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Risk Aversion and Optimal Trade Restrictions

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If the representative consumer of a country is risk averse then the choice of trade controls must take account of their effects on the fluctuations of domestic real income. If the world price of the importable is uncertain and risk aversion is high then the optimal policy for achieving a ceiling on expected imports involves a reduction in imports and a rise in the domestic price as the world price falls. Moreover, a quota is superior to a tariff in achieving the ceiling. Under domestic uncertainty, a tariff is superior to a quota but it could be optimal to reduce the domestic price as imports increase.

1. INTRODUCTION

Economists have long recognized the equivalence of price and quantity controls under certainty. The comparison of these control modes and the characterization of the optimal form of control under uncertainty has been undertaken by two groups of authors. Weitzman (1974, 1978) considered a planning authority which faces uncertainty about the costs and benefits of producing a good. He showed that the ranking of price and quantity controls depends on the curvature of the cost and benefit functions and that an optimal policy can be considered as a mixture of price and quantity controls. His work was developed by Laffont (1977), Ireland (1977), Malcomson (1978) and Yohe (1978). In the context of international trade, the comparison of tariffs and quotas under uncertainty has been undertaken by Fishelson and Flatters (1975), Pelcovits (1976), Dasgupta and Stiglitz (1977), Young (1979, 1980a,b) and Helpman and Razin (1980). Recently, Young and Anderson (1980) showed that the policy maximizing expected consumer’s surplus given a constraint on expected imports (expected import expenditure) is a specific tariff (an ad valorem tariff). In contrast to Weitzman’s planning model, the price instrument is superior to the quantity instrument—whatever the curvature of the benefits function.

The conclusion of Young and Anderson appears to provide a strong argument against using quantity instruments to restrict trade. However, the expected surplus criterion is a valid welfare measure only if the marginal utility of income is constant. In this paper we characterize the optimal form of trade restriction if the representative consumer of the importing country exhibits risk aversion toward income fluctuations. The optimal policy can have surprising properties—properties which, with high risk aversion, are better approximated by a quota rather than a tariff. Hence the “second best” justification for using quantity controls to restrict trade turns out to depend on the curvature of utility as a function of income rather than its curvature as a function of consumption of the imported good.

Section 2 motivates our model and presents an intuitive discussion. Section 3 sets out the general equilibrium model of a trading country facing a random world price for
imports. Section 4 characterizes the optimal trade restriction given a ceiling on expected imports. Section 5 gives conditions under which a quota is superior to a tariff and conditions under which the reverse is true. Section 6 considers the case where the source of uncertainty is in domestic production. Quotas are then always inferior to tariffs but the optimal policy could involve a schedule of tariffs whose level falls as imports increase. Conclusions and possible extensions are presented in Section 7.

2. MOTIVATION AND INTUITIVE DISCUSSION

Most textbook comparisons of tariffs and quotas (e.g. Caves and Jones (1977, p. 226), Richardson (1980, p. 357)) argue that tariffs are superior because they permit imports to respond "appropriately" to changes in world prices and in domestic demand and supply conditions. We evaluate this argument for a small country, assuming ex post choices in production and consumption. In these circumstances, free trade is optimal and the motivation for the trade restriction (and the basis for comparing tariffs and quotas) must be sought in some "non-economic" constraint imposed on the government by subgroups threatened by imports. Political expediency is likely to lead such a subgroup to advance claims, not directly in terms of its own welfare level, but indirectly by focussing on the highly visible and emotive issue of the level of foreign imports.

If the non-economic constraint were on the level of imports per period and world supply conditions were uncertain, then this could be achieved only by a quota. In practice, such rigidity in the face of random shocks is rare. For example, in 1973 and again in 1974, U.S. dairy product quotas were substantially increased by the Nixon administration to dampen rises in food prices. The balance of political forces which creates import restrictions appears to allow—or even to require—some averaging across periods. Clearly, the argument that tariffs permit a desirable flexibility in imports assumes that the non-economic constraint permits such flexibility. We shall therefore assume that the non-economic constraint is on the average level of imports. While the government might also be concerned with fluctuations in the level of imports, an argument that "rigid" quotas can be superior to "flexible" tariffs would have particular force if it applies even when the government is neutral toward import fluctuations.

Consider first the case where the country faces fluctuations in world prices. The fact that tariffs encourage (restrict) imports when the world price is low (high) means that it leads to "arbitrage" of imports across states of the world. Hence expected import expenditure is lower under a tariff. However, this arbitrage also implies that real income fluctuations are greater under the tariff because the country's "gains from trade" become further restricted just when it is being impoverished by the income effects of a rise in world price.

Under a quota, the reduction in real income from high world prices is not accompanied by further restriction of trade. With sufficiently high risk aversion, this greater income stability under a quota can outweigh the "arbitrage" benefits of tariffs. These considerations also imply that, as the world price rises, the optimal trade restriction could involve an increase in imports and a fall in the domestic price.

The above argument is incomplete. It ignores for example, income effects on demand and the effects of changes in domestic prices on domestic production and on the marginal utility of income. We now set out a general equilibrium model of a trading country which captures such effects.

3. THE MODEL

We assume that there are two goods and take the exportable as numeraire. As in Dixit and Norman (1980) and Woodland (1980), domestic production possibilities are represented by a revenue function \( R(P) \) where \( P \) is the domestic price of the importable.
The domestic supply function of the importable is then:

\[ Y(P) = R_P(P). \]  

(1)

(A subscript denotes partial differentiation with respect to the corresponding variable.)

