RANKING OPTIMAL TARIFFS AND QUOTAS FOR A LARGE COUNTRY UNDER UNCERTAINTY

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Optimal tariffs and quotas are compared for a large country under uncertainty. If the import supply schedule has constant elasticity and is subject to multiplicative uncertainty and domestic demand is random then the optimal policy is a fixed ad valorem tariff. If the supply schedule has constant elasticity but this elasticity is random then the optimal tariff is superior to the optimal quota. If the demand and supply schedules are linear then the optimal quota is superior to the optimal tariff if and only if the supply schedule is inelastic and the degree of uncertainty in the demand and supply schedules is small.

1. Introduction

Bhagwati (1965, p. 53) initiated the analysis of the equivalence of quotas and tariffs. He defined equivalence as obtaining when the replacement of a quota (tariff) by a tariff (quota) at the level defined by the implicit tariff (import level) in the quota (tariff) situation leads to an identical equilibrium. He showed that equivalence obtains if there is perfect competition, certainty, no retaliation or distribution effects and quotas are auctioned. However, as Bhagwati (1978, ch. 2) has noted, if one of these conditions is violated then it is important to compare the welfare effects of these two instruments when they are 'equivalent' in the weaker sense that both satisfy some constraint, e.g. on the resulting level of domestic production or imports.

Recent papers in the latter tradition have compared tariffs and quotas under uncertainty. For a large country, Fishelson and Flatters (1975) compared the optimal quota with the tariff yielding the same expected imports.¹ For a small country, Pelcovits (1976) compared quotas with tariffs yielding the same expected imports while Dasgupta and Stiglitz (1977) compared quotas with tariffs yielding the same expected tariff revenue.

This paper compares the optimal tariff with the optimal quota under

¹The title of their paper 'The (non-) equivalence of optimal tariffs and quotas under uncertainty' (italics inserted) is therefore not completely accurate. However, as we shall see in section 3, their results comparing optimal quotas with mean equivalent tariffs lead immediately to conclusions about optimal quotas and optimal tariffs in two important cases.
uncertainty for a large country without imposing additional constraints, such as those above on expected imports or tariff revenue. We present models where the optimal tariff is always superior to the optimal quota and also models where the reverse can hold. In a related paper, Ohta (1978) has compared quotas and export price controls for a large country in terms of expected export profits.

Our choice of models and method of analysis is best motivated by recalling Fishelson and Flatters' initial challenge to the widely held presumption that tariffs are preferable to quotas under uncertainty because they permit imports to respond to domestic and foreign disturbances. They pointed out, very perceptively, that 'what is ignored in this reasoning is the possibility that under a given tariff structure the responsiveness of imports to a change in demand may be such that the new level of imports overshoots the new optimal level by an amount greater than the amount by which the new optimum exceeds the old (quota) level of imports'. In their linear model this point is illustrated by their proposition that if the demand schedule is fixed but there is uncertainty about the height of the supply schedule then the optimal quota is superior to the mean equivalent tariff if and only if the elasticity of the expected import supply is less than unity.

It is intuitively plausible that the magnitude of the elasticity of import supply should be related to the possibility of 'overshooting' and hence to the ranking of tariffs and quotas. We take up this suggestion in a model where the elasticity of supply is constant. Corresponding to Fishelson and Flatters' analysis of stochastic shifts in the height and slope of a linear supply schedule we analyse the effects of multiplicative shifts in the supply function and of shifts in the magnitude of the elasticity itself. (This corresponds to stochastic shifts in the height and slope of the supply schedule when expressed as a linear relationship between the logarithms of price and quantity.) This model of supply uncertainty is of intrinsic interest because the behaviour of supply over a wide price range is plausible, even given large stochastic shifts in the parameters. As we shall see, this is not the case in the linear model.

In section 2 we show that, in the case of multiplicative shifts of a constant elasticity supply function, a fixed ad valorem tariff is not only superior to a fixed quota but is also the optimal policy - even in the face of simultaneous uncertainty in demand. In the case of shifts in the elasticity of supply the

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2Ohta (1978) has used a special case of our model of supply uncertainty. He assumes that import supply is of constant elasticity with a multiplicative, lognormally distributed random term; that the domestic production set is Ricardian; and that the home country does not consume the exported good. Thus, his objective function is essentially expected profits from exports while our objective function is expected domestic welfare. Moreover, he considers controls on export prices while we consider the choice of a tariff rate - which would still allow export prices to fluctuate.
optimal tariff is again superior to the optimal quota, because a fixed tariff can be chosen which always leads to imports which are closer to the optimal imports than the optimal fixed quota: 'overshooting' never occurs. Our analysis also identifies the general circumstances under which 'overshooting' can be excluded.

