Optimal trade policies for a developing country under uncertainty

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Abstract: This paper investigates optimal trade policies for a developing small open economy which faces international price uncertainty. Trade taxes are used to finance provision of a public good, which enters the utility function of consumers. If demands for private goods are independent of the public good, the optimal composite tariff dominates the optimal quota. If the optimal state-contingent tariff increases with the foreign price, the optimal specific tariff also dominates the optimal quota, regardless of risk aversion. However, the ranking of the optimal specific tariff and the optimal quota generally depends on risk attitudes as well as ordinal preferences.

1. Introduction

Free trade is rarely practiced by developing countries despite its well-known optimality for small open economies. Staiger and Tabellini (1987, p. 823) observed that 'active protectionist programs are widely pursued by countries with little or no apparent world market power'. The tariff was the principal source of federal revenue in the U.S. during the nineteenth century, and was not displaced until the income tax was adopted. The works of Boadway, Maital and Prachowny (1973), Vanek (1971), and Feehan (1988) suggest that tariff revenue may be the main source of financing public goods in LDCs. Ethier (1988) cited for example that the government of Uganda derived two-thirds of revenue from trade taxes in 1984, and Lesotho raised about 69 percent of its revenue from trade restrictions in 1983. Thus, trade taxes may be justified for the provision of public goods in LDCs. An important question is whether the governments of LDCs should use taxes or quotas.

A substantial amount has been written concerning the ranking of tariffs and quotas under uncertainty. In a seminal paper, Fishelson and Flatters...
(1975) pointed out that the equivalence between policy instruments breaks down under uncertainty, and initiated a study of ranking tariffs and quotas. The optimal tariff for a small country is zero except under factor market distortions [Batra and Naqvi (1987)] or under variable returns to scale [Choi and Yu (1984)]. Thus, the literature – apart from that dealing with the large-country case – has developed by imposing a constraint on expected imports or on expected government revenue, and comparing consumer surpluses under alternative regimes. For example, Pelcovitz (1976) and Young and Anderson (1980) compared quotas with tariffs that yield the same level of expected import. 

Young and Anderson's (1982) more recent work represents a landmark in the literature of ranking of trade policies under uncertainty for two reasons. First, they employed expected utility analysis, rather than expected surplus area which is known to be a valid welfare criterion under restrictive conditions. Second, they showed that the ranking of ex ante policies depends crucially on the property of the optimal state-contingent policy. Their analysis suggests that the policy with an implicit tariff which moves in the same direction, rather than in the opposite direction, as the optimal state-contingent policy is the dominant policy.

An important criticism of the existing literature concerns the lack of rationale for trade restrictions. If the tariff revenue is rebated to consumers, as is conventionally assumed, there is no intrinsic rationale for trade restrictions and hence the optimal tariff is zero. On the other hand, if the tariff or quota revenue is used to finance government purchases of public goods, the ranking of tariff and quota must be based, not just on the mean, but on the entire distribution of government revenue.

The purpose of this paper is to investigate optimal trade policies for an LDC which uses trade taxes for provision of the public good. The model extends Young and Anderson (1982) and Feehan (1988) in two important ways. First, we employ Feehan's (1988) assumption that the government provides a public good which enters the utility function of consumers and

1 Bhagwati (1965, 1968) has shown that under certain competitive conditions import quotas and tariffs are equivalent. Equivalence does not hold under retaliation [Rodriguez (1974)]. However, equivalence holds if foreign demand is uncertainty-free even when domestic markets are subject to uncertainty [Ohta (1978)].

2 However, Batra and Naqvi (1987) emphasize that the first-best policy is a wage subsidy, rather than a tariff. For further analyses of variable returns to scale, see for example Eaton and Panagariya (1979).

3 The ranking of trade policies under an expected revenue constraint has also been investigated. For instance, using social surplus areas Dasgupta and Stiglitz (1977) and Young (1980) compared quotas and ad valorem tariffs that yield the same expected revenue.

4 Turnovsky, Shalit and Schmitz (1980) showed that expected surplus criterion is a valid welfare measure only if the marginal utility of income is constant. For this reason Young and Anderson (1982) employ expected utility.
that all trade taxes are used to purchase the public good.\textsuperscript{5} Second, we adopt Young and Anderson's (1982) expected utility framework to investigate the ranking of alternative policies.

The plan of this paper is as follows. Section 2 constructs a general equilibrium model which links trade and provision of the public good. Section 3 investigates the properties of the optimal state-contingent policy while section 4 analyzes the properties of ex ante optimal quota and tariff schedules. Section 5 shows that under weak separability the optimal composite tariff dominates the optimal quota. When a single instrument is used, the ranking of ex ante second-best policies requires information about cardinal, as well as ordinal, preferences. Section 6 provides a brief summary and concluding remarks.

2. Trade and provision of the public good

Consider an open economy which uses trade taxes to finance provision of the public good. A general equilibrium model is developed employing the following assumptions.

(i) The domestic economy consists of $N$ identical consumers.\textsuperscript{6}

(ii) Two private goods, the exportable $Z$ (numeraire) and the importable $Y$ are produced and consumed domestically.

