Problemset 1

1. Let \( X = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \) and \( y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \).

(a) Calculate \( P_X \) and \( M_X \).
(b) Calculate \( P_{X^y} \) and \( M_{X^y} \).
(c) Calculate \( P_{X^X} \) and \( M_{X^X} \). Explain why you get the results you do.
(d) Draw a graph with \( X, y, P_{X^y} \) and \( M_{X^y} \).

2. Intelligent Electronics, Inc., manufactures LCD screens. The number of hours an LCD screen functions without failures is represented by a random variable \( X \) with pdf \( f(x) = 0.1e^{-\frac{x}{100}}1_{(0,\infty)} \). The value of \( x \) is measured in thousands of hours. The company has a one-year warranty on its LCD screen, during which the LCD screen will be replaced free of charge should it fail to function.

(a) Assuming that the LCD screen is used for 10,000 hours in a year, what is the probability that the company will have to replace the LCD screen under the warranty?
(b) What is the probability the screen functions for at least 50,000 hours?
(c) Given that the screen has already functioned for 50,000 hours, what is the probability it will function for at least another 50,000 hours?

3. Which of the following are valid pdfs? Justify your answer.

(a) \( f(x) = (0.2)^x (0.6)^{1-x} 1_{(0,1)}(x) \)
(b) \( f(x) = 0.3 \cdot (0.7)^x 1_{(0,1,2,\ldots)}(x) \)
(c) \( f(x) = 0.6 \cdot e^{-x/4} 1_{(0,\infty)}(x) \)
(d) \( f(x) = x^{-1} 1_{[1,e]}(x) \)
4. A small nursery has seven employees, three of whom are sales persons, and four of whom are gardeners who tend to the growing and caring of nursery stock. With such a small number of staff, employee absenteeism can be critical. The number of salespersons and gardeners absent on any given day is the outcome of a bivariate random variable \((X, Y)\). The nonzero values of the joint density function are given in the table below:

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.75</td>
<td>0.025</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>1</td>
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<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.003</td>
</tr>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.01</td>
<td>0.005</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
</tr>
</tbody>
</table>

(a) What is the probability that more than two employees will be absent on any given day?

(b) Find the marginal density function for the number of gardeners that are absent. What is the probability that more than two gardeners will be absent on any given day?

(c) Find the conditional density function for the number of salespersons that are absent given that no gardeners are absent. What is the probability that no salespersons are absent given that no gardeners are absent? Is the conditional probability higher or lower given that there is at least one gardener absent?

5. People Power Inc., is a firm that specializes in providing temporary help to various businesses. Job applicant are administered an aptitude test that evaluates mathematics, writing and manual dexterity skills. After analyzing the data from thousands of applicants, the firm found that the scores on the three tests could be viewed as outcomes of random variables with the following joint density function (the test are graded on a 0-1 scale, with 0 the lowest score and 1 the highest):

\[
f(x_1, x_2, x_3) = 0.80 (2x_1 + 3x_2) x_3 \prod_{i=1}^{3} 1_{[0,1]} (x_i)
\]

(a) A job opening occurred for an office manager. This requires scores of > 0.75 on both the mathematics and writing tests for an applicant to be offered the position. Find the marginal density for the mathematics and writing scores. Then calculate the probability that an applicant who took the test will qualify for the job as office manager.

(b) A job opening has occurred for a warehouse worker. This requires a score of > 0.8 on the manual dexterity test. Find the marginal density for the dexterity score. Then calculate the probability that an applicant who took the test will qualify for the job as warehouse worker.

(c) Find the conditional density of the writing test given that an applicant achieves a score of > 0.75 on the mathematics test. Given that an applicant scores > 0.75 on the mathematics test, what is the probability that she score > 0.75 on the writing test?