The constant elasticity model comprises an important class of cases where the presumption that tariffs are superior to quotas under uncertainty is vindicated despite the considerations raised by Fishelson and Flatters. Within it, a low expected elasticity of supply does not mean that the optimal quota is superior to the optimal tariff. In contrasting this conclusion with that of Fishelson and Flatters above, it should be noted that, not only do they use a different model of supply, but they compare the optimal quota with the mean equivalent tariff while we compare the optimal quota with the optimal tariff. In section 3 we show that in their linear model, the optimal quota is superior to the optimal tariff if and only if the expected supply is inelastic and the degree of uncertainty in supply and demand is sufficiently small. We shall relate this result to the importance of overshooting in the linear model.

2. The model when the supply elasticity is independent of the quantity imported

2.1. The general problem

The analysis of this section uses the standard two-country, two-good, general equilibrium model of trade. We thereby avoid the use of consumers surplus and evaluate the welfare of the tariff-levying country (the 'home' country) directly in terms of its indifference map over the quantities $x, q$ of goods exported and imported. The home country’s indifference map depends on a random variable $\theta$ with a fixed distribution and is assumed to be convex. We can suppose that the indifference curves arise from a utility function $V(x, q, \theta)$. The supply curve of imports is given by

$$p_s = p_s(q, x),$$

where $p_s$ is supply price using domestic exports as numeraire, $q$ is domestic imports and $x$ is a random variable with a fixed distribution.

In the arguments in this section we show that, under the tariff rate specified, domestic utility is higher than under the optimal quota for each value of $x$ and $\theta$. At no point shall we trade off gains and losses under different values of $x$ and $\theta$. Thus, our conclusions on the ranking of tariffs and quotas are valid not only for the expected utility criterion but also for any welfare criterion which is a nondecreasing function of the utility levels attained under each value of $x$ and $\theta$ and is an increasing function of the
utility level attained under at least one pair \((x, \theta)\). For example, our conclusions hold under the maxmin criterion (applied to utility levels) which corresponds to extreme risk aversion. Note also that we begin by assuming that there is no domestic production. The extension of our arguments to include domestic production is elementary and is given in Appendix A.

Since the supply curve of imports is given by \(p_s(q, x)\) the foreign offer curve is given by \(x_q = q p_s(q, x)\), where \(x\) is domestic exports. The marginal cost of domestic imports expressed in terms of exports is

\[
\frac{\partial x}{\partial q} = p_s + q \left( \frac{\partial p_s}{\partial q} \right) = p_s \left( 1 + \frac{1}{\eta(q, x)} \right),
\]

where \(\eta(q, x)\) is the elasticity of the import supply function.

For any given value of \(x\) and \(\theta\) the optimal choice for the home country is the point \(Q^* = (x^*(x, \theta), q^*(x, \theta))\) on the offer curve diagram where the home country's marginal rate of substitution equals the slope of the foreign offer curve, i.e.

\[
MRS_q(Q^*) = \frac{\partial x}{\partial q} = p_s(q^*) \left( 1 + \frac{1}{\eta(q^*, x)} \right).
\]

For the given \(x, \theta\) this could be achieved by a quota \(q^*(x, \theta)\) or by a tariff

\[
t^*(x, \theta) = \frac{1}{\eta(q^*(x, \theta), x)}.
\]

In general, there is no fixed tariff or quota which ensures that (2) will be satisfied for all values of \(x\) and \(\theta\). Hence, in general, a fixed tariff or quota will lead to lower welfare than would be possible if the policies were contingent on \(x\) and \(\theta\). However, the country is assumed to be constrained to adopt either a fixed tariff or a fixed quota. We must compare the welfare levels resulting from these two policies, taking account of all possible realisations of \(x\) and \(\theta\).

2.2. Constant supply elasticity and multiplicative uncertainty

We first identify a case where a fixed ad valorem tariff \(t\) is the optimal policy. The tariff \(t\) would lead to the optimal imports if the resulting domestic relative price of imports equals their marginal cost, which by (1) equals \(p_s(q, x)(1 + 1/\eta(q, x))\). Hence, if \(\eta(q, x)\) were independent of \(q\) and \(x\) then a fixed tariff would be the optimal policy.