(iii) For all realizations of the foreign price of the importable, $p^*$, the economy imports $Y$.

(iv) The economy is small and the distribution of the world price is exogenous.

(v) One unit of the exportable produces one unit of the public good. Provision of the public good is solely financed by tariff/quota revenue. Since $N$ is large, each consumer ignores the impact of his consumption decisions on government revenue.

(vi) Domestic demands for traded goods are independent of the public good, i.e., consumer preferences are weakly separable in private goods and the public good.

(vii) The public good is not traded.

\textsuperscript{5} Naturally, a first-best policy for raising revenue would involve taxes on resources in fixed supply. Alternatively, if we imagine an economy with a production possibility curve using resources in fixed supply, a simultaneous consumption (or production) tax on both goods would raise revenue without distorting relative prices. Presumably, the rationale for trade taxes is either that such domestic taxes are infeasible or that factor supplies are endogenous. If factor taxes are feasible and factor supplies are endogenous, then optimal policy would in general require a mix of trade and factor taxes as in Boadway, Maital and Prachowny (1973).

The exportable $Z$ is the numeraire and its price is unity. Let $p^*$ and $p$ denote the world price and the domestic price of the importable $Y$, respectively. Production decisions are made after the foreign and domestic prices are known. The production possibility frontier of the private goods, in per capita form, is given by

$$Z = F(Y), \quad F' < 0, \quad F'' < 0, \quad (1)$$

where $Y$ and $Z$ denote the per capita domestic production of the importable and the exportable, respectively. Recall that one unit of the exportable is required to produce one unit of the public good, $G$, i.e., $Z$ is an intermediate input to produce the public good $G$. Thus, $Z$ represents the gross production of the exportable, the net output of the exportable is $Z - G$, and private income is $(Z - G) + G + pY = Z + pY$. Producers of the private goods choose $Y$ and $Z$ to maximize private income, $I = Z + pY$. The first-order condition is

$$p + F' = 0. \quad (2)$$

The domestic per capita supply of the importable, $Y(p)$, is positively sloped since $Y'(p) = -1/F'' > 0$. While consumer income is endogenous, $dI/dp = Y$ by the Envelope Theorem, and hence private income $I = Z + pY(p)$ increases with $p$.

Consumer preferences are represented by a monotone increasing von Neumann–Morgenstern utility function $u(C, X, G)$, where $C$ and $X$ denote the individual consumption of the exportable and the importable, respectively. If consumer demands for private goods are independent of the public good, consumer preferences can be represented by a weakly separable function

$$U(C, X, G) = U[f(C, X), G]. \quad (3)$$

Unlike the conventional trade models, tariff revenue is not rebated to consumers but is used to purchase the public good $G$. The representative consumer makes consumption decisions after observing the domestic price $p$. Since consumer income is spent only on private consumption goods, the budget constraint is

$$C + pX = I. \quad (4)$$

The first-order condition is

$$U_X(C, X, G)/U_C(C, X, G) = p.$$
However, the marginal rate of substitution between private goods is independent of the public good by weak separability. Thus, the demand functions of the representative consumer can be written as

\[ C(p, I, G) = C(p, I), \quad X(p, I, G) = X(p, I) \]  

and \( C_G = X_G = 0 \). Substituting the demand functions into (3) gives the indirect utility function,

\[ V[p, I, G] = U[f(C(p, I), X(p, I)), G]. \]  

Using the endogenous income, \( I(p) = F(Y) + pY(p) \), the per capita import demand function can be written as

\[ Q(p) = M(p, I(p)) = X(p, I(p)) - Y(p). \]  

Observe that since the domestic supply \( Y(p) \) depends only on its price, \( M_I = X_I \). The per capita net export of the exportable is \( (Z - G - C) \). Recall that trade taxes are the only source of financing the provision of the public good. Thus, the total government revenue, in terms of the numeraire, is \( N(p - p^*)Q(p) \). Moreover, one unit of the numeraire \( Z \) is required to produce one unit of the public good. Thus, the total quantity of the public good provided is equal to government revenue

\[ G = tNQ(p), \]  

where \( t = p - p^* \) is the specific tariff.

3. Optimal state-contingent policy

The first-best policy for optimal provision of the public good may be taxes on income, production or consumption. However, tariffs are the major revenue sources for the governments of most LDCs. Thus, we begin by considering the optimal state-contingent tariff. It should be noted that since

\[ \text{The income of the consumer is } I = Z + pY(p). \]  

If \( T \) denotes the amount of \( Z \) exported, domestic consumption of the exportable is \( C = Z - G - T \). If trade is balanced, \( T = p^*Q \), then the total expenditure is \( C + pX = (Z - G - T) + p(Q + Y) = Z + pY \), equal to national income.

\[ \text{This assumption of constant marginal rate of transformation (MRT) between the exportable and the public good is not as restrictive as it appears. If } G = tNQ \text{ is government revenue and } g \text{ represents the quantity of the public good produced, then for a general transformation function } g = g(G) \text{ MRT varies as } G \text{ changes, } g'(G) > 0. \]  

In this case, the direct utility function can be rewritten \( U[f(C, X), g(G)] = u[f(C, X), G] \), where the new utility function \( u(\cdot) \) is expressed in terms of government expenditure \( G \), rather than the public good \( g \). The corresponding indirect utility can also be written \( V[p, I, g(G)] = v[p, I, G] \). With this modification, all the results of the paper hold if \( V \) is replaced by \( v \).
trade restriction by a tariff or a quota is assumed to be the sole means of raising revenue, the 'optimal' policy here is a second-best policy.