The preferences of the representative consumer are embodied in the indirect utility function \( V(P, I) \) where \( I \) is income. The domestic demand function for the importable, \( X(P, I) \), satisfies Roy’s Formula:

\[ V_P(P, I) = -X(P, I) V_I(P, I). \]  

(2)

The country is small and faces a random world price \( \pi \) for the importable. When imports are \( Q \) and the domestic price of the importable is \( P \), consumer income is:

\[ I(P, \pi, Q) = R(P) + (P - \pi)Q \]

i.e. production revenue (profits plus factor payments) plus tariff revenue. Domestic excess supply of the importable is then:

\[ Z(P, \pi, Q) = Y(P) - X(P, I(P, \pi, Q)). \]

Let \( \tilde{P}(Q, \pi) \) be the equilibrium domestic price when imports are \( Q \) and the world price is \( \pi \). Then \( \tilde{P} \) is the solution to:

\[ Q + Z(P, \pi, Q) = 0. \]  

(3)

The slope of the general equilibrium excess supply function for the importable is:

\[ Z_P = Y_P - X_P - YX_I - QX_I \]

by (1)

\[ = Y_P - X_P - XX_I \quad \text{when } P = \tilde{P}(Q, \pi). \]

The supply function has slope \( Y_P > 0 \) and the compensated demand function has slope \( X_P + XX_I < 0 \) so:

\[ Z_P > 0. \]  

(4)

Our arguments use the “indirect utility function over imports”:

\[ \tilde{V}(Q, \pi) = V(\tilde{P}(Q, \pi), I(\tilde{P}(Q, \pi), \pi, Q)). \]  

(5)

This equals domestic utility in equilibrium when imports are \( Q \) and the world price is \( \pi \). Differentiating (5) and substituting from (1) yields:

\[ \tilde{V}_I(Q, \pi) = \tilde{P}_Q V_P + \tilde{P}_Q (Y + Q) V_I + (\tilde{P} - \pi) V_I \]

(3) and (2) then imply that:

\[ \tilde{V}_I(Q, \pi) = (\tilde{P} - \pi) V_I(\tilde{P}, I(\tilde{P}, \pi, Q)). \]  

(6)

Thus, the marginal utility of imports in state \( \pi \) equals the rent \((\tilde{P} - \pi) \) on a quota licence for imports in that state, multiplied by the marginal utility of income in that state. The analysis of the behaviour of these two terms permits us to express precisely the trade-off between the arbitrage and the income stabilizing (or de-stabilizing) effects of trade restrictions which was discussed in Section 2.

We frequently refer to the effect of a rise in the domestic price \( P \) on the marginal utility of income. We have:

\[ dV_I(P, I(P, \pi, Q))/dP = V_{PI} + (Y + Q)V_{II} \]

\[ = V_{PI} + XV_{II} \quad \text{when } P = \tilde{P}. \]

Differentiating (2) with respect to \( I \):

\[ V_{PI} = -X_I V_I - XV_{II}. \]  

(7)
Therefore:
\[ dV_t(P, I(P, \pi, Q))/dP = -X_t V_t \quad \text{when } P = \hat{P}(Q, \pi) \] (8)
i.e. a rise in the domestic price of a non-inferior good always lowers the marginal utility of income when the effects on domestic production and tariff revenue are taken into account.

We shall assume that:
\[ V_{II}(P, I) \leq 0 \quad \text{for all } P, I \] (9)
i.e. the representative consumer is always risk averse or risk neutral. A simple calculation (see Appendix) then shows that:
\[ \hat{V}_{Q0}(Q, \pi) < 0 \] (10)
i.e. the utility function over imports \( Q \) is concave in \( Q \) for a given world price.

4. THE OPTIMAL TRADE RESTRICTION

As discussed in Section 2, we suppose that the government wishes to maximize domestic expected utility subject to a ceiling \( \hat{Q} \) on expected imports. If only the world price \( \pi \) of the importable is uncertain, then any policy restricting trade is equivalent to a rule \( Q(\pi) \) for determining the quantity to be imported in each state \( \pi \). (Note that \( Q(\pi) \) is actual imports rather than a ceiling on imports in state \( \pi \).) The optimal policy ex ante is the solution, \( Q^0(\pi) \), to the problem:
\[ \max_{Q(\pi)} E[\hat{V}(Q(\pi), \pi)] \quad \text{subject to } E[Q(\pi)] \leq \hat{Q}. \] (11)
The first-order condition is:
\[ \hat{V}_{Q}(Q, \pi) = \lambda \] (12)
where \( \lambda \) is the Kuhn–Tucker multiplier corresponding to the constraint on expected imports. By (10), \( \hat{V}_{Q0}(Q, \pi) < 0 \) so the solution to (12) is unique and the first order condition indeed defines an optimum. The optimal domestic price in state \( \pi \) is:
\[ P^0(\pi) = \hat{P}(Q^0(\pi), \pi) \]

To characterize the optimal policy we define:
\[ \rho = -IV_{II}/V_t \]
the Arrow–Pratt coefficient of relative risk aversion,
\[ \eta = IX_t/X \]
the income elasticity of demand for the importable,
\[ T = (P - \pi)Q/I \]
the share of tariff revenue in national income and
\[ M = 1 - (P - \pi)X_t. \]

Note that \( M = 1 - PX_t/(1+t) \) where \( t \) is the implicit ad valorem tariff rate. If neither good is inferior, then \( 0 < PX_t < 1 \) and \( 0 < M < 1 \). Given any \( \pi \), we say that the above parameters are "evaluated at \( Q(\pi)" \) when \( Q = Q(\pi) \), \( P = \hat{P}(Q(\pi), \pi) \) and \( I = I(\hat{P}(Q(\pi), \pi), \pi, Q(\pi)). \) Only the parameter \( \rho \) involves cardinal properties of the utility function. Hence, at any \( Q(\pi) \), we can make assumptions about the value of \( \rho \) independently of assumptions about the other parameters. The following result shows that, for
sufficiently high values of $\rho$, the perverse features of the optimal policy mentioned in
Section 2 indeed occur.