Theorem 1. Suppose that the home country's indifference curves over quantities of imports and exports are convex and the indifference map is dependent
on a random variable \( \xi \) with a fixed distribution. Suppose also that the import supply schedule is of the form

\[
q = \beta p_\xi^{\eta_0},
\]

where \( \eta_0 \) is a fixed positive number and \( \beta \) is a positive random variable with a fixed distribution. Then the optimal policy for improving the terms of trade for the home country is a fixed ad valorem tariff \( t = 1/\eta_0 \). This policy is strictly better than any fixed quota except in one special case.

**Proof.** When the world price of imports is \( p_\xi \), the marginal cost of imports is \( p_\xi (1 + 1/\eta_0) \). If a tariff rate \( t = 1/\eta_0 \) is set then the domestic relative price of imports is \( p_\xi (1 + 1/\eta_0) \), i.e. it equals the marginal cost of imports in terms of home country exports or the slope of the foreign offer curve. At a competitive equilibrium the home country’s marginal rate of substitution between imports and exports equals the domestic price ratio. Hence, for any value of \( \beta \) and \( \theta \), equilibrium occurs at a point where a home country indifference curve is tangential to the foreign offer curve.

The constant elasticity import supply schedule implies a foreign offer curve which is concave to the home exports axis. Since the home indifference curve is convex the point of tangency represents the highest indifference level attainable on the foreign offer curve for the current value of \( \beta \) and \( \theta \). Thus, given a tariff \( t = 1/\eta_0 \), the equilibrium attained maximises the home country’s welfare for all values of \( \beta \) and \( \theta \). Hence it constitutes the optimal policy.

In general, the adoption of a fixed quota in the face of uncertainty will lead the home country to a point on the ex poste foreign offer curve which, for some values of \( \beta \) and \( \theta \), is not a point of tangency with an indifference curve. For such values the quota will yield strictly lower welfare than the tariff above. The exception to this general conclusion clearly involves highly specialised assumptions on the home country’s indifference map, the way in which \( \theta \) affects this map and the correlation between \( \theta \) and \( \beta \). This completes the proof of the theorem.

Notice that the foreign country’s import demand is \( \beta p_\xi^{1+\eta_0} \) so the elasticity \( \hat{\eta} \) of its offer curve is \( 1 + \eta_0 \). Thus, the optimal tariff rate is

\[
t = 1/(\hat{\eta} - 1).
\]

This is just the optimal tariff formula derived in textbooks [e.g. (Caves and Jones, p. 239)]. In general, \( \hat{\eta} \) depends on the quantity imported so application of the optimal tariff formula under certainty involves actually solving the problem of maximising the home country’s welfare given the foreign offer curve. The exception is when \( \hat{\eta} \) is independent of the quantity imported. Theorem 1 simply involves noticing that, in this case, the tariff remains
optimal in the face of shifts in domestic preferences and multiplicative shifts in the foreign offer curve. No assumptions are required about the stochastic structure of domestic preferences apart from the convexity of the indifference curves. Moreover, it is immaterial whether the supply curve is elastic or inelastic.

2.3. Supply elasticity random but independent of quantity supplied

If the elasticity \( \eta(q, \alpha) \) varies with \( q \) and \( \alpha \) then a fixed ad valorem tariff cannot always provide the margin between the terms of trade and the marginal cost of imports required to ensure an optimal choice by domestic consumers for all values of \( \alpha \) and \( \theta \). Hence, any fixed tariff will be inferior to some policy contingent on \( \alpha \) and \( \theta \). However, given the usual convexity assumptions, a fixed tariff \( t \) would be superior to a fixed quota \( \bar{q} \) provided the following condition holds, for all \( \alpha \) and \( \theta \):

\[
q^*(\alpha, \theta) > \bar{q} \quad \text{implies} \quad q^*(\alpha, \theta) \geq \bar{q}(\alpha, \theta, t) > \bar{q}
\]

and

\[
q^*(\alpha, \theta) < \bar{q} \quad \text{implies} \quad q^*(\alpha, \theta) \leq \bar{q}(\alpha, \theta, t) < \bar{q},
\]

where \( \bar{q}(\alpha, \theta, t) \) is the quantity imported under tariff \( t \). It should be a simple matter to choose a \( t \) so that \( q^*(\alpha, \theta) > \bar{q} \) implies \( \bar{q}(\alpha, \theta, t) > \bar{q} \) and \( q^*(\alpha, \theta) < \bar{q} \) implies \( \bar{q}(\alpha, \theta, t) < \bar{q} \), i.e. so that under the tariff imports respond in the right direction to changes in \( \alpha \). The additional requirements in (3) preclude the possibility of 'overshooting' raised by Fishelson and Flatters.