Weitzman (1974, p. 481) points out that an ideal instrument of central control would be a contingent message whose instructions depend on which state of the world is revealed. Since the optimal state-contingent policies—specific and ad valorem tariffs and quotas—are chosen under conditions of certainty, they are all equivalent for any desired level of the public good.

After the world price is known, the policy maker's problem is to choose \( p \) to maximize

\[
 J(p, p^*) = V[p, F(Y) + pY, N(p - p^*)Q(p)].
\]  

(9)

Note that by Roy's identity, \( V_p = -V_X \), and \( p + F' = 0 \). Differentiating (9) with respect to \( p \) yields \( J_p = V_p + V_l(I_p) + V_d(G_p) \). Moreover, that for a given level of the public good \( G \), \( dV/dp + V_p + V_l(I_p) = -V_lQ \), and \( d^2V/dp^2 = V_l(X_lQ - Q') \). If \( Q \) is given and the consumer is risk neutral (\( V' = 0 \)), then \( V \) is convex (concave) in \( p \) if \( X_lQ - Q' \) is positive (negative) or \( \eta + \varepsilon/s > <0 \), where \( \varepsilon = -(dQ/dp)(p/Q) \) is the price elasticity of import demand, \( \eta = (\partial Q/\partial I)(1/Q) \) is the income elasticity of import demand, and \( s = pQ/I \) is the budget share of the imports. Thus, the first-order condition is written

\[
 J_p = -V_lQ + V_d(G_p) = 0, \quad G_p = N(Q + (p - p^*)Q').
\]  

(10)

This implies that \( G_p \) is positive in equilibrium. The second-order condition requires that \( J_{pp} < 0 \). If the left side of (10) is evaluated at the revenue maximizing tariff, then \( G_p = 0 \) and \( J_p < 0 \). Thus, the optimal state-contingent tariff is always less than the revenue-maximizing tariff, i.e.,

\[
 t < p/\varepsilon.
\]  

(11)

Rearranging (10) yields an alternative expression for the equilibrium condition,

\[
 N(V_G/V_l) = Q/(Q + tQ') \quad \text{or} \quad V_G/V_l = \Theta = Q/N(Q + tQ'),
\]  

(12)

where \( V_G/V_l \) is the individual marginal rate of substitution (MRS) of the public good \( G \) for private expenditure \( I \), and \( \Theta(Q) = Q/(Q + tQ') \) reflects the social marginal cost of 'providing' the public good via trade taxes. The presence of \( N \) in (12) indicates the pure public good characteristic of \( G \).
Optimality requires equating the sum of individual MRS to the social marginal cost of the public good.\textsuperscript{9} Alternatively, the condition $V_G/V_I = \Theta$ means that the individual MRS must be equated to the per capita social marginal cost of the public good $\Theta$.

Since one unit of the numeraire good produces one unit of the public good, the social marginal cost of providing the public good should be unity if a non-distortionary method of financing $G$ were available. Except when the import demand is totally price inelastic ($Q' = 0$), a tariff necessarily distorts the relative price between the private goods. The fact that only a distortionary tariff or quota is employed implies that $N(V_G/V_I)$ exceeds unity at an optimum.

To illustrate the link between the optimal state-contingent tariff and optimal provision of the public good, consider the following problem of a policy maker to choose the optimal mix of the public good $G$ and private expenditure $I$:

$$\text{maximize } V(p, I, G) \text{ subject to } B = I + \Theta G, \quad (13)$$

where $\Theta$ is the price or tax each consumer must pay to consume the public good or the per capita social marginal cost of the public good each consumer bears, and $B$ is full income. In addition to money income, 'full income' includes the value of non-market goods (e.g. public goods). Unlike private consumption goods, the price of the public good $\Theta$ is endogenous, and depends on the prices, $p$ and $p^*$. However, for given $p$ and $p^*$, the price of the public good $\Theta$ is constant, and the full income constraint is linear in private expenditure $I$ and the public good $G$.\textsuperscript{10} Optimality requires that the indifference curve $V(p, I, G)$ be tangent to the full income constraint in $(I, G)$ space, i.e., $V_G/V_I = \Theta$. Thus, the per capita social marginal cost of the public good in (12) can be viewed as the price of the public good each consumer pays through trade taxes. It can be shown that $I$ is a normal good ($\partial V/I \geq 0$) if $\Theta V_{IG} - V_{GG} \geq 0$ and $G$ is a normal good ($\partial G/B \geq 0$) if $V_{IG} - \Theta V_{GG} \geq 0$.

