Clarida, Gertler, and Gali (2000, *QJE*) formulate a model of the federal funds rate in period $t$, $r_t$, which can be written in the form

$$ (*) \quad r_t = \alpha_0 + \alpha_1 E(\pi_{t+k} | \Omega_t) + \alpha_2 E(x_{t+q} | \Omega_t) + \beta(L) r_{t-1} + \varepsilon_t $$

where

- $\pi_t$ is the inflation rate in period $t$,
- $x_t$ is the “output gap,” (GDP – potential GDP)/potential GDP, in period $t$,
- $\varepsilon_t$ is a white noise error term
- $\beta(L)$ is a polynomial in the lag operator $L$, $\beta_0 + \beta_1 L + \ldots + \beta_p L^p$
- $E(\Omega_t)$ denotes the conditional expectation formed in period $t$
- $\alpha_0, \alpha_1, \alpha_2, \beta_0, \ldots, \beta_p, k, q, \text{ and } p$ are constants.

Quarterly U.S. data on $r$, $\pi$, and $x$ are available in excel format through the class homepage. The data are the same data as the data used in CGG except that 1) the data set includes an additional 10 years of observations and 2) whereas CGG measure $r_t$ using the monthly average daily Fed Funds rate in the first month of quarter $t$, our measure of $r_t$ is the average of the three monthly averages for the three months that make up quarter $t$.

Following CGG, assume that $k = 1$, $q = 1$, and $p = 1$. That is, the interest rate is selected to target inflation and the output gap one period ahead and the smoothing polynomial is a first-order polynomial. Thus, the unknown parameters are $\alpha_0, \alpha_1, \alpha_2, \beta_0 \text{ and } \beta_1$.

**Part I - Looking at the Data**

I.1. In three separate graphs, plot $r_t$, $\pi_t$, and $x_t$ for $t = 1,\ldots,185$ (i.e., $t = 1960:I-2006:I$) against $t$.

I.2. Construct a table providing the summary statistics for each series, i.e., their sample means, their sample variances and standard deviations, and the $3 \times 3$ sample contemporaneous correlation matrix for $[r_t, \pi_t, x_t]$.

I.3. Do these series look like realizations from covariance stationary processes? Explain.
II. Forecasting $\pi_{t+1}$ and $x_{t+1}$

One approach to fitting the regression model above is to generate forecasts of $\pi_t$ and $x_t$ using time series models.

II.1. Assume $\pi_t$ and $x_t$ follow stationary AR($p_1$) and AR($p_2$) processes, respectively. Fit each series to AR($p$) models (with an intercept) for $p = 1, \ldots, 8$ over the sample period ending in 1996:IV. (Using a shortened sample period will allow you to evaluate out-of-sample forecasts and also will facilitate comparisons with the CGG results later in this problem set.) Use the AIC to select $p_1$ and $p_2$. Report the full AIC results and report the regression results for the two selected models: point estimates, estimated standard errors, $R^2$. (Remember to use the same sample to fit each of the regressions to select $p$. Then, if the selected $p$ is less than 8, refit the model using the maximum number of initial observations.) Do the AR coefficients in the final models satisfy the stationarity condition?

II.2. Using the autoregressions selected above, produce one-step ahead forecasts of the output gap and the inflation rate for 1997:I-2006:I. In constructing this sequence of one-step ahead forecasts, use the actual values of $r$ and $x$ on the right-hand-side of the forecasting equations. Compare the one-step ahead forecasts for each series against the realized values by

i. graphing the actual values (1997:I-2006:I) against the forecasted values.

ii. graphing the forecast errors (1997:I-2006:I)

iii. tabulating the sample mean and standard deviation of each forecast error series and tabulating the correlation between the two forecast error series.

Comment on the results as appropriate.

II.3. Fit $\pi_t$ and $x_t$ to bivariate VARs for $p = 1, \ldots, 8$ over the sample period ending in 1996:IV. Use the AIC to select $p$. Report the full AIC results. (Remember to use the same sample to fit each of the VARs to select $p$. Then, if the selected $p$ is less than 8, refit the VAR using the maximum number of initial observations. Do the estimated VAR coefficients in the final model satisfy the stationarity condition?

II.4. Using the VAR you selected in problem III.3, test the null hypothesis that $\pi_t$ does not Granger-cause $x_t$ and test the null hypothesis that $x_t$ does not Granger $\pi_t$. Report the appropriate test statistics and asymptotic $p$-values. Interpret your results.

II.5. Using the VAR selected in problem III.3, produce one-step ahead forecasts of the output gap and the inflation rate for 1997:I-2006:I. In constructing this sequence of one-step ahead forecasts, use the actual values of $\pi$ and $x$ on the right-hand-sides of the forecasting equations. Compare the one-step ahead forecasts for each series against the realized values by

i. graphing the actual values (1997:I-2006:I) against the forecasted values.

ii. graphing the forecast errors (1997:I-2006:I)

iii. tabulating the sample mean and standard deviation of each forecast error series and tabulating the correlation between the two forecast error series.

Comment on the results as appropriate.
II.6. Compute the RMSEs of the sequences of one-step ahead forecasts for the inflation rate generated by the AR and VAR approaches. Do the same for the sequences of one-step ahead forecasts for the output gap generated by the AR and VAR approaches. Discuss the results.
III. Fit the CGG model

Note that equation (*) in Part I is a reduced-form equation derived from CGG’s structure:

\[ r_t^* = r^* + \beta(E(\pi_{t+k} \mid \Omega_t) - \pi^*) + \gamma(E(x_{t+q} \mid \Omega_t) \]  
\[ r_t = \beta(L)r_{t-1} + (1 - \beta_0 - \beta_1 - \ldots - \beta_p)r_t^* \]

where \( r^* \) and \( \pi^* \) are the Fed’s “long-run” targets for \( r \) and \( \pi \), respectively.

So, the \( \alpha \) parameters in (3) are related to those in (1)-(2) according to:

\[ \alpha_0 = (1 - \beta_0 - \beta_1 - \ldots - \beta_p)(r^* - \beta \pi^*) \]
\[ \alpha_1 = (1 - \beta_0 - \beta_1 - \ldots - \beta_p)\beta \]
\[ \alpha_2 = (1 - \beta_0 - \beta_1 - \ldots - \beta_p)\gamma \]

Fit (*) by OLS using your forecasts of inflation and the output gap and using the CGG assumptions that \( k = 1 \), \( q = 1 \), and \( p = 1 \). Fit (*) over each of the following sample periods:

- 1960:I-1996:IV (full sample)
- 1960:I-1979:II (Pre-Volcker sample)
- 1979:III-1996:IV (Greenspan sample)

III.1 Summarize the reduced form equation estimates in a table which includes, for each of the three sample periods, the point estimates, standard errors, and p-values (for the significance tests) of the OLS estimates of \( \alpha_0, \alpha_1, \alpha_2, \beta_0 \), and \( \beta_1 \).

III.2. For each of the three sample periods, use the reduced-form estimates and the structural restrictions to produce a table containing point estimates of the structural parameters: \( r^* \), \( \pi^* \), \( \beta \), \( \gamma \), \( \beta_0 \), and \( \beta_1 \). Note that \( r^* \) and \( \pi^* \) are not identified without an additional restriction. Use the following identifying restriction, which was also used by CGG: \( r^* = \) sample mean of \( r_t \). Discuss the economic interpretation of these results. Include a comparison of your results to the results reported by CGG in their Table II.

III.3. Apply a Chow test to test the null of structural stability over the full sample against the alternative of a structural break in 1979:III. Report and discuss your results.