Let us assume that domestic preferences are fixed so that \( \theta \) can be omitted from the arguments of \( q^* \) and \( \bar{q} \). Then \( q^*(\alpha) > \bar{q} \) would imply that \( q^*(\alpha) > \bar{q}(\alpha, t) \), provided that a change in \( \alpha \) which increases \( q^*(\alpha) \) above \( \bar{q} \) also ensures that \( t \) overestimates the marginal cost of imports. (Similar remarks hold mutatis mutandis for the case \( q^*(\alpha) < \bar{q} \).) Our next theorem is based on the observation that this would certainly be the case if the random variable \( \alpha \) were simply the elasticity of import supply \( \eta \) and this elasticity is independent of \( q \). The restriction \( \bar{q} > \beta_0 \) in the theorem ensures that all equilibria occur in the region \( q > \beta_0 \) where the more elastic supply curve results in more domestic imports for given exports. This ensures that an increase in elasticity increases the quantity imported.

Theorem 2. Suppose that the home country's indifference curves are fixed and convex and that the import supply function has the form

\[
q = \beta_0 p^\eta,
\]

where \( \beta_0 \) is a constant and \( \eta \) is a positive random variable with a fixed
distribution. If the optimal quota \( q > \beta_0 \) then the optimal tariff is superior to the optimal quota.

**Proof.** The foreign offer curve \( OR_\eta^* \) corresponding to the import supply function \( q = \beta_0 p^*_q \) is given by

\[
x = \beta_0^{-1/\eta} q^{1+1/\eta},
\]

where \( x \) is the exports to the foreign country required to pay for imports \( q \). This offer curve is concave to the home export axis. Choose a tariff rate \( t \) such that the foreign offer curve corresponding to \( \eta = 1/t \) is tangential to a home indifference curve at \( q = \bar{q} \). This tariff defines a tariff-ridden offer curve \( OR_t \) for the home country. Given tariff \( t \), the point \( \bar{Q}(\eta, t) \) chosen by the home country when facing the import supply function with elasticity \( \eta \) is given by the intersection of \( OR_t \) with \( OR_\eta^* \).

If \( \eta > 1/t \) then the quantity imported satisfies

\[
MRS(\bar{Q}(\eta, t)) = p_s(\bar{q}, \eta)(1 + t) > p_s(\bar{q}, \eta) \left( 1 + \frac{1}{\eta} \right) = \frac{\partial x(\bar{q}, \eta)}{\partial q}.
\]

Thus, if \( \eta > 1/t \) then the home country's indifference curve at \( \bar{Q}(\eta, t) \) is less steep than the foreign offer curve \( OR_\eta^* \) at \( \bar{Q}(\eta, t) \). But the indifference curves are convex and the foreign offer curve is concave from below. From fig. 1 it can be seen that \( \bar{Q}(\eta, t) \) lies on a higher indifference curve than any point \( \bar{Q} \) on \( OR_\eta^* \) which involves fewer imports than \( \bar{q}(\eta, t) \). We now show that the quota \( \bar{q} \) leads to such a point \( \bar{Q}(\eta) \) on \( OR_\eta^* \).

![Diagram](image)
The tariff rate \( t \) is chosen so that \( OR_t \) passes through the point \( Q(1/t, t) \) with co-ordinates \((\beta_0(q/\beta_0)^{1/t}, q)\). Therefore \( OQ(1/t, t) \) has slope \((\beta_0/q)^{1/t}\). This is less than 1 since \( q > \beta_0 \). In the region \( q < \beta_0 \) the foreign offer curves \( OR^*_q \) all lie above the 45° line and hence above the line \( OQ(1/t, t) \) while the home tariff-ridden offer curve \( OR_t \) lies below this line. Hence \( OR_t \) must intersect the foreign offer curves in the region \( q > \beta_0 \). In this region, if \( q > q' \) then the offer curve \( OR^*_q \) lies above \( OR^{*}_{q'} \). Hence if \( q > 1/t \) then \( q(q, t) > q(1/t, t) = q \).

We have shown that if \( q > 1/t \) then \( Q(\eta, t) \) is superior to \( \bar{Q}(\eta) \). If \( \eta < 1/t \) then it can likewise be shown that

\[
MRS(Q^{*}(\eta, t)) = \frac{\partial x(q, \eta)}{\partial q}
\]

and that \( q(\eta', t) < \bar{q} \) so that \( Q(\eta', t) \) is again superior to \( \bar{Q}(\eta') \). The theorem is proved.