We now investigate how an increase in the world price of the importable affects the optimal state-contingent policy. Let $t'$ be the optimal state-contingent tariff and $p' = p^* + t'(p^*)$ be the optimal state-contingent domestic

\textsuperscript{9}It is interesting to note that $\sum MRS = N(V_G/V_I) = 1/(1 + e^\Theta)$, where $e^\Theta = (Q'/\partial t)/(t/Q)$ is the elasticity of imports with respect to the tariff. This is a special case of a more general result in Feehan (1988, p. 160). However, we sacrifice some generality here to obtain a ranking of second-best policies under uncertainty.

\textsuperscript{10}See Becker (1965) for the notion of full income. For the typical consumer utility maximization problem, money income is independent of price changes. However, with the utility maximization problem with the full income constraint, 'full income' is dependent on the relative price $\Theta$ of the public good.
price of the importable. Observe that the signs of $\frac{dt}{dp^*}$ and $\frac{dp}{dp^*}$ are independent of $N$. Without loss of generality, we assume $N=1$ in the rest of the paper. Differentiating (10) totally gives $J_{pp^*}\frac{dp^*}{dp} + J_{pp}\frac{dp}{dp} = 0$, or

$$\frac{dp}{dp^*} = -\frac{J_{pp^*}}{J_{pp}},$$

where $J_{pp} < 0$ by the second-order condition, and

$$J_{pp^*} = -QV_{1G}(G_p) + V_{GG}(G_p,G_p) - V_GQ'.$$

Using $Q_p = Q/\Theta$ from (10) and $G_p = -Q$, we obtain

$$J_{pp^*} = (\Theta V_{1G} - V_{GG})Q^2/\Theta - V_GQ'.$$  \hspace{1cm} (14)

Recall from the implicit maximization problem in (13), the first term in (14) is greater than or equal to zero if private expenditure $I$ is a normal good. The following proposition indicates that $\frac{dt}{dp^*}$ is bounded from below, $\frac{dt}{dp^*} > -1$.

**Proposition 1.** Assume that private expenditure $I$ is a normal good ($\frac{\partial I}{\partial B} \geq 0$). Then the optimal state-contingent domestic price of the importable increases with the world price, $\frac{dp}{dp^*} > 0$.

Next, how does the optimal state-contingent tariff respond to a change in the world price $p^*$? Using $t^f - p^f - p^*$, we have

$$\frac{dr}{dp^*} = -\frac{J_{pp^*} + J_{pp}}{J_{pp}},$$

where, using $V_{ip} = -V_{1I}X - V_{1I}I$ and $V_{Gp} = -V_{1G}X$ (due to weak separability),

$$J_{pp^*} = (\Theta V_{1I} - V_{1G})Q^2/\Theta + V_i(X_IQ - Q') + (-\Theta V_{1G} + V_{GG})Q^2/\Theta^2 + V_{1G}(2Q' + tQ'').$$  \hspace{1cm} (15)

Using $(\partial I/\partial B) + \Theta(\partial G/\partial B) = 1$ and rearranging terms, we have

$$J_{pp} + J_{pp^*} = [\Theta(\partial I/\partial B) - 1]Q^2H/\Theta^2 + V_i(X_IQ - Q') + V_{1G}(2Q' + tQ'').$$

$$= (\tau e - m)Q^2H/\Theta + V_i(X_IQ - Q') + V_{1G}(Q' + tQ''),$$  \hspace{1cm} (16)

where $\tau \equiv t/p$ is the ad valorem tariff in terms of the domestic price, $m \equiv \Theta(\partial G/\partial B)$ is the marginal propensity to consume the public good, and $H \equiv 2\Theta V_{1G} - V_{GG} - \Theta^2V_{1I} > 0$ and is the Hessian of the maximization problem.
in (13). The sum of the terms in (16) is positive, if $\tau e - m > 0$, $\eta + e/s > 0$ and $-tQ''/Q' > 1$.

Proposition 2. Sufficient conditions for the optimal state-contingent tariff to increase (decrease) with $p^*$ are:

(i) $\tau e$ is greater (less) than $m$,
(ii) $-tQ''/Q'$ is greater (less) than unity, and
(iii) $\eta + e/s$ is greater (less) than zero.

Intuitively, if the product of the ad valorem tariff rate $\gamma$ and the price elasticity of the import demand $\varepsilon$ is large relative to the marginal propensity to consume ($\gamma e > m$), the import demand function is sufficiently convex in price ($-tQ''/Q' \geq 1$), and the weighted sum of elasticities ($\eta + e/s$) is positive, then the optimal state-contingent tariff increases with $p^*$.

4. Properties of ex ante second-best policies

The main difficulty with implementing the optimal state-contingent policy is that the policy maker must have full information, without delay, about the state of the world. If information is costly to obtain or is not available instantaneously, the burden of full information is likely to outweigh the potential gains from the optimal state-contingent policy. Weitzman (1974, p. 481) thus observed that 'it is infeasible for the centre to transmit an entire schedule of ideal prices or quantities', because 'the contingent message is complicated, expensive to draw up and hard to understand'.