**Theorem 1.** Suppose that $\lambda > 0$, i.e. expected utility would be increased if the con-
straint on expected imports were relaxed. Then:

(A) \[ \rho \overset{\text{def}}{=} (1 + QMx_{1}/Z_{p})/T \quad \text{implies} \quad Q^{0}_{\pi} \overset{\text{def}}{=} 0 \]

(B) \[ \rho \overset{\text{def}}{=} M/T \quad \text{implies} \quad 0 \overset{\text{def}}{=} P^{0}_{\pi} \]

(C) \[ \rho \overset{\text{def}}{=} \eta XM/(Q + (\bar{P} - \pi)Z_{p}) \quad \text{implies} \quad 1 \overset{\text{def}}{=} P^{0}_{\pi} \]

(All functions are evaluated at $Q = Q^{0}(\pi)$.)

**Proof.** If $\lambda > 0$ then (12) and (6) imply that, in all states $\pi$, $(\bar{P} - \pi)V_{t} > 0$. It follows
that, for all states $\pi$:

\[ \bar{P}(Q^{0}(\pi), \pi) - \pi > 0, \quad Q^{0}(\pi) > 0 \quad \text{and} \quad T > 0 \]

i.e. we have positive denominators in the first inequalities in (A), (B) and (C) above.

Applying the Implicit Function Theorem to (12):

\[ Q^{0}_{\pi}(\pi) = -\bar{V}_{O\pi} / \bar{V}_{OQ}. \]

(13)

Differentiating (6) and substituting from (8):

\[ \bar{V}_{O\pi} = (\bar{P} - 1)V_{t} + (\bar{P} - \pi)(-\bar{P}_{\pi}X_{1}V_{t} - QV_{H}). \]

(14)

Applying the Implicit Function Theorem to (3):

\[ \bar{P}_{\pi}(Q, \pi) = -X_{1}Q/Z_{p}. \]

(15)

Therefore:

\[ \bar{V}_{O\pi} = (\rho T - 1 - QX_{1}M/Z_{p})V_{t}. \]

(16)

(A) follows from (13), (10) and (16). Moreover:

\[ P^{0}_{\pi}(\pi) = \bar{P}_{\pi}(Q^{0}, \pi) + \bar{P}_{Q}(Q^{0}, \pi)Q^{0}_{\pi}. \]

(17)

(B) and (C) then follow by substituting from (15) and (13) and simplifying (see
Appendix).

To see the underlying intuition, note that by (6) the first order condition (12) can
be written as:

\[ \{\bar{P}(Q, \pi) - \pi\}V_{t}(\bar{P}(Q, \pi), I(\bar{P}(Q, \pi), \pi, Q)) = \lambda \]

(18)

i.e. the contingent quotas $Q(\pi)$ should be allocated so that the rent on a quota license
times the marginal utility of income is the same in all states $\pi$. If the marginal utility of
income is constant, i.e. expected consumer's surplus is a valid welfare measure, then
(18) implies that the rents $\bar{P} - \pi$ on quota licenses should be the same in all states. As
Young and Anderson (1980) have noted, this condition can be met by setting a specific
tariff equal to the common value of these rents. Under this policy, imports decrease
(increase) as $\pi$ rises (falls).

To see how risk aversion affects the optimal response of imports to changes in $\pi$,
consider the effect of $\pi$ on the marginal utility of imports, $\bar{V}_{O}(Q, \pi)$, when $Q$ is fixed.
A rise in $\pi$ lowers the rent on the quota $Q$ (unless the importable is very inferior).
However, it also lowers the country's real income and, given risk aversion, this tends to
raise the marginal utility of income $V_{t}$. Moreover, for fixed $Q$ and a non-inferior
importable, the income effect of the increase in $\pi$ lowers the domestic price $P$ and this
increases \( V_I \) also (see (8)). If the last two effects are sufficiently strong, then the increase in \( \pi \) will increase \( V_O(Q, \pi) \). Optimal imports \( Q^0(\pi) \) would then increase with the increase in \( \pi \). Theorem 1(A) shows that this becomes more likely with higher values of \( T \), the share of tariff revenue in national income; \( \rho \), the coefficient of relative risk aversion and \( Z_P \), the slope of the domestic excess supply function. This is in line with the discussion in Section 2, since \( T \) is associated with the impact of the world price on national income, \( \rho \) with the aversion to variations in income and \( Z_P \) with the additional "gains from trade" from an increase in imports. Theorems 1B and 1C show that this increase in optimal imports can lead to the optimal domestic price to fall—or, at least, to increase more slowly than \( \pi \).

The optimal policy, characterized in Theorem 1, could be achieved by imposing a schedule of specific tariffs \( P^0(\pi) - \pi \) depending on the world price \( \pi \). Such a tariff schedule is operational, in principle, since \( \pi \) is readily monitored. However, the cost of collecting the information required to calculate the schedule could be prohibitive. We therefore consider second-best comparisons between simpler forms of trade restriction. Our analysis of the optimal ex-ante policy illuminates this issue also.

5. SECOND BEST COMPARISONS

Theorem 1(A) showed that if there is a constraint on expected imports and the representative consumer is highly risk averse, then it is optimal to restrict imports when the world price is low. In these circumstances, we would expect a fixed quota to be superior to a tariff—which leads to an increase in imports when the world price is low. In fact, the condition (19) below which ensures that the quota is superior is essentially the condition in Theorem 1(A) which ensures that \( Q^0(\pi) > 0 \). The only difference is that it is evaluated at the quota level \( \bar{Q} \) rather than at \( Q^0(\pi) \).

**Theorem 2.** Suppose that the quota \( \bar{Q} \) on imports is binding in all states \( \pi \) and that:

\[
\rho \geq (1 + QX_M)/Z_P \quad \text{for all} \quad \pi \quad \text{when} \quad Q = \bar{Q}.
\]

Then the quota \( \bar{Q} \) yields higher expected utility than any other policy \( Q(\pi) \) such that:

\[
E[Q(\pi)] = \bar{Q} \quad \text{and} \quad Q_\pi(\pi) \leq 0.
\]

**Proof.** By (10), \( \hat{V}_{oo}(Q, \pi) < 0 \) so the Second Mean Value Theorem implies that:

\[
\hat{V}(\bar{Q}, \pi) - \hat{V}(Q(\pi), \pi) \geq \hat{V}_O(\bar{Q}, \pi)[\bar{Q} - Q(\pi)].
\]

The inequality will be strict on a set of positive measure if the policy \( Q(\pi) \) differs from the quota \( \bar{Q} \). Therefore:

\[
E[\hat{V}(\bar{Q}, \pi) - \hat{V}(Q(\pi), \pi)] > E[\hat{V}_O(\bar{Q}, \pi)[\bar{Q} - Q(\pi)]].
\]

If (19) holds then by (16):

\[
\hat{V}_O(\bar{Q}, \pi) \equiv 0.
\]

Therefore, if \( Q_\pi(\pi) \equiv 0 \) then \( \hat{V}_O(\bar{Q}, \pi) \) and \( \bar{Q} - Q(\pi) \) are non-negatively correlated as \( \pi \) varies and:

\[
E[\hat{V}_O(\bar{Q}, \pi)[\bar{Q} - Q(\pi)]] \geq E[\hat{V}_O(\bar{Q}, \pi)]E[\bar{Q} - Q(\pi)]
\]

\[
= 0 \quad \text{by hypothesis (20)}.
\]

The Theorem then follows from (21). 

The intuition is similar to that given for Theorem 1(A). A high \( \pi \) lowers the rent associated with the fixed quota \( \bar{Q} \). However, it also raises the marginal utility of income \( V_I \) —both through the direct effect of \( \pi \) in lowering real income and through its indirect
effect in lowering the domestic price. If the latter effects are sufficiently strong, then a rise in \( \pi \) raises the marginal utility of imports \( V_0(Q, \pi) \). Hence, the quota will be superior to any policy which reduces imports as \( \pi \) rises. It is readily shown that the latter occurs under a specific or an ad valorem tariff or under a foreign exchange quota.

We next give conditions under which a tariff is superior to the mean-equivalent quota. We consider a specific tariff because this is the optimal trade restriction given constant marginal utility of income. (An ad valorem tariff can be inferior to a quota—even with risk neutrality (Pelcovits (1976)).)

Let \( Q^S(\pi) \) be imports in state \( \pi \) under a specific tariff of \( S \) dollars. The condition (23) below for the specific tariff to dominate the mean-equivalent quota is essentially the condition in Theorem 1(C) which ensures that \( P^S_{\pi} \geq 1 \). The only difference is that it is evaluated at \( Q^S(\pi) \) rather than at \( Q^D(\pi) \).

**Theorem 3.** Suppose that neither good is inferior, \( Q^S(\pi) > 0 \) for all \( \pi \) and:

\[
\rho \leq \eta XM/(Q + SZ_p) \quad \text{for all } \pi \text{ when } Q = Q^S(\pi).
\]  

(23)

Then the specific tariff \( S \) yields higher expected utility than any policy \( Q(\pi) \) such that:

\[
E[Q(\pi)] = E[Q^S(\pi)] \quad \text{and} \quad Q^S_{\pi}(\pi) \geq 0.
\]  

(24)

**Proof.** We first show that the hypotheses imply that \( Q^S_{\pi}(\pi) - Q^S_{\pi}(\pi) < 0 \). \( Q^S_{\pi}(\pi) \) satisfies:

\[
Q + Z(\pi + S, \pi, Q) = 0.
\]

By the Implicit Function Theorem:

\[
Q^S_{\pi}(\pi) = -(Z_p + QX_I)/(1 - SX_I).
\]  

(25)

If neither good is inferior, then \( 1 - (\pi + S)X_I > 0 \) and \( X_I > 0 \) so:

\[
M = 1 - SX_I > 0
\]  

(26)

(25), (4), (26) and hypothesis (24) then imply that:

\[
Q^S_{\pi}(\pi) - Q^S_{\pi}(\pi) < 0.
\]  

(27)

By (10) \( \tilde{V}_{OO}(Q, \pi) < 0 \) so by the Second Mean Value Theorem:

\[
\tilde{V}(Q^S(\pi), \pi) - \tilde{V}(Q(\pi), \pi) \geq \tilde{V}_O(Q^S(\pi), \pi)[Q^S(\pi) - Q(\pi)].
\]

(28)

implies that this inequality is strict on a set of positive measure so:

\[
E[\tilde{V}(Q^S(\pi), \pi) - \tilde{V}(Q(\pi), \pi)] > E[\tilde{V}_O(Q^S(\pi), \pi)[Q^S(\pi) - Q(\pi)].
\]

We now consider how \( \tilde{V}_O(Q^S(\pi), \pi) \) changes with \( \pi \). By (16):

\[
\tilde{V}_O(Q^S(\pi), \pi) = SV_I(\pi + S, R(\pi + S) + SQ^S(\pi)).
\]

By (1):

\[
d\tilde{V}_O(Q^S(\pi), \pi)/d\pi = S[V_{IP} + YV_{II} + SQ^S_{\pi}V_{II}].
\]

Substituting from (7) and (25):

\[
d\tilde{V}(Q^S(\pi), \pi)/d\pi = S[-X_IV_I - V_{II}[Q + S(Z_p + QX_I)/(1 - SX_I)]
\]

\[
= SV_I[-X_I + \rho(Q + SZ_p)/(1 - SX_I)].
\]

By (26), \( 1 - SX_I > 0 \) so hypothesis (23) implies that:

\[
d\tilde{V}_O(Q^S(\pi), \pi)/d\pi \equiv 0.
\]  

(29)
(27) and (29) imply that:

\[ E[\tilde{V}_Q(Q^S(\pi), \pi)(Q^S(\pi) - Q(\pi))] \equiv E[\tilde{V}_Q(Q^S(\pi), \pi)]E[Q^S(\pi) - Q(\pi)] \]

\[ = 0 \quad \text{by (24)}. \]

The conclusion then follows from (28).

Under a specific tariff, an increase in \( \pi \) implies an equal increase in \( P \). The rise in \( \pi \) raises \( V_t \) by reducing real income but by (8) the rise in \( P \) lowers \( V_t \) when the importable is not inferior. For sufficiently low \( \rho \), the latter effect dominates so that the marginal utility of imports \( V_t - SV_t \) is negatively correlated with \( \pi \). Thus the specific tariff decreases (increases) imports when their marginal utility is low (high)—and is superior to any policy which fails to do this, such as a quota.

6. THE MODEL WITH DOMESTIC UNCERTAINTY

In this section we suppose that the world price of the importable is fixed at some level \( \tilde{\pi} \) but domestic production possibilities are influenced by a random variable \( \theta \). Production decisions are made after \( \theta \) is known, so \( \theta \) generally affects the domestic supply function \( Y(P, \theta) \) of the importable, the domestic supply function \( y(P, \theta) \) of the exportable and the revenue function \( R(P, \theta) \). We shall assume that:

\[ R_\theta(P, \theta) > 0, \quad Y_\theta(P, \theta) > 0 \quad \text{and} \quad y_\theta(P, \theta) < 0 \quad (30) \]

i.e. an increase in \( \theta \) increases production revenue and shifts the supply schedule of the importable (exportable) to the right (left). This assumption holds under the following two interpretations of \( \theta \):

**Lemma 1.** In the neo-classical 2×2 trade model, (30) holds if either:

(A) \( \theta \) is the labour supply and the importable is relatively labour intensive or:

(B) \( \theta \) is a parameter affecting the production function of the importable in a multiplicative (Hicks neutral) fashion.

**Proof.** See Appendix.

When imports are \( Q \) and domestic price is \( P \), consumer income is:

\[ I(P, Q, \theta) = R(P, \theta) + P - \tilde{\pi})Q \quad (31) \]

and domestic excess supply of the importable is:

\[ Z(P, Q, \theta) = Y(P, \theta) - X(P, I(P, Q, \theta)). \quad (32) \]

Let \( \tilde{P}(Q, \theta) \) be domestic equilibrium price in state \( \theta \) when imports are \( Q \). This is the \( P \) satisfying:

\[ Q + Z(P, Q, \theta) = 0. \quad (33) \]

An argument like that yielding (4) shows that:

\[ Z_P > 0 \quad \text{when} \quad P = \tilde{P}(Q, \theta). \quad (34) \]

Let \( \tilde{V}(Q, \theta) \) be domestic utility in state \( \theta \) in equilibrium when imports are \( Q \).

Any trade restriction is equivalent to a rule determining the quantity \( Q(\theta) \) to be imported in any state \( \theta \). Let \( Q^*(\theta) \) be the policy maximizing domestic expected utility subject to a constraint on expected imports and let \( \lambda \) be the Lagrange multiplier associated with this constraint.
Theorem 4. If $\lambda > 0$ and neither good is inferior, then $Q^0(\theta) < 0$.

Proof. It is readily checked that (6) and (10) remain valid when foreign uncertainty is replaced by domestic uncertainty. Hence $Q^0(\theta)$ is determined by the first order condition:

$$\lambda = \hat{V}_Q(Q, \theta) = (\hat{p}(Q, \theta) - \bar{\sigma})V_I(\hat{p}(Q, \theta), I(\hat{p}(Q, \theta), Q, \theta)).$$  \hspace{1cm} (35)

If $\lambda > 0$ then by (35):

$$\hat{p}(Q, \theta) - \bar{\sigma} > 0.$$  \hspace{1cm} (36)

Applying the Implicit Function Theorem to (35):

$$Q^0_\theta = -\hat{V}_{Q\theta}/\hat{V}_{QQ}$$  \hspace{1cm} (37)

Differentiating (6) with respect to $\theta$ and substituting from (8):

$$\hat{V}_{Q\theta} = \hat{p}_\theta V_I(1 - (\hat{p} - \bar{\sigma})X_I) + (\hat{p} - \bar{\sigma})V_{II}R_\theta.$$  \hspace{1cm} (38)

Applying the Implicit Function Theorem to (33):

$$\hat{p}_\theta = -(Y_\theta - X_\theta R_\theta)/Z_p.$$  \hspace{1cm} (39)

But

$$R_\theta = y_\theta + PY_\theta$$

so

$$\hat{p}_\theta = (y_\theta X_I - Y_\theta (1 - \hat{p}X_I))/Z_p.$$  \hspace{1cm} (40)

If neither good is inferior then $X_I > 0$ and $1 - \hat{p}X_I > 0$ so by (30) and (34):

$$\hat{p}_\theta < 0.$$  \hspace{1cm} (41)

Moreover, if neither good is inferior then $1 - (\hat{p} - \bar{\sigma})X_I > 0$ so by (38), (41), (36) and (30):

$$\hat{V}_{Q\theta} < 0.$$  \hspace{1cm} (42)

The Theorem then follows from (37), (42) and (10).

There is no counterpart to the possibly perverse effect of $\sigma$ on optimal imports. A rightward shift in the supply schedule of imports increases the representative consumer’s income and hence reduces the marginal utility of income $V_I$. Moreover, its effect in reducing the rent on a quota license outweighs the rise in $V_I$ due to the fall in $P$. Hence as $\theta$ increases, the marginal utility of imports always decreases, so that optimal imports always decrease also.