The hypothesis \( \bar{q} > \beta_0 \) of Theorem 2 will be satisfied provided that in the region \( q < \beta_0 \) the home country's marginal rate of substituting imports for exports is greater than \( 1 + 1/\eta \), where \( \eta \) is the smallest value that \( \eta \) can take. This condition, corresponding to a high marginal valuation of imports when imports are low, ensures that in region \( q < \beta_0 \) all foreign offer curves are steeper than all the home indifference curves so that the optimal quota \( \bar{q} \) is greater than \( \beta_0 \).

The argument of Theorem 2 also applies when the supply function is of the form

\[
q = \beta p^\eta,
\]

where both \( \beta \) and \( \eta \) are random, provided that

\[
\eta \geq 1/t \quad \text{implies} \quad q(\eta, \beta, t) \geq \bar{q}.
\]

This will be the case if (a) the possible pairs \((\beta, \eta)\) are confined to a subset \( Z \) of \( \mathbb{R}^2 \) such that for any \((\beta_1, \eta_1, \beta_2, \eta_2)\) in \( Z, \eta_1 \geq \eta_2 \) implies \( \beta_1 \geq \beta_2 \) and (b) the optimal quota \( \bar{q} \) is greater than any \( \beta \) such that \((\beta, \eta)\) is in \( Z \) for some \( \eta \).

Let us see how far the argument of Theorem 2 can be adapted to the general model of 2.1. We suppose that the supply price \( p(q, \alpha) \) depends on a random variable \( \alpha \) with \( \dot{c}p/c\alpha < 0 \). Let \( \bar{q} \) be the optimal quota. Define \( \ddot{\alpha} \) as the value of \( \alpha \) such that the optimal quota leads to a trade point \( \bar{Q} \) on the foreign offer curve associated with \( \alpha \) satisfying:

\[
MRS(\bar{Q}) = \frac{\dot{c}x(\bar{q}, \ddot{\alpha})}{\dot{c}q} = p_s(\bar{q}, \ddot{\alpha}) \left( 1 + \frac{1}{\eta(\bar{q}, \ddot{\alpha})} \right).
\]
Choose \( t = 1/\eta(\tilde{q}, \bar{x}) \) so that

\[
MRS(\bar{Q}) = p_s(\tilde{q}, \bar{x})(1 + t).
\]

The quantity \( \tilde{q}(\alpha, t) \) imported leads to a point \( \bar{Q}(\alpha, t) \) on the foreign offer curve associated with \( \alpha \) such that

\[
MRS(\bar{Q}) = p_s(\tilde{q}, \alpha)(1 + t) = p_s(\tilde{q}, \alpha)(1 + 1/\eta(\tilde{q}, \bar{x})).
\]

For \( \alpha > \bar{x}, \tilde{q}(\alpha, t) > \tilde{q} \). In order to conclude that

\[
MRS(\bar{Q}) > p_s(\tilde{q}, \alpha)(1 + 1/\eta(\tilde{q}, \alpha)) = \frac{\partial x(\tilde{q}, \alpha)\partial q}{\partial q}
\]

and hence to duplicate the reasoning of Theorem 2, we must have

\[
\eta(\tilde{q}, \alpha) > \eta(\tilde{q}, \bar{x}), \quad \text{if and only if} \quad \alpha > \bar{x}.
\]

Thus, tariffs are superior to quotas in the general case provided that (4) holds. This will certainly be the case if \( \eta \) is nondecreasing with respect to \( \alpha \) and \( \tilde{q} \). Expression (4) can also hold if \( \eta \) is increasing with respect to \( \alpha \) and decreasing with respect to \( \tilde{q} \) – provided that the direct impact of \( \alpha \) on \( \eta \) overcomes its indirect impact through \( \tilde{q}(\alpha, t) \). This requires assumptions on domestic preferences as well as on the supply function for imports.

3. Linear supply and demand functions

We now compare the optimal tariff and the optimal quota in the partial equilibrium model of Fishelson and Flatters which assumes linear demand and supply functions. In the linear model it can be shown that the optimal quota \( \tilde{q} \) is determined by the intersection of the expected demand and marginal cost schedules for imports. Fishelson and Flatters show that if the supply schedule is fixed then, under stochastic shifts in the demand schedule, the quota \( \tilde{q} \) is inferior to the mean-equivalent tariff. They also show that if the demand schedule is fixed then under stochastic shifts in the height of the supply schedule a quota \( \tilde{q} \) is inferior to the mean-equivalent tariff if and only if the expected supply schedule is elastic. Inspection of their arguments reveals that the above results remain valid when the quantity \( \tilde{q} \) is replaced by any \( \tilde{q} \) which leads to a binding quota.

If the optimal quota is inferior to the mean-equivalent tariff then it is inferior to the optimal tariff. However, if the optimal quota is superior to the mean-equivalent tariff – as in the case of supply uncertainty and inelastic expected supply – then it does not follow that it is superior to the optimal
tariff. This section characterises the circumstances in which the optimal quota is superior to the optimal tariff.

We suppose that the supply curve has the form

\[ p_s = \alpha + bq, \tag{5} \]

where \( b \) is a fixed positive number and \( \alpha \) is a random variable with mean \( \mu \). The import demand function is

\[ p_D = c - dq, \tag{6} \]

where \( c \) and \( d \) are fixed positive numbers. The case where there is also uncertainty in import demand is dealt with subsequently. We shall assume that, in the absence of intervention, the expected equilibrium quantity imported \((b-a)/(b+d)\) is positive, i.e.

\[ c-a>0, \tag{7} \]

and that the expected equilibrium price \((bc+ad)/(b+d)\) is positive, i.e.

\[ bc+ad>0. \tag{8} \]

The welfare criterion is expected net consumers’ surplus. If quantity \( q \) is imported at price \( p_s \), then the consumers’ surplus is

\[ W(q, \alpha) = cq - \frac{1}{2}dq^2 - p_s q. \]

By (5)

\[ W(q, \alpha) = (c-\alpha)q - (b+d/2)q^2. \]

For a given \( \alpha \), \( W(q, \alpha) \) is maximised by

\[ q^*(\alpha) = \frac{c-\alpha}{2b+d}. \tag{9} \]

Suppose a quota must be set before \( \alpha \) is known. The optimal quota \( \bar{q} \) is that which maximises

\[ E[W(q, \alpha)] = W(q, \alpha). \]

By (9)

\[ \bar{q} = \frac{c-a}{2b+d}. \tag{10} \]
Now suppose that an ad valorem tariff \( t \) is set. By (5) and (6) the resulting imports \( q = \hat{q}(\alpha, t) \) satisfy

\[
(\alpha + b\hat{q})(1 + t) = c - d\hat{q}.
\]

Therefore

\[
\hat{q}(\alpha, t) = \frac{c - \alpha(1 + t)}{d + \alpha(1 + t)}.
\]

The following result characterises the conditions under which the optimal quota is inferior to some tariff \( t \) and hence to the optimal tariff. Let \( G = c/d \), the intercept of the demand schedule with the quantity axis; let \( F = -a/b \), the intercept of the expected supply schedule with the quantity axis and let \( \sigma^2 \) be the variance of \( \alpha \).

**Theorem 3.** If there is uncertainty about the height of the import supply function then the optimal tariff is superior to the optimal quota if and only if

\[
\frac{\sigma^2}{a^2} > \frac{2G}{F} - 1.
\]  \( (11) \)

**Proof.** The proof uses the theory of quadratic inequalities. The difference between the expected surpluses under the tariff rate \( t \) and the optimal quota \( \hat{q} \) is \( E[W(\hat{q}(\alpha, t), x) - W(\hat{q}, x)] \). In Appendix B we show that this equals

\[
-\left((A^2 + D)t^2 + 2A(A - B)t + (A - B)^2 - D\right)/2(b + d)(d + b(1 + t))^2.
\]  \( (12) \)

where \( A \equiv ab + bc + ad, B \equiv 2bc + ad, \) and \( D \equiv d\sigma^2(2b + d) \).

It follows that the tariff \( t \) is superior to the optimal quota if and only if

\[
0 > (A^2 + D)t^2 + 2A(A - B)t + (A - B)^2 - D \equiv L(t).
\]

The optimal tariff is superior to the optimal quota if and only if \( L(t) \) is negative for some values of \( t \), i.e. the graph of \( y = L(t) \) crosses the \( t \) axis. This will be the case if and only if the equation

\[
(A^2 + D)t^2 + 2A(A - B)t + (A - B)^2 - D = 0
\]

has real roots. By the theory of quadratic equations, this will be true if and only if

\[
0 < A^2(A - B)^2 + (A^2 + D)[D - (A - B)^2]
\]
This condition is equivalent to each of the following:

\[
0 < D[D - (A - B)^2 + A^2],
\]

\[
0 < D[D - B(B - 2A)],
\]

\[
d\sigma^2(2b + d) > (2bc + ad)(-2ab - ad),
\]

\[
d\sigma^2 > -a(2bc + ad),
\]

\[
\frac{\sigma^2}{a^2} > \frac{-2bc}{ad} - 1,
\]

\[
\frac{\sigma^2}{a^2} > \frac{2G}{F} - 1.
\]

Thus, we have shown that the optimal tariff is superior to the optimal quota if and only if the last inequality holds. The theorem is proved.