For this reason, we focus on two widely used second-best policies, ex ante tariffs and quotas.11 The policy maker is assumed to choose, before observing the foreign price, either (a) a fixed quota, or (b) a combination of a specific tariff and an ad valorem tariff. We now investigate the properties of second-best policies, and demonstrate that the effects of price uncertainty on the levels of second-best instruments depend on cardinal preferences.

4.1. Optimal quota

When a quota is imposed, government revenue $(p - p^*)Q$ is raised by auctioning the quota rights. Although government revenue depends on the realized foreign price $p^*$, the import demand is independent of $G$ due to weak separability, and hence is unaffected by random fluctuations of the foreign price $p^*$. Thus, fixing an import quota is equivalent to fixing the domestic price. Let $Q_o$ denote the optimal quota and $p_o$ be the corresponding

11See Fishelson and Flatters (1975) for non-equivalence of tariffs and quotas under uncertainty.
domestic price of the importable, i.e., $Q_o = Q(p_o)$. The policy maker's problem is to choose $p_o$ to maximize the expected indirect utility

$$E[J(p_o, p^*)] = E[V[p_o, l_o, (p_o - p^*)Q(p_o)]]$$

(17)

where $l_o = F(Y(p_o)) + p_o Y(p_o)$ is the private expenditure under the optimal quota. The first-order condition is

$$E[J, L_p] = -E[I_o, L_q]Q_o + E[V_o(Q_o + (p_o - p^*)Q')\] = 0.$$

(18)

Two interrelated questions are: how do increases in price uncertainty and risk attitudes affect the optimal quota? Differentiating (17) with respect to $p^*$ twice yields

$$J_{pp^*} = QV_{1G} - (Q + tQ')V_{GG} - V_GQ'.$$

(19)

$$J_{pp^*p^*} = V_{GGG}Q^2 - Q^3V_{1GG} + 2V_{GGQ'Q'}.$$  

(20)

Note that $J_{pp^*}$ in (19) is evaluated at the optimal quota $Q_o$ [whereas $J_{pp^*}$ in (14) is evaluated at the optimal state-contingent tariff].

Let $p_c$ and $Q_c$ denote the optimal domestic price and quota, respectively, when the world price equals $E(p^*)$ with certainty. If $J_{pp^*p^*}$ is everywhere positive (negative), then $E[J(p, p^*)]$ is greater (less) than $J(p, E(p^*))$, and hence the left side of the equation in (18) is positive (negative) when evaluated at $p_c$. It follows immediately that $p_o \leq (<) p_c$ and $Q_o \leq (>) Q_c$ as $J_{pp^*p^*} \geq (<) 0$. If the consumer is 'risk neutral' in the public good ($V_o = 0$), then $V_{1GG} = V_{GGG} = 0$ and $J_{pp^*p^*} = 0$. In this case, $p_o = p_c$ and $Q_o = Q_c$. On the other hand, if the consumer is 'risk averse' in the public good ($V_{GG} < 0$), and $V_{GGG} \geq 0 \geq V_{1GG}$, then $J_{pp^*p^*} > 0$.

Proposition 3. Assume that the optimal quota $Q_o$ is chosen before the foreign price is known. Then (i) if the consumer is risk neutral in the public good ($V_{GG} = 0$), then $p_o = p_c$ and $Q_o = Q_c$, and (ii) if the consumer is risk averse in the public good ($V_{GG} < 0$) and $V_{GGG} \geq 0 \geq V_{1GG}$, then $p_o > p_c$ and $Q_o < Q_c$.

Observe that the conditions for the second part of the proposition are not as restrictive as they might appear. For instance, decreasing absolute risk aversion in the public good implies $V_{GGG} \geq 0$, and strong separability between private goods and the public good implies $V_{1G} = V_{1GG} = 0$. Thus, strong separability and decreasing absolute risk aversion in the public good suffice to imply $p_o > p_c$.

Next, consider a monotone increasing and concave transformation of $V(p, l, g)$ that preserves ordinal preferences but increases risk aversion:
\[ H(p, p^*) = w[J(p, p^*)]; \quad w' > 0 > w''. \] (21)

Then
\[ E[H_p] = E[w'J_p] = E[w']E[J_p] + \text{Cov}(w', J_p). \] (22)

Note that \( dw'/dp^* = w''J_. > 0. \) Evaluating \( E[H_p] \) at \( p_o \) yields
\[ E[H_p(p_o, p^*)] \geq (\text{<})0 \quad \text{as} \quad J_{pp^*} \geq (\text{<})0. \] (23)

where \( J_{pp^*} \) is given in (19).

**Proposition 4.** Assume that \( V_{1G} \geq 0 \geq V_{GG}. \) Then as the consumer becomes more (less) risk averse, the optimal domestic price \( p_o \) increases (decreases) and the optimal quota \( Q_o \) decreases (increases).

Observe that if \( V_{1G} \geq 0 \geq V_{GG} \), then from the implicit maximization problem in (13) for the optimal state-contingent policy, private expenditure \( I \) becomes a normal good.