In contrast to Section 4, the optimal policy cannot be effected by a schedule of tariffs depending on the world price $\pi$, since $\pi$ is now fixed. However, the optimal policy could be effected by a schedule of tariffs depending on the level of imports $Q$. By Theorem 4, if neither good is inferior then, for each $Q$, there is a unique $\theta = \hat{\theta}(Q)$ such that $Q = Q(\theta)$. The optimal policy is equivalent to levying a specific tariff:

$$\hat{S}(Q) = \hat{p}(Q, \hat{\theta}(Q)) - \bar{\sigma}$$

when imports are $Q$. We now identify an unexpected feature of this optimal tariff schedule.

Theorem 5. Suppose that $\lambda > 0$, neither good is inferior, the representative consumer is risk averse and, for all $\theta$, $\hat{p}(Q^0(\theta), \theta) \leq 2\bar{\sigma}$. Then $\hat{S}(Q^0(\theta), \theta) < 0$. 

**Proof.** Since $\hat{\theta}(Q)$ is the inverse function of $Q^0(\theta)$:

$$\hat{\theta}_Q(Q) = 1/Q^0_\theta(\hat{\theta}(Q)).$$

Therefore:

$$\hat{S}_Q(Q) = \hat{P}_Q + \hat{P}_\theta/Q^0_\theta.$$  

Noting that $\hat{P}_Q = -M/Z_P$ and substituting from (37) and (40):

$$\hat{S}_Q = -M/Z_P + (Y_\theta(1 - \hat{P}_X) + \hat{P}_X Y_\theta) V_{O\theta} Z_P.$$  

(43)

In the Appendix we show that this reduces to:

$$\hat{S}_Q = - (\hat{P} - \bar{\pi}) V_{II} ((\hat{P} - \bar{\pi}) X_1 Y_\theta + \bar{\pi} Y_\theta) / Z_P V_{O\theta}.$$  

(44)

We now determine the sign of the term in curly brackets in (44). If $\hat{P} \leq 2\bar{\pi}$ then $\bar{\pi} \geq \hat{P} - \bar{\pi}$ so:

$$\hat{P} - \bar{\pi} Y_\theta + \bar{\pi} Y_\theta \geq (\hat{P} - \bar{\pi})(X_1 Y_\theta + Y_\theta).$$  

(45)

By (30), $R_\theta = \hat{P} Y_\theta + y_\theta > 0$ so:

$$(\hat{P} - \bar{\pi})(X_1 Y_\theta + Y_\theta) > (\hat{P} - \bar{\pi}) (\bar{\pi} - 1)/\hat{P}.$$  

(46)

Since $\lambda > 0$ and the exportable is not inferior,

$$\hat{P} - \bar{\pi} > 0 \quad \text{and} \quad \hat{P}_X - 1 < 0.$$  

(47)

By (30) $y_\theta < 0$ so by (47) the right side of (46) is positive and by (45) and (46) the term in curly brackets in (44) is positive. Since the representative consumer is risk averse, $V_{II} < 0$. Therefore by (44), (47), (34) and (42):

$$\hat{S}_Q < 0. \quad \blacksquare$$

Theorem 5 shows that if the constraint on expected imports is not so restrictive that the optimal domestic price can be over double the world price, then income stability is again more important than arbitrage considerations. Tariffs should be increased when the supply schedule of the importable shifts to the left (right) in order to buffer the domestic economy against the associated shifts in national income.

It may be difficult to administer a tariff schedule which varies continuously with the level of imports. However, the optimal tariff schedule could be approximated by a multiple-block schedule in which the tariff rate steps downward as imports exceed certain critical levels. Multiple-block tariff schedules are, in fact, quite common but the tariff rate usually steps upward as imports increase. 11

Finally, we compare a quota with a fixed tariff. Since the world price $\bar{\pi}$ is fixed, any fixed tariff is equivalent to some specific tariff $S$. Let $Q^*(\theta)$ be imports in state $\theta$ under this tariff.

**Theorem 6.** Suppose that the representative consumer is risk averse, neither good is inferior, $Q^*(\theta) > 0$ for all $\theta$ and

$$\bar{\pi} Y_\theta (\bar{\pi} + S, \theta) + y_\theta (\bar{\pi} + S, \theta) > 0.$$  

(48)

Then the specific tariff $S$ yields higher expected utility than any trade restriction $Q(\theta)$ such that:

$$E[Q(\theta)] = Q^*(\theta) \quad \text{and} \quad Q_\theta(\theta) \geq 0.$$  

(49)

**Proof.** To apply the argument of Theorem 3, we show that $V_Q(Q^*(\theta), \theta)$ and $Q^*(\theta) - Q(\theta)$ are positively correlated. $Q^*(\theta)$ is the solution to:

$$Q + Y(\bar{\pi} + S, \theta) - X(\bar{\pi} + S, R(\bar{\pi} + S, \theta) + SQ) = 0.$$
By the Implicit Function Theorem:

\[ Q^*_\theta(\theta) = -(Y_\theta - X_\theta R_\theta)/(1 - SX_\theta). \]

By (39):

\[ Q^*_\theta(\theta) = [-Y_\theta(1 - (\bar{\pi} + S)X_\theta) + y_\theta X_\theta]/(1 - SX_\theta). \]  

(50)

Since neither good is inferior, \(1 - (\bar{\pi} + S)X_\theta > 0\) and \(1 - SX_\theta > 0\) so by (30), (50) and hypothesis (49):

\[ Q^*_\theta(\theta) - Q_\theta(\theta) < 0. \]  

(51)

By (6):

\[ V_Q(Q^*(\theta), \theta) = SV_I(\bar{\pi} + S, R(\bar{\pi} + S, \theta) + SQ^*(\theta)). \]

Therefore:

\[ d\bar{V}_Q(Q^*(\theta), \theta)/d\theta = S[R_\theta + SQ^*_\theta(\theta)] V_{II}. \]  

(52)

By (39) and (50):

\[ R_\theta + SQ^*_\theta = y_\theta[1 + SX_\theta/(1 - SX_\theta)] + y_\theta[\bar{\pi} + S - S(1 - (\bar{\pi} + S)X_\theta)/(1 - SX_\theta)] \]

\[ = (y_\theta + \bar{\pi}y_\theta)/(-1 - SX_\theta). \]  

(53)

Since the exportable is not inferior, \(1 - SX_\theta > 0\). Therefore, by (53) and hypothesis (48):

\[ R_\theta + SQ^*_\theta > 0 \]

(52) then implies that:

\[ d\bar{V}_Q(Q^*(\theta), \theta)/d\theta < 0. \]  

(54)

(54) and (51) imply that \(\bar{V}_Q(Q^*(\theta), \theta)\) and \(Q^*(\theta) - Q(\theta)\) are positively correlated. Therefore, the argument of Theorem 3 shows that:

\[ E[\bar{V}(Q^*(\theta), \theta) - \bar{V}(Q(\theta), \theta)] > 0. \]

Hypothesis (48) states that the value of domestic production at world prices increases with \(\theta\). Since \(0 < R_\theta = (\bar{\pi} + S)Y_\theta + y_\theta\), this always holds provided that the tariff \(S\) is not too high. In these circumstances, an increase in \(\theta\) increases domestic income and hence decreases the marginal utility of income. The rightward shift in the domestic supply schedule also lowers the rent on quota licenses. Both these effects lead optimal imports to decrease. Since a tariff indeed decreases imports in these circumstances, it dominates any policy which would fail to decrease imports—such as a quota.

For any quota \(\bar{Q}\) we can show that \(d\bar{V}_Q(\bar{Q}, \theta)/d\theta \equiv 0\). Hence we cannot use the argument of Theorem 2 to derive circumstances under which a quota is superior to a tariff under domestic uncertainty.

7. CONCLUSIONS

If the representative consumer of a country is risk averse, then the choice of trade controls must take account of their effects on fluctuations of domestic real income. The source of uncertainty is then critical. If the disturbances arise from abroad then, with high risk aversion, it would be optimal to increase imports and to lower the domestic price as the world price rises. Furthermore, a quota would be superior to a tariff. Under domestic disturbances, not only do we confirm the superiority of a tariff over a quota, but we also conclude that a fixed tariff does not go far enough in encouraging import responses to domestic changes: the optimal domestic price falls as imports increase. The intuition for these results was given in Section 2 and was confirmed by our analysis in
terms of the marginal utility of imports. If both domestic and foreign disturbances occur and risk aversion is high then the ranking of tariffs and quotas would, of course, depend on which source of disturbances is predominant.

It would be a routine matter to extend the above analysis to the case of a large country, or where the "non-economic constraint" is on expected foreign exchange expenditure or expected tariff revenue or where the policy-maker is concerned with higher moments of the distribution of imports. A more challenging problem would be to consider the optimal trade restriction given a constraint on the expected welfare of a "protected" subgroup within the country.

APPENDIX

Derivation of (10). Differentiating (6) and substituting from (8) yields:

\[ \tilde{V}_{oo} = \tilde{P}_o V_I - (\tilde{P} - \pi) \tilde{P}_o X_I V_I + (\tilde{P} - \pi)^2 V_{II}. \]

Applying the Implicit Function Theorem to (3) yields:

\[ \tilde{P}_o = -\{1 - (\tilde{P} - \pi) X_I\}/Z_p. \tag{55} \]

Therefore:

\[ \tilde{V}_{oo} = -\{1 - (\tilde{P} - \pi) X_I\}^2 V_I/Z_p + (\tilde{P} - \pi)^2 V_{II}. \tag{56} \]

By (4) if \( V_{II} \leq 0 \) then \( V_{oo} < 0. \)

Proof of Theorems 1(B) and 1(C). In (17), substituting from (15), (55), (13), (16) and (56):

\[ P^0_\pi = \frac{-X_I Q}{Z_p} + \frac{Q X_I M^2/Z_p - (\rho T - 1)M}{M^2 + \rho Z_p (\tilde{P} - \pi)^2/I} \]

\[ = \frac{M(1 - \rho T) - Q X_I (\tilde{P} - \pi)^2 p/I}{M^2 + \rho Z_p (\tilde{P} - \pi)^2/I} \]

\[ = \frac{M - \rho T [M + (\tilde{P} - \pi) X_I]}{M^2 + \rho Z_p (\tilde{P} - \pi)^2/I} \]

\[ = \frac{M - \rho T}{M^2 + \rho Z_p (\tilde{P} - \pi)^2/I}. \]

Since \( Z_p > 0 \) and \( T > 0 \), Theorem 1(B) follows immediately. Moreover, the condition \( P^0_\pi < 1 \) is equivalent to each of the following inequalities:

\[ M - \rho T < M^2 + \rho Z_p (\tilde{P} - \pi)^2/I \]

\[ M(1 - M) < \rho [T + Z_p (\tilde{P} - \pi)^2/I] \]

\[ M(\tilde{P} - \pi) X_I < \rho [(\tilde{P} - \pi) Q/I + Z_p (\tilde{P} - \pi)^2/I] \]

\[ M X_I < \rho [Q + Z_p (\tilde{P} - \pi)]. \]