There are three points to note about this result. We first check when the optimal quota can be dominated by a positive tariff. The coefficient of \( t \) in \( L(t) \) is

\[
2A(A - B) = 2(ab + bc + ad)b(a - c).
\]  \hspace{1cm} (13)

If the expected supply price of the optimal quota \( \bar{q} \) is positive, i.e.

\[
a + \frac{b(c - a)}{2b + d} = \frac{ah + bc + ad}{2b + d} > 0,
\]

then by (7) and (13) \( 2A(A - B) < 0 \). It follows that at least one of the zeros of \( L(t) \) is positive. For some \( t_0 > 0 \) sufficiently close to this zero, \( L(t_0) < 0 \). We thus conclude that if the tariff implicit in the optimal quota \( \bar{q} \) is positive then the optimal quota is inferior to some positive tariff if (11) is satisfied.

Secondly, notice that if \( a > 0 \), i.e. the expected supply curve is elastic, then \( F = -a/b < 0 \) and (11) is satisfied so that tariffs are superior to quotas. However, if \( a < 0 \), i.e. the expected supply curve is inelastic, then (11) will still be satisfied provided that \( \sigma^2/a^2 > 2G/F - 1 \), i.e. provided the proportional uncertainty in the height of the supply schedule is sufficiently large. On the other hand if \( a < 0 \) and \( \sigma^2/a^2 \) is sufficiently small then (11) is violated and quotas are superior to tariffs. We have thus established that the optimal quota can indeed be superior to the optimal tariff in improving the terms of trade: an important possibility raised, but not in fact demonstrated, by Fishelson and Flatters. The following corollary to Theorem 3 identifies a broad class of situations where this will be the case.
Corollary. If \( \alpha < 0 \) (except on a set of measure zero), i.e. almost all the random supply curves are inelastic, then the optimal quota is superior to the optimal tariff.

Proof. By assumption (8)

\[
(-2bc/ad) - 1 > 1.
\]

Thus, if tariffs are superior to quotas then it is necessary that

\[
\frac{\sigma^2}{a^2} > 1.
\]

But this implies that the set of \( \alpha > a \) satisfying \( |\alpha - a| > |a| \) is of positive measure, i.e. the set of \( \alpha > 0 \) is of positive measure. Therefore if \( \alpha < 0 \) except on a set of measure zero then the optimal quota is superior to the optimal tariff.

Thirdly, suppose that the import demand curve is random and of the form

\[
p_d = \beta - dq,
\]

where \( \gamma \) is a random variable, uncorrelated with \( \alpha \), with mean \( c \) and variance \( s^2 \). Then the argument of Theorem 3 can be extended to obtain:

Theorem 4. If there is uncertainty about the heights of both the demand and the import supply functions then the optimal tariff is superior to the optimal quota if and only if

\[
\frac{\sigma^2 + s^2}{a^2} + \frac{s^2}{\sigma^2} \left[ \frac{dA^2 + 2bA(c - a) + b^2(2b + d)^2s^2}{d^2a^2(2b + d)} \right] > \frac{2G}{F} - 1.
\]

By (7) the term in the square brackets is positive. Therefore the optimal quota will be superior to the optimal tariff only if expected supply is inelastic and the degree of uncertainty in both demand and supply is small and the variance of \( \gamma \) is small compared to that of \( \alpha \).

We conclude by relating the results of this section to the analysis of section 2. The tariff-augmented supply price \( (\alpha + bq)(1 + t) \) will generally differ from the marginal cost of imports \( \alpha + 2bq \). Since the optimal tariff \( t \) must be chosen taking account of all values of \( \alpha \) there will always be values of \( \alpha \) such that \( q^*(\alpha) > \bar{q} \) and \( (\alpha + bq(\alpha, t))(1 + t) < \alpha + 2b\bar{q}(\alpha, t) \). Hence, the optimal tariff will lead imports to overshoot for some values of \( \alpha \).
The ranking of tariffs and quotas in the linear model is thus a matter of comparing the welfare losses from overshooting under tariffs with the losses from the rigidity under quotas. This has been done in Theorem 3. From this theorem we see that the sign of \( a \) (hence whether the expected supply curve is elastic) is not the only factor: the degree of uncertainty in \( a \) is also critical. The inflexibility of quotas will always render them inferior to tariffs if the degree of uncertainty is sufficiently large — despite the possibility of the overshooting of imports under tariffs.