### 4.2. Optimal tariff

Since the foreign price \( p^* \) is readily verifiable ex post, some simple state-contingent rules can be established ex ante. Popular forms of state-contingent policies are (i) a specific tariff, (ii) an ad valorem tariff, and (iii) a combination of a specific tariff and an ad valorem tariff. Since the former two instruments are special cases of a composite tariff, we will consider the latter. The domestic price of the importable is given by
\[ p = k + (1 + \alpha)p^*, \]
where \( k \) is a specific tariff, \( \alpha \) is an ad valorem tariff, and the total tariff schedule is \( t = k + \alpha p^*. \)

Expected utility with an optimal tariff schedule is
\[ E[J(t, p^*)] = E\{V[p^* + t, I(p^* + t, tQ(p))]; \quad t = k + \alpha p^*. \] (24)

Optimizing over \( k \) and \( \alpha \) gives
\[ E[J_k] = E[J_\alpha] = E[-V_I Q + V_o(Q + tQ')] = 0, \] (25a)
\[ E[EJ_t] = E[p^*J_t] = E[p^*(-V_I Q + V_o(Q + tQ'))) = 0. \] (25b)

We now investigate the properties of the second-best tariff schedule.
Suppose that only a specific tariff can be used (i.e., $x$ is constrained to zero), and denote the optimal ex ante specific tariff by $k_o$. Assume that the optimal state-contingent tariff $t'(p^*)$ is monotone in $p^*$. Then there exists a world price, denoted $p_o^*$, for which $k_o$ is the optimal state-contingent tariff, i.e., $k_o = t'(p^*)$ at $p_o^*$. Assume also that the second-order condition, $J_{tt}(t'(p^*), p^*) = 0$, is globally satisfied. Then $J_t(k_o, p^*) \geq (>)p$ as $t'(p^*) \geq (>)k_o$, since $J_t(t'(p^*), p^*) = 0$. If $t'(p^*)$ is monotonically increasing in $p^*$, then $J_t(k_o, p^*) \geq (>)0$ as $p^* \geq (>)p_o^*$. On the other hand, if $t'(p^*)$ is monotonically decreasing in $p^*$, then $J_t(k_o, p^*) \geq (>)0$ as $p_o^* \geq (>)p^*$. Evaluating the left side of the equality in (25b) at $(x, k) = (0, k_o)$ and using (25a), we obtain

$$E[J_t] = E[p^*J_t] = E[(p^* - p_o^*)J_t]$$

(26)

since $p_o^*E[J_t] = 0$ for the optimal specific tariff $k_o$. If the optimal state-contingent tariff is monotone increasing (decreasing) in $p^*$, then $(p^* - p_o^*)J_t$ is positive (negative) for all $p^* \neq p_o^*$. Thus, $E[J_t]$ evaluated at $(x, k) = (0, k_o)$ is positive (negative) if $dt'/dp^*$ is positive (negative).

**Proposition 5.** The second-best linear tariff schedule includes a positive (negative) ad valorem tariff if $\gamma \geq (>)m$, $-\tau Q'/Q > (>)1$ and $\eta + \epsilon/s > (>)0$.

Intuitively, this proposition indicates that the second-best tariff schedule tends to mimic the optimal state-contingent tariff by moving in the same direction as the latter. It should also be noted that since the proof of the proposition depends only upon the properties of the optimal state-contingent tariff, the qualitative properties of the second-best tariff schedule are determined only by ordinal preferences.

### 5. Ranking of second-best policies

We have indicated earlier the difficulty of implementing the optimal state-contingent policy in response to changes in the foreign price. Although the preceding analysis of the optimal state-contingent tariff is interesting in its own right, its main value is not in implementation but in facilitating a comparison of second-best policies. The ranking of second-best policies depends on how each policy behaves relative to the optimal state-contingent policy. The policy that maximizes expected utility will most closely approxi-
mate the optimal state-contingent policy and will be deemed the superior second-best policy.

We have assumed that consumer preferences are weakly separable in private goods and the public good. If weak separability is relaxed, marginal rate of substitution between the private goods is generally affected by changes in the quantity of the public good. Thus, in the general case, the public good is a determinant of the domestic demand for the importable, i.e., $X = X(p, I, G)$. This implies that a change in the foreign price $p^*$ — which affects the level of the public good $(p - p^*)Q$ — results in a 'shift' in the import demand schedule. If weak separability is assumed, the domestic demand for the importable reduces to $X(p, I)$. Accordingly, the import demand function, $Q(p) = X(p, I(p)) - Y(p)$, is independent of the foreign price. Thus, under weak separability an import quota $Q_o$ is equivalent to fixing the domestic price of the importable at $p_o$ such that $Q_o = Q(p_o)$.

From the expression of the domestic price, $p = k + (1 + x)p^*$, if the ad valorem tariff $x$ were equal to $(-1)$, then the composite tariff schedule results in a stable target price, $p = k$. That is, an import quota, which is equivalent to (domestic) target pricing of the importable, can be viewed as a special case of the composite tariff where $x = -1$. Thus, the optimal composite tariff schedule dominates the optimal quota.

We now investigate the ranking of a second-best specific tariff $k_o$ and a second-best quota $Q_o$ (which stabilizes the domestic price at $p_o$). Suppose that the optimal state-contingent tariff $t'(p^*)$ is monotonically increasing in $p^*$. From the analysis of the previous section, it is immediately apparent that the second-best specific tariff $k_o$ dominates the second-best quota, since (i) a quota is equivalent to a composite tariff with the ad valorem tariff $x = -1$, whereas (ii) the optimal composite tariff entails a positive ad valorem tariff.

More formally, let $p_o$ be the domestic price under the optimal quota, and $p^f(p^*)$ be the optimal state-contingent domestic price of the importable. Then there exists a world price, denoted $p^*$, for which $p_o$ is the optimal state-contingent price, as shown in fig. 1. Let $k = p^f(p^*) - p^*$, and let $p^k = (p^* + k)$ denote the domestic price under a constant specific tariff $k$. Since $dt/dp^* > 0$, it follows that $p_o > p^k > p^f$ for $p^* < p^*$ and $p^f > p^k > p_o$ for $p^* > p^*$. Thus, the domestic price $p^k$ under the specific tariff $k$ is everywhere closer to the optimal state-contingent price $p^f$ than is the target price $p_o$ under the optimal quota $Q_o$. Since this specific tariff $k$ dominates the optimal quota, the optimal specific tariff $k_o$ must also dominate the latter.

**Proposition 6.** If $\gamma \epsilon > m$, $-tQ''/Q' > 1$ and $\eta + \epsilon/s > 0$, then the optimal specific tariff $k_o$ dominates the optimal quota $Q_o$, regardless of risk aversion.

Note that the proposition is independent of cardinal preferences, as the behavior of the optimal state-contingent tariff is determined only by ordinal
preferences. Changes in risk attitudes do not affect \( (dt'/dp^*) \), and hence cannot reverse the ranking of second-best policies where \( (dt'/dp^*) \) is positive.

5.1. Risk aversion

If the optimal state-contingent tariff is monotone decreasing or not monotone in \( p^* \) everywhere, then information about ordinal preferences is insufficient to rank the second-best policies. We now illustrate how cardinal preferences may affect the ranking of second-best policies. Let \( J^*=J(p^*+k, p^*) \), \( J^o=J(p_o, p^*) \) and \( J'=J(p^*+t', p^*) \) denote the realized utility under a specific tariff \( k \), a quota \( Q_o \), and the optimal state-contingent tariff \( t' \), respectively. First, we investigate the general conditions under which the optimal specific tariff dominates the optimal quota. If \( Q_o \) is the optimal quota, there exists a world price, denoted \( \tilde{p}^* \), for which \( (p_o-\tilde{p}^*) \) is the optimal state-contingent tariff, i.e., \( t'(\tilde{p}^*)=p_o-\tilde{p}^*=k \). Moreover, \( \tilde{p}^* \) is unique, since \( dp'/dp^*>0 \) by Proposition 1. If the constant specific tariff \( k \) dominates the optimal quota, then the optimal specific tariff must also dominate the latter.

Differentiating the realized utilities with respect to \( p^* \) yields
Recall that at \( \hat{p}^* \), \( p_o = \hat{p}^* + k = \hat{p}^* + t^f(\hat{p}^*) \), and hence \( J^k = J^o = J^f \). Moreover, when evaluated at \( \hat{p}^* \), \( J^k_{p^*} = J^o_{p^*} = J^f_{p^*} \); but \( J^f \geq J^k \) and \( J^f \geq J^o \) everywhere. Thus, these functions are monotone decreasing in \( p^* \) and are tangent at \( \hat{p}^* \), as shown in fig. 2. Thus, a comparison of the curvatures of \( J^k \) and \( J^o \) could reveal which policy yields a higher utility ex post.

Differentiating (27a) and (27b) with respect to \( p^* \) yields

\[
J^k_{p^*} = -V_{1}Q + V_G k Q', 
\]

(27a)

\[
J^o_{p^*} = -V_G Q, \quad (27b)
\]

\[
J^f_{p^*} = -V_G Q(p'). \quad (27c)
\]

Recall that at \( \hat{p}^* \), \( p_o = \hat{p}^* + k = \hat{p}^* + t^f(\hat{p}^*) \), and hence \( J^k = J^o = J^f \). Moreover, when evaluated at \( \hat{p}^* \), \( J^k_{p^*} = J^o_{p^*} = J^f_{p^*} \); but \( J^f \geq J^k \) and \( J^f \geq J^o \) everywhere. Thus, these functions are monotone decreasing in \( p^* \) and are tangent at \( \hat{p}^* \), as shown in fig. 2. Thus, a comparison of the curvatures of \( J^k \) and \( J^o \) could reveal which policy yields a higher utility ex post.

