Lecture 31 – Structural VARs: I

Let’s assume that $y_t$ (or $\Delta y_t$) is an $n$-dimensional I(0) process with a VAR($p$) representation, i.e.,

$$y_t = A_0 + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \varepsilon_t$$

where $\varepsilon_t \sim w.n. (\Omega)$.

Note that the $n$ elements of $\varepsilon_t$ can be contemporaneously correlated (but not serially correlated). Note too that, for now, we are ruling out CI.

Earlier this semester we talked about estimation, inference (e.g., causality tests), and forecasting with the VAR. In that discussion we also addressed practical issues like variable selection and lag length selection.
In a major paper, that has had the sort of impact on macroeconometrics that Lucas’s work had on macroeconomics, Sims (1980, Econometrica, “Macroeconomics and Reality”) made two important arguments.

1. The standard simultaneous equations modeling approach being used in empirical macroeconomics was not likely to be very useful because it relied on “incredible” identifying restrictions: exogeneity and exclusion restrictions (including restriction on dynamics, i.e., lag structures). While these restrictions might seem sensible when looked at from a partial equilibrium approach, they seem much less sensible from a general equilibrium approach.

2. Constructive alternative: Use the VAR as the basis for doing structural analysis. (Does not rely on exogeneity or exclusion restrictions.)
How to use the VAR for structural analysis when the VAR itself is a reduced-form model?

**Innovation Accounting**: the response of the system to structural shocks.

It is straightforward to use the estimated VAR to compute the impulse response functions:

$$\frac{dy_{i,t+s}}{d\varepsilon_{j,t}}$$ for \(i,j = 1,\ldots,n\) and \(s = 0,1,2,\ldots\)

However, the \(\varepsilon\)'s are reduced-form shocks (evidenced by the nonzero contemporaneous correlations) and, therefore, these impulse response functions don’t have a natural economic interpretation.

Structural VAR analysis – formulate the \(n\) \(\varepsilon\)'s in terms of \(n\) uncorrelated structural disturbances, say, \(v_{1t},\ldots,v_{nt}\), and compute the impulse response functions:

$$\frac{dy_{i,t+s}}{dv_{j,t}}$$ for \(i,j = 1,\ldots,n\) and \(s = 0,1,2,\ldots\)
Step back for more motivation –

Recall the standard textbook simultaneous equation model of, e.g., supply and demand:

\[
Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 I_t + v_{1t} \quad \alpha_2 < 0 \quad \text{(demand)}
\]
\[
Q_t = \beta_1 + \beta_2 P_t + