rate of transformation is \(-\frac{dmP}{dx}=c_x\), where marginal cost, \(c_x\), is measured in units of \(m\). Therefore (19) becomes:

\[
dZ=(x-z)dp+(p-c_x)dx.
\]

Under pure competition \(p=c_x\), so a subsidy, which causes \(dp\) to be negative, produces a negative change in real income.

Starting from the equilibrium determined by Cournot rivalry between domestic and foreign firms, however, a subsidy increases real income. Substituting \(p-c_x=-(xp'-s)\) [from (2)] into (20), we obtain:

\[
dI=(x-z)dp-(xp'+s)dx.
\]

With a general utility function, the comparative static effects derived in section 2 do not apply directly, because of income effects. In general, a change in the subsidy affects profit and income in the producing countries and therefore may affect demand arising from producing countries. (These effects tend to be small because the loss to one country is partially offset by the gain to the other as subsidy levels change.) At any rate, the main point can be made by using the utility function, \(u(z,m)=U(z)+m\), since, in this case, all income effects are absorbed by the perfectly competitive numeraire good. With this utility function previous comparative static results apply exactly so dividing (21) by \(ds\) (measured in units of good \(m\)) and using \(ps=p'(x_s+y_s)\) yields:

\[
dI/ds=(x-z)p_s-(xp'+s)x_s
\]

\[=zp_s+xp'y_s-sx_s.
\]

It is clear that \(dI/ds\) is positive at \(s=0\) since \(p_s\) and \(y_s\) are negative, indicating that there is an incentive to raise the subsidy to positive levels. The optimal subsidy can be obtained by setting \(dI/ds\) to zero:

\[
s=xp'/y_s/x_s-zp_s/x_s>0.
\]

The general equilibrium effect of the subsidy is illustrated in fig. 2. Point C represents the competitive production point with the associated

\[8\] Similar diagrams are available in the literature to illustrate other imperfectly competitive situations. For example, Rieber (1982) illustrates the case of a domestic monopoly producing an export good with a competitive fringe in the foreign country.
consumption at $Q^c$ and utility level $u^c$. A subsidy from this point must lower welfare since production moves down the production possibility frontier (PPF) and the terms of trade move adversely.

The quantity of good $x$ produced by the domestic firm at the noncooperative Cournot equilibrium is indicated by point $N$. Consumption occurs at $Q''$, indicating that quantity $NA$ of good $x$ is traded for $Q''A$ of good $m$. The absolute value of the slope of $Q''N$, which represents the terms of trade, then exceeds the absolute value of the slope of the PPF, which represents the marginal rate of transformation. (As drawn, utility at the noncooperative equilibrium, $u''$, exceeds $u^c$, but this need not be the case and is not necessary for our argument showing the advantage from a subsidy.)

The imposition of a subsidy then shifts production to point $S$, increasing the output of good $x$ but worsening the terms of trade as represented by the

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9Whether this is the case depends on the tradeoff between the consumer loss from the restriction of domestic consumption of $x$ versus the additional profits earned from exports.
absolute value of the slope of $Q^S$. Nevertheless (as we have shown) a small subsidy at the noncooperative equilibrium always increases domestic welfare through an expansion of profitable exports. As illustrated in fig. 2, domestic consumption occurs at $Q^s$ and consumers enjoy higher level of utility $u^s$ rather than $u^a$.

4. Two governments: Nash export subsidy equilibrium

The actions of the two foreign governments must also be considered. Surely the two producing nations face similar incentives and there also may be some response by the importing nation. In this section we examine the noncooperative Nash equilibrium in subsidies in which each exporting country is assumed to choose its subsidy level given the subsidy level of the other exporting country. The actions of the importing nation are considered in the next section. As in section 3, we assume that any consumption of the good by the producing nations is at a common world price and that the subsidy applies to all units of production. If these consumption levels are zero, the subsidies reduce to pure export subsidies of the type considered in section 2.

Assume the utility of the consumers in the foreign exporting country can be represented by $u(z^*,m^*)= U(z^*) + m^*$, where $z^*$ is consumption of the imperfectly competitive good and $m^*$ is consumption of a numeraire competitive good. With this utility function the marginal utility of income is constant and equal to 1, so that consumer benefit from the consumption of $z^*$ is consistently measured by $U^*(z^*) - pz^*$. The 'gain' function of the foreign producing nation from a per unit production subsidy, $s^*$, given $s$, is then:

$$G^*(s, s^*) = U^*(z^*) - pz^* + \pi^*(x, y; s^*) - s^* y,$$  \hspace{1cm} (24)

where $\pi^* = yp(x + y) - c(y) + s^* y$ and $z^*, x$ and $y$ all depend on $s$ and $s^*$.

Using $dU^*/dz^* = p$ and $d\pi^*/ds^* = yp'x^* + y$ [see (9)], the first-order condition for a maximum of $G^*$ with respect to $s^*$ is:

$$G^*_{s^*} = -z^* p + yp'x^* - s^* y = 0,$$ \hspace{1cm} (25)

Similarly, the gain of the domestic country at subsidy levels $s$ and $s^*$ is:

$$G(s, s^*) = U(z) - pz + \pi(x, y; s) - sx,$$ \hspace{1cm} (26)

so that the first-order condition for the choice of $s$ is:

$$G_s = -zp + xp'y - sx = 0.$$ \hspace{1cm} (27)

This expression is the same as (22) in section 3, reflecting the fact that the
foreign subsidy \( s^* \) affects \( G \) only indirectly through the values of \( z, x \) and \( y \). Expressions (25) and (27) define the noncooperative Nash equilibrium in subsidies and imply the following proposition.

**Proposition 4.** The noncooperative Nash subsidy equilibrium is characterized by positive production subsidies in both exporting countries.

**Proof.** Rearranging (25) and (27) yields:

\[
s = \frac{xp'y_s}{x_s} - zp_s/x_s \quad \text{and} \quad s^* = \frac{yp'x_s}{y_s} - z^*p_s/y_s^* \tag{28}
\]

Introducing \( s^* \) does not affect the signs of the comparative static effects \( y_s < 0, x_s > 0 \) and \( p_s < 0 \) given by (8), (8*) and proposition 1 (although the magnitudes are altered) so that \( s \) is still positive. Comparative static effects \( x_s, y_s, \) and \( p_s \) are symmetric implying that \( s^* \) is also positive.

If the exporting countries do not consume the good, then \( z = z^* = 0 \) and it is clear from (28) that both the pure export subsidies are still positive: the incentive for subsidization remains. Regardless of the level of consumption by the exporting nations, however, their joint welfare would be higher if subsidy levels were reduced below the Nash equilibrium levels. The noncooperative solution is jointly suboptimal for the producing countries, but if one country sets a zero subsidy, it cannot expect this to deter the other country and it will then be worse off than if it had joined the subsidy game.

**Proposition 5.** At the noncooperative Nash subsidy equilibrium given by (25) and (27), joint welfare of the producing nations would rise if subsidy levels were reduced.

**Proof.** The joint gain function is \( g = G(s,s^*) + G^*(s,s^*) \). Since \( g_s = G_s + G_s^* \), at the noncooperative equilibrium, by (27) \( G_s = 0 \), and using (24):

\[
g_s = G_s^* = -z^*p_s + \pi_s^* - s^*y_s.
\]

From (10) and \( p_s = p'(x_s + y_s) \):

\[
g_s = -z^*p'(x_s + y_s) + yp'x_s - s^*y_s. \tag{29}
\]

Substituting (28) for \( s^* \), using \( p_s = p'(x_s + y_s) \) and gathering terms yields:

\[
g_s = (y - z^*)p'x_s(1 - x_s y_s / y_s x_s). \tag{30}
\]

From (8) and (8*) and similar expressions for \( x_s^* \) and \( y_s^* \):

\[
g_s = (y - z^*)p'x_s(1 - \pi_{x,y}^*/\pi_{x,x}^* \pi_{y,y}^*), \tag{31}
\]
which with some exports \((y-z^* > 0)\), is negative by (6) and (8). The effect of \(s^*\) is symmetric. Therefore a reduction in \(s\) or \(s^*\) will increase joint welfare.

The subsidy levels which maximize joint welfare are:\textsuperscript{10}

\[
s = (y - (z + z^*))p', \quad s^* = (x - (z + z^*))p'.
\]  

These jointly optimal subsidy levels are positive provided total consumption by producing nations exceeds their individual export levels. If there were no consumption of the good by producing nations, the jointly optimal policy would be to tax exports. These optimal taxes would ensure that each firm would produce the same output that a member of a two-firm monopoly cartel would, enabling the two producing countries to act as a monopoly against the rest of the world. The countries are, we assume, unable to make binding agreements of this sort.

5. Three governments: Nash tariff and subsidy equilibrium

The consuming nation also has an incentive to set a tariff or subsidy on the import of the imperfectly competitive good, so as to extract some of the rent earned by its producers. The optimal tariff (or subsidy) for the importing nation has been analysed by Brander and Spencer (1984a) for the case of a foreign Cournot oligopoly, but no consideration was given to the way export subsidies might affect the optimal import tariff or to the nature of the Nash tariff and subsidy equilibrium between the three governments.

For this purpose we again assume a simple additive utility function in good \(x\) and the numeraire good. Then let \(G^3(s, s^*, t)\) represent the gain to the importing nation (country 3) from a tariff, \(t\), on total imports \(X = x + y\) of the imperfectly competitive good. Then,

\[
G^3(s, s^*, t) = U(X) - q(X)X + tX,
\]

where \(q(X)\) is the consumer price including the tariff. The first-order

\textsuperscript{10}At the jointly optimal solution \(\partial g/\partial s = 0\) and \(\partial g/\partial s^* = 0\). The solutions for \(s\) and \(s^*\) can be found directly as follows. Using (27) and its analogue \(G^*_s = -z^*p_y + y p'x - s^*y\), yields:

\[
\frac{\partial g}{\partial s} = G_s + G^*_s = - (z + z^*)p_x + xp'y_s - y s_x + s^*y = 0.
\]

There is a corresponding expression for \(\partial g/\partial s^*\). Then using (8) and (8*) to substitute for \(x_s\) and \(y_s\), we obtain:

\[
\frac{\partial g}{\partial s} = (\text{expression for } \frac{\partial g}{\partial s}) = 0,
\]

\[
\frac{\partial g}{\partial s^*} = (\text{expression for } \frac{\partial g}{\partial s^*}) = 0.
\]

These can be solved by inspection for \(s = (y - (z + z^*))p'\) and \(s^* = (x - (z + z^*))p'\).
condition for the choice of \( t \) given \( s \) and \( s^* \) is:

\[
G_t = -Xq'(X)X_t + tX_t + X = 0,
\]

(34)

where \( X_t = x_t + y_t \). The comparative static effects \( x_t \) and \( y_t \) [obtained by the same method as in (7), (7*), (8) and (8*)] are:

\[
x_t = (\pi^*_y - \pi_{xy})/D; \quad y_t = (\pi_{xx} - \pi^*_{yx})/D.
\]

(35)

From condition (6), \( D \) is positive and at least one of \( \{x_t, y_t\} \) is negative. In the symmetric case in which the two firms face the same costs, both \( x_t \) and \( y_t \) are negative ensuring that \( X_t < 0 \). From (34):

\[
t = X(1 - q'(X)X_t)/X_t.
\]

(36)

From (36), with \( X_t < 0 \), the optimal tariff is positive if the rate of change of the consumer price with respect to the tariff is less than one. This is the 'usual' case.\(^{11}\)

With a tariff on imports, the price function \( p(X) \) becomes \( p(X; t) \) and must be re-interpreted as the producer price received by firms net of the tariff: \( p(X; t) = q(X) - t \). With this interpretation, there is no change in formulas (25) and (27) which, together with (34), are the first-order conditions for the (three) government subsidy and tariff equilibrium.

To see more clearly the nature of this Nash tariff and subsidy equilibrium, consider the case of linear demand and constant marginal cost. Assume also that exporting nations do not consume the imperfectly competitive good. In this case \( \pi_{xx} = \pi_{yy}^* = 2p' \) and \( \pi_{xy} = \pi_{yx}^* = p' \), so that from (8), (8*) and (35):

\[
x_t = y_t = 1/3p', \quad x_s = y_s^* = -2/3p' \quad \text{and} \quad x_s^* = y_s = 1/3p'.
\]

(37)

Hence using the fact that \( q' = \partial p/\partial x \equiv p' \), from (28) and (36) (with \( z = z^* = 0 \)).

\[
s = -xp'/2, \quad s^* = -yp'/2 \quad \text{and} \quad t = -Xp'/2.
\]

(38)

In this example, the tariff set by the importing country is just the sum of the two subsidy levels set by the exporting countries. Also, taking the derivative of \( t = -Xp'/2 \) recognizing that \( X \) depends on both \( s \) and \( t \) yields:

\[
dt/ds = -p'X_s/(2 + p'X_t);
\]

\(^{11}\)The rate of change of the consumer price with respect to the tariff is less than one if the demand curve is not too convex to the origin. Constant elasticity demand curves violate this condition, but linear demand curves and most other plausible demand curves satisfy it. [See Brander and Spencer (1984a).]
therefore, a small increase in the subsidy set by one exporting nation serves to increase the import tariff one-eighth as much [using (37)] enabling the importing country to shift additional rent to itself. (An equal increase in both $s$ and $s^*$ leads to an increase in $t$ by one-quarter of this increase.) In this example the introduction of export subsidies increases both the import tariff and the level of world output so that the importing country increases its tariff revenue by 30 per cent relative to the situations where $t$ is set optimally but both $s$ and $s^*$ are zero.

Conversely, if both $s$ and $s^*$ are chosen optimally, an increase in the import tariff serves to reduce export subsidy levels by one-fifth. The tariff reduces the potential gains from (unilateral) export subsidization and in this linear example, rather than offsetting the cost increase from the tariff, the exporting countries reduce subsidy levels.

One further possible direction of generalization suggests itself: if markets in the two producing countries are segmented rather than unified, then there may be two-way trade in the imperfectly competitive commodity. Dixit (1984) analyzes the motives for and consequences of tax/subsidy, tariff, and anti-trust policies in the presence of such trade between producing countries, but without third markets.

6. Concluding remarks

There are a couple of caveats that should be stressed. First, any policy involving subsidies should be viewed with suspicion because the marginal opportunity cost of government revenue may be much higher than the value of unity assumed in simple surplus analysis. Secondly, in a world of imperfect information and imperfect governments, any argument indicating a plausible national motive for subsidies may open the door for various kinds of socially wasteful rent-seeking. These are important concerns, but it seems worthwhile to us to separate out the logical implication of imperfect competition per se for international trade policy incentives. What the paper shows is that noncooperative behavior provides incentives for such policies, but these policies are jointly suboptimal from the point of view of producing nations taken together.

The paper is built around what seems to us an important part of the modern international environment: countries perceive themselves as being in competition with each other for profitable international markets. In such a world the credibility of governments can confer strategic advantages on

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12 Assuming $p = a + p'x$ and $c = c^*$, if $s = s^* = 0$, but $t = -Xp'/2$ is set optimally $x = y = -(a - c)/4p'$. Tariff revenue is $tX = -(a - c)^2/8p'$. With the introduction of subsidies, $s = s^* = -xp'/2$, output rises to $x = y = -2(a - c)/7p'$ and tariff revenue to $tX = -8(a - c)^2/49p'$, which is a 30 percent increase.

13 From (38), $ds/dt = (−p'/2)(x_d, ds/dt + x_d, ds^*/dt + x_i)$. Using $ds/dt = ds^*/dt$, we obtain $ds/dt = −p'x_i/[2 + p(x_i + x_i)]$, which, from (37), reduces to $−1/5$. 
domestic firms. In particular, export subsidies can appear as attractive weapons because they improve the relative position of the domestic firm in noncooperative rivalries with other firms, and allow it to expand its market share. The terms of trade will move against the subsidizing country but price still exceeds the marginal resource cost of exports so that the resulting expansion of exports can actually raise domestic welfare. Producing countries have cooperative incentives to get together to agree not to use such subsidies, but they also have an incentive to cheat on any resulting agreements, suggesting that international regulations which attempt to discourage subsidization, such as GATT regulations, are likely to require regular reinforcement if they are to survive.

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