1. Critically evaluate the following claim, explaining whether it is either true or false and why.

“It is known that, in both countries A and B, the decision to purchase a home follows a standard probit specification; i.e.,

$$\Pr\{y_i^k = 1 | X_{ii}^k, X_{2i}^k; \beta\} = \Phi(\beta_0 + \beta_1 X_{ii}^k + \beta_2 X_{2i}^k) \quad k = A, B$$

where $\Phi(\cdot)$ denotes the standard normal cdf and the coefficients are known to be the same for the two countries. However, the proportion of homeowners in country A is 50%, while in country B only 25% of households own homes. Using this information, you can conclude that the marginal impact of a change in $X_{ii}^k$ will be larger in country A than in country B.”

2. It is illegal in the U.S. to discriminate on the basis of race in deciding whether or not to approve a home mortgage application. As an analyst, you decide to investigate this issue. Before proceeding, however, you want to be sure that your program to estimate your specified model is working properly, so you conduct a pseudo-data experiment. Specifically, you set up a data-generating process in which you assume that whether or not an individual’s loan is approved depends upon the following factors:

- $C_i$: The household’s credit score;
- $R_i$: The ratio of total monthly debt payments to total monthly income;
- $D_{Ai}$: A dummy variable that equals 1 if the applicant is an African-American and equals 0 otherwise; and
- $D_{Pi}$: A dummy variable that equals 1 if the applicant has any public record of credit problems (e.g., bankruptcy, etc.) and equals zero otherwise.

You assume that there is a latent variable that determines loan approval, with

$$y_i^* = \beta_0 + \beta_1 C_i + \beta_2 R_i + \beta_3 D_{Ai} + \beta_4 D_{Pi} - \eta_i$$

where $\eta_i$ follows the standard normal distribution (i.e., $\eta_i \sim N(0,1)$). The observed discrete choice variable is given by

$$y_i = \begin{cases} 1 & y_i^* > 0 \text{ (loan approved)} \\ 0 & y_i^* \leq 0 \text{ (loan denied)} \end{cases}$$

Finally, assume that in the population:

- $C_i \sim TN_{(3.00,8.50)}(6.75,0.64)$; i.e. truncated normal (truncated below at 3.00 and above at 8.50.
- $R_i \sim TN_{(0.09)}(0.25,0.04)$
• $D_{Ai} = \begin{cases} 1 & \text{with probability 0.15} \\ 0 & \text{with probability 0.85} \end{cases}$

• $D_{Pi} = \begin{cases} 1 & \text{with probability 0.10} \\ 0 & \text{with probability 0.90} \end{cases}$

a) Generate a sample of 1000 observations on $y_i, y_i^*, C_i, R_i, D_{Ai}$ and $D_{Pi}$ assuming that

\[
\begin{align*}
\beta_0 &= -10 \\
\beta_C &= 2 \\
\beta_R &= -1.5 \\
\beta_A &= -1.0 \\
\beta_P &= -0.8
\end{align*}
\]

Provide summary statistics (i.e., means, minimums, maximums, and standard deviations) for each of these variables.

b) Write a GAUSS program to estimate these parameters using the maximum likelihood routine MAXLIK. Your program does not have to provide fancy output, but I would like you to provide me with both a copy of your code and its output, with the relevant estimates highlighted.

c) Using your parameter estimates, compute point estimates and standard errors for the following:

- The probability that the following “types” of individuals would have their loan approved:
  - Type I: $C_i = 5.50, R_i = 0.6, D_{Ai} = 1, D_{Pi} = 1$
  - Type II: $C_i = 7.50, R_i = 0.2, D_{Ai} = 0, D_{Pi} = 0$
  - Type III: $C_i = 6.50, R_i = 0.3, D_{Ai} = 1, D_{Pi} = 0$

- The marginal effect that the person’s credit score has on the probability that each of the three “types” of individuals will have their loan approved.

- The impact that a sudden public bad credit event would have on the Pr[$y_i = 1$] for Type II and Type III individuals.

- The racial bias in the loan application decision implied by your model.

d) Using the same data, repeat parts (b) and (c) using a logit specification. Compare and contrast the implications of your probit and logit results.