Differentiating (27a) and (27b) with respect to \( p^* \) yields

\[
J^k_{p^*} = [V_{11}Q^2 + V_{1}X_{1}Q - V_{1}Q'] - 2V_{1}kQQ' + V_{GG}(kQ')^2 + V_G kQ^*, \quad (28a)
\]

\[
J^o_{p^*} = V_{GG}Q^2; \quad V_{GG} = V_{GG}[p_o, l_o, (p_o - p^*)Q_o]. \quad (28b)
\]
If the consumer is risk averse (neutral) in the public good, then the indirect utility $J^\circ$ under the optimal quota is concave (linear) in $p^\circ$, but the curvature of $J^k$ is generally ambiguous. To compare the curvatures of $J^k$ and $J^\circ$, let $\Delta \equiv J^k_{p^\circ p^\circ} - J^\circ_{p^\circ p^\circ}$. Then

$$\Delta = (\eta + \varepsilon/s - R) V_k Q^2/2 - 2V_{kq} kQ' + [V_{gg}(kQ')^2 - V_{gg}Q]^2$$

where $R \equiv IV_1/IV$ is the relative risk-aversion index. If $\Delta$ is positive, then $(J^k - J^\circ)$ is convex in $p^\circ$. Since $J^k$ and $J^\circ$ are tangent at $\tilde{p}^\circ$, $(J^k - J^\circ)$ obtains a (global) minimum at $\tilde{p}^\circ$. Moreover, $J^k > J^\circ$ for all $p^\circ \neq \tilde{p}^\circ$ and hence $J^k = J^\circ$ at $\tilde{p}^\circ$, the specific tariff $k$ dominates the optimal quota (see fig. 2). A fortiori, the optimal specific tariff dominates the optimal quota.

**Proposition 7.** If $V_{gg} = 0 \leq V_{ig}$, $Q^\circ \geq 0$, and $\eta + \varepsilon/s \geq R$, then the optimal specific tariff dominates the optimal quota. Alternatively, if $V_{gg} = 0 \leq V_{ig}$, $Q^\circ \leq 0$, and $\eta + \varepsilon/s \leq R$, then the optimal tariff dominates the optimal specific tariff.

In contrast to Proposition 6, this proposition allows the ranking of ex ante second-best policies when the representative consumer is risk averse. Moreover, its assumptions correspond to those in the expected revenue constraint models considered by Dasgupta and Stiglitz (1977), and Young (1980a, b). If the policy maker is comparing policies with the same expected revenue and dispersion of revenue does not matter, the representative consumer is implicitly assumed to be risk neutral in the public good ($V_{gg} = 0$). Moreover, for expected consumer surplus to be a valid welfare measure under uncertainty, the marginal utility of income must be invariant with respect to the random price, i.e., $V_kp = -V_kX - V_X X = 0$ or $R = \eta$ [Turnovsky, Shalit and Schmitz (1980)], as well as with respect to the random public good, i.e., $V_{ig} = 0$ [see Rogerson (1980)]. Finally, if the import demand is linear, then $Q'' = 0$. With these assumptions and additive disturbances Young (1980b) demonstrate that the optimal specific tariff is superior to the optimal quota. While the sufficient conditions in Proposition 7 are more general, they are certainly satisfied by the implicit assumptions in the expected revenue constraint models. Observe also that when $V_{ig} < 0 = V_{gg}$, private expenditure is an inferior good ($\hat{c}I, \hat{c}B < 0$). This suggests that in

13Young (1980a) demonstrated that the ranking of ad valorem tariffs and quotas under an expected revenue constraint is generally ambiguous.

14Rogerson (1980) further states that for expected consumer surplus to be a valid welfare criterion under uncertainty the marginal utility of income must be invariant with respect all random variables. Choi and Johnson (1987) show that expected equivalent variation is a better measure than expected consumer surplus.
so far as private expenditure is a normal good it is unlikely for the optimal quota to dominate the optimal specific tariff.

6. Concluding remarks

Despite its well-known optimality free trade is rarely practiced by LDCs. The literature has compared tariffs and quotas under uncertainty for a small country with a constraint on expected import or expected government revenue. This paper emphasizes the revenue motive for trade taxes in LDCs, and investigates optimal trade policies when trade taxes are used to finance public goods. Specifically, we investigated the properties of optimal state-contingent policy and ex ante policies. While the optimal state-contingent policy is difficult to implement, its properties provide valuable information on the ranking of second-best policies.

If demands for private goods are independent of the public good, a change in the foreign price does not shift the import demand function. An import quota is then equivalent to a fixed domestic price of the importable even in the presence of the foreign price uncertainty. Since the specific tariff is state-independent, a supplementary ad valorem tariff can be utilized to enable the second-best tariff schedule to better approximate the optimal state-contingent tariff. Moreover, the optimal composite tariff always dominates the optimal quota. When a single instrument is used, the ranking of specific tariffs and quotas is generally ambiguous, and depends on the curvatures of the indirect utility function (with respect to income and the public good) and the import demand function. When the optimal state-contingent tariff is monotonically increasing in the foreign price, the optimal specific tariff dominates the optimal quota, regardless of risk aversion. If the optimal state-contingent tariff is not increasing in the foreign price everywhere or the behavior of the optimal state-contingent tariff is difficult to ascertain, risk attitudes and ordinal preferences jointly determine the ranking of second-best policies.

References
Choi, J.Y. and E.S.H. Yu, 1984, Gains from trade under variable returns to scale, Southern Economic Journal 50, 979–992.