Since \( Q > 0 \) and \( \tilde{P} - \pi > 0 \), Theorem 1(C) now follows. ||

Proof of Lemma 1. In both cases (A) and (B), an increase in \( \theta \) leads to an outward shift in the production possibility set of the home country. For any \( P \), this leads to an increase in the maximum value of domestic production at domestic prices, i.e. \( R_\theta P \neq 0 \). In case (A) the remaining conclusions in (30) follow from the Rybczynski Theorem.
In case B, let $C^I(W, R, \theta)$ and $C^E(W, R)$ be the minimum unit costs of the importable and exportable in state $\theta$ when the wage is $W$ and the rental on capital is $R$. Let $A^L(W, R, \theta)$, $A^K(W, R, \theta)$ and $A^{EL}(W, R)$, $A^{EK}(W, R)$ be the cost-minimizing inputs of labour and capital required to produce a unit of importable and exportable respectively. If the country has $L^0$ units of labour and $K^0$ units of capital and production is not specialized, then, in state $\theta$ at price $P$ for the importable, $Y(P, \theta)$ and $y(P, \theta)$ are determined simultaneously with $W$ and $R$ by the equations:

$$C^I(W, R, \theta) = P, \quad C^E(W, R) = 1$$

$$A^L(W, R, \theta) Y(P, \theta) + A^{EL}(W, R) y(P, \theta) = L^0$$ (57)

$$A^K(W, R, \theta) Y(P, \theta) + A^{EK}(W, R) y(P, \theta) = K^0.$$

Since $\theta$ affects the production function of the importable in a multiplicative fashion:

$$C^I(W, R, \theta) = C^I(W, R, 1)/\theta$$


Therefore (57) becomes:

$$C^I(W, R, 1) = P\theta, \quad C^E(W, R) = 1$$

$$A^L(W, R, 1) Y(P, \theta)/\theta + A^{EL}(W, R) y(P, \theta) = L^0$$ (58)

$$A^K(W, R, 1) Y(P, \theta)/\theta + A^{EK}(W, R) y(P, \theta) = K^0.$$

Therefore $Y(P, \theta)/\theta$ and $y(P, \theta)$ satisfy the same equations (58) as those which determine $Y(P\theta, 1)$ and $y(P\theta, 1)$, i.e.

$$Y(P, \theta) = \theta Y(P\theta, 1) \quad \text{and} \quad y(P, \theta) = y(P\theta, 1).$$

Therefore:

$$Y_\theta(P, \theta) = Y(P\theta, 1) + \theta P Y_\theta(P\theta, 1) > 0$$

and

$$y_\theta(P, \theta) = P y_\theta(P\theta, 1) < 0. \quad ||$$

Derivation of (44). By (43):

$$-\tilde{S}_{QZ} \tilde{V}_{Q\theta} = y_\theta X_{\tilde{p}} \tilde{V}_{QQ} - Y_\theta (1 - \tilde{p} X_{\tilde{p}}) \tilde{V}_{Q\theta} + M \tilde{V}_{QQ}.$$

Substituting from (56), (38) and (40) and regrouping:

$$-\tilde{S}_{QZ} \tilde{V}_{Q\theta} = y_\theta X_{\tilde{p}} [-M^2 V_{ll} + M^2 X_{ll} - (\tilde{p} - \tilde{\pi})^2 V_{ll}]$$

$$+ Y_\theta [-((1 - \tilde{p} X_{\tilde{p}}) M^2 X_{ll} + M(\tilde{p} - \tilde{\pi}) V_{ll}]$$

$$+ ((1 - \tilde{p} X_{\tilde{p}}) M^2 V_{ll} - (1 - \tilde{p} X_{\tilde{p}})(\tilde{p} - \tilde{\pi})^2 V_{ll}]$$

$$= y_\theta X_{\tilde{p}} (\tilde{p} - \tilde{\pi})^2 V_{ll} + Y_\theta (\tilde{p} - \tilde{\pi}) V_{ll} [(1 - (\tilde{p} - \tilde{\pi}) X_{\tilde{p}}] - (1 - \tilde{p} X_{\tilde{p}})(\tilde{p} - \tilde{\pi})]$$

$$= (\tilde{p} - \tilde{\pi}) V_{ll} [1 - (\tilde{p} - \tilde{\pi}) X_{\tilde{p}} + Y_\theta \tilde{\pi}]$$

(44) follows immediately.

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NOTES
1. Helpman and Razin (1980) focus on uncertainty in production and assume that production decisions are made before the uncertainty is resolved, while consumption decisions are made afterwards. The other authors cited assume that both production and consumption decisions are made ex post. The latter assumption is also made in this paper. Helpman and Razin assume that intervention is targeted at the levels of inputs allocated to the import-competitor sector while the other authors assume constraints on expected imports, import expenditure or tariff revenue.
2. Provided only that consumer's surplus is a concave function of imports, i.e. the excess demand schedule is downward sloping.
3. This resembles Weitzman's (1977) conclusion that quantity rationing of a good amongst individuals will be preferable to price rationing if the variance of their marginal utilities of income is large compared to the variance of their marginal utilities from consumption of the good.
4. With ex ante choices in production, the outcome will depend on the opportunities for producers to trade in risk and to enter and exit from the uncertain industry.
6. Anderson and Young (1981) consider the optimal policy when the government is also concerned with higher moments of the import distribution.
7. Turnovsky (1974), Anderson and Riley (1976) and Flemming, Turnovsky and Kemp (1977) use a similar model to determine the effect of exogenous price uncertainty on the optimal ex ante choice of production when only consumption choices are made ex post.
8. This can be proved formally using the Second Mean Value Theorem as in Young (1980a, p. 427).
9. Suppose that the implicit tariff rate is 100% [(P - π)/P = 1/2], 50% of national income is spent on the importable [PX/I = 1/2], 25% of the importable is domestically produced [Q/X = 3/4], the price elasticity of the general equilibrium excess supply of imports is 2[PQ/0Q = 2] and the income elasticity of demand for the importable is 1. Then a simple calculation shows that the breakeven values of ρ in A, B and C are respectively, 6²/3, 4 and ½. By comparison, Friend and Blume's (1975) empirical analysis of portfolio choices using the capital asset pricing model suggests values of ρ of about 3 or above.
10. It might appear that an indeterminacy arises because domestic consumers and producers could not make their decisions without knowing the domestic price while the government could not set the tariff level (and hence the domestic price) without knowing final imports. However, the government need only monitor the level of imports and levy $S(Q)$ on the marginal unit as it enters. Earlier units might enter under a different tariff but arbitrage would ensure that all units have the same price as the last unit imported. Tariff revenue plus arbitrage gains or losses would equal $QS(Q)$.

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