You have 2 hours to write this exam. The exam consists of 7 questions, they are NOT weighted equally. The maximum possible score is 100 points. There are 3 pages to this exam including this one. If there is any question you do not understand please ask me for clarification. Also remember that partial credit is given.

PLEASE remember to put your name and university ID on the answers!!

Please remember that I treat all violations of academic integrity seriously and report every incident to the appropriate committee. Therefore, do not consult any human sources other than me during the course of this exam.

GOOD LUCK!!!!!
1. (5 points) Identify whether the following outcomes are discrete or continuous.

(a) The total time a London cab driver is spent stopped at traffic lights in any given day.
(b) The number of strikes in the car industry per year.
(c) The number of left-handed people in the population.
(d) The number of eggs laid by a chicken per day.
(e) The quantity of milk produced by a cow per day.

2. (10 points) Suppose \( R_t \) is distributed log-normally, i.e., \( \ln R_t \sim N(\mu, \sigma^2) \).
Then prove that
\[
E_t \left[ R_t^{1-\theta} \right] = e^{(1-\theta)[\mu+(1-\theta)\sigma^2/2]},
\]
where \( E \) is the expectations operator and \( \theta \) is a parameter. **SHOW ALL STEPS.**

3. (10 points) A particular type of printer cartridge is produced by only two companies (A and B). Company A produces sixty-five per cent of these cartridges and company B produces the remaining thirty-five percent. Eight per cent of cartridges produced by company A are defective as are twelve percent of those produced by company B. A customer purchases a new printer cartridge.

(a) What is the probability that company A produced the cartridge?
(b) What is the probability that company B produced the cartridge?
Now suppose the cartridge is found to be defective
(c) what is the probability that company A produced the cartridge?
**SHOW ALL STEPS.**
(d) What is the probability that company B produced the cartridge?
**SHOW ALL STEPS.**

4. (20 points) Is unbiasedness either a necessary or a sufficient condition for consistency? Discuss in detail.
5. (20 points) Recall that if $X_n$ is a vector, $\text{plim } X_n = Y$ if and only if $\text{plim} \{X_n\}_i = \{Y\}_i$, where $\{\cdot\}_i$ denotes the $i$'th entry of the vector. Prove that $\text{plim } X_n = X$ if and only if
\[
\lim_{n \to \infty} P \left( (x_n - y)' (x_n - y) > \varepsilon \right) = 0 \text{ for any } \varepsilon > 0
\]

SHOW ALL STEPS.

6. (15 points) Suppose $\sqrt{n} (\hat{\alpha}_n - \alpha) \xrightarrow{d} N (0, 1)$. Does it follow that $\text{plim } \hat{\alpha}_n = \alpha$? SHOW ALL STEPS.

7. (20 points) Prove that if
\[
E \left[ \frac{(X_n - \bar{X}_n)^2}{1 + (X_n - \bar{X}_n)^2} \right] \to 0
\]
then
\[
\lim_{n \to \infty} P (|\bar{x}_n - \bar{\mu}_n| < \varepsilon) = 1, \forall \varepsilon > 0
\]

SHOW ALL STEPS.