Problemset 4

1. Assume that \( \{ y_t \} \) is generated according to the simple trend model

\[
y_t = \alpha + \delta t + \varepsilon_t
\]

where \( \varepsilon_t \) is white noise with finite fourth moments.

(a) Suppose you estimate the model

\[
y_t = \alpha + \varepsilon_t
\]

what are the asymptotic properties of \( \hat{\alpha} \)?

(b) Suppose you estimate the model

\[
y_t = \delta t + \varepsilon_t
\]

what are the asymptotic properties of \( \hat{\delta} \)?

2. Let \( x_t \) be generated by:

\[
\begin{align*}
x_t &= y_t + e_t \\
y_t &= \rho y_{t-1} + a_t
\end{align*}
\]

where \( e_t \) and \( a_t \) are independent white noise processes with variances \( \sigma_e^2 \) and \( \sigma_a^2 \) respectively.

(a) What is the univariate ARMA representation for \( x_t \)?

(b) Show the relationships between the "population" estimate of the ARMA parameters and the structural parameters of the data generation process.

(c) How would you test the hypothesis that the variance of \( e_t \) equals zero?
3. This part prepares you for Monte Carlo exercises to be performed later.

(a) Suppose you toss a coin and a tetrahedon. For the coin you get 1 point for a heads and 2 points for a tail. The faces of the tetrahedon are labelled 1 through 4 and you are awarded the number of points shown on the downward face.

i. Calculate the probabilities of getting 2,3,4,5 and 6 points.

ii. Describe how you would construct a Monte Carlo simulation to calculate the probabilities.

(b) Let

\[ y_t = \alpha + \delta t + u_t \]

where

\[ u_t = 0.5u_{t-1} + \varepsilon_t - 0.2\varepsilon_{t-1} \]

and \( \varepsilon_t \) is a standard normal white noise process.

i. Describe how you would generate the data series \( \{y_t\} \) on the computer.

ii. How would you verify, using simulations, verify that you can use 1.96 as the 5% critical value for the standard t-test for hypotheses on \( \delta \)?