4. Conclusions

We have shown that optimal tariffs are superior to optimal quotas, if (1) import supply demands on a random variable which affects the quantity supplied and the supply elasticity in the same direction, and (2) the supply elasticity is constant or increasing with the quantity supplied. No assumptions are required on demand apart from convexity of the home country’s indifference map. The possibility, raised by Fishelson and Flatters, that optimal quotas can be superior to optimal tariffs through ‘overshooting’ has been established in the linear model when the degree of uncertainty is sufficiently small.

In comparing the two models of import supply above we note that, for large stochastic shifts of the type analysed, the assumption that the elasticity of supply is independent of quantity is more plausible than the assumption that the slope of the supply function is independent of quantity. With large shifts in the height of the supply function the linear model yields implausible behaviour for small prices and quantities. However, the possibility that quotas can be superior to tariffs cannot be dismissed on this account since the linear model would be a good approximation to some supply situations when stochastic shifts are small. It is in just such situations that we have identified conditions under which quotas can be superior to tariffs. It would be interesting to establish general conditions on supply and demand under which this would be true.

Appendix A: Theorems 1 and 2 when there is domestic production

Suppose production possibilities in the tariff-imposing country are given by the production function

\[
g(y_1, y_2, \theta_1) \leq 0,
\]

where \( \theta_1 \) is a random variable with a fixed distribution. Suppose also that domestic preferences over the two goods are given by a utility function
$U(z_1, z_2, \theta_2)$. An indirect utility function over the quantities of good 1 exported and of good 2 imported is given by

$$V(x, q, \theta_1, \theta_2) = \max \{U(z_1, z_2, \theta_2) : g(z_1 + x, z_2 - q, \theta_1) \leq 0\}.$$ 

This defines a family of indifference maps over $(x, q)$ which are contingent on $\theta = (\theta_1, \theta_2)$. The indifference curves will be convex provided that $U$ is quasiconcave and $g$ is convex. The reasoning of Theorems 1 and 2 will hold for the utility function $V$ provided that when the domestic price ratio is $p_4(q, x)(1 + t)$ competitive choices lead to a trade point $Q$ such that

$$\frac{\partial V}{\partial x} \bigg| \frac{\partial V}{\partial q} = MRS(Q) = p_4(q, x)(1 + t).$$

This in fact follows immediately from the fact that in the face of this price ratio the representative consumer chooses a bundle such that

$$\frac{\partial U}{\partial z_1} \bigg| \frac{\partial U}{\partial z_2} = p_4(q, x)(1 + t)$$

and the profit maximising firms choose output so that

$$\frac{\partial g}{\partial y_1} \bigg| \frac{\partial g}{\partial y_2} = p_4(q, x)(1 + t).$$

**Appendix B: Proof of eq. (12)**

$$W(\bar{q}, x) - W(\bar{q}, x) = W(q^*, x) - W(\bar{q}, x) - [W(q^*, x) - W(\bar{q}, x)]$$

$$= \frac{1}{2}(2b + d)[(q^* - \bar{q})^2 - (q^* - \bar{q})^2].$$

$$q^* - \bar{q} = \frac{c - x}{2b + d} - \frac{c - x(1 + t)}{d + b(1 + t)}$$

$$= \frac{bcT - 2bc + zd + zdT + zdT}{(2b + d)(d + bT)}$$

$$= \frac{AT - B + (z - a)[d + bT]}{(2b + d)(d + bT)},$$

where $T \equiv 1 + t$, $A \equiv ab + bc + ad$, and $B \equiv 2bc + ad$. Also

$$q^* - \bar{q} = \frac{(z - a)(d + bT)}{(2b + d)(d + bT)}.$$
Since $E[x-a]=0$ and $E[(x-a)^2]=\sigma^2$

$$(2b+d)^2(d+bT)^2E[(q^*-q)^2-(q^*-\bar{q})^2]$$

$$=-(AT-B)^2+\sigma^2[(d+bT)^2-(-d+bT)^2-2dT(-d+bT)-d^2T^2]$$

$$=-(AT-B)^2+\sigma^2[4bdT+2d^2T-2bdT^2-d^2T^2]$$

$$=-(AT-B)^2+D(2T-T^2)$$

$$=-(A^2+D)T^2+2(AB+D)T-B^2$$

$$=-(A^2+D)t^2-2A(A-B)t-(A-B)^2+D,$$  \hspace{1cm} (15)

where $D=d\sigma^2(2b+d)$.

By (14) and (15) $E[W(\bar{q},z)-W(\bar{q},\bar{z})]$ equals

$$-[\frac{(A^2+D)t^2+2A(A-B)t+(A-B)^2-D}{2(2b+d)(d+b(1+t))}].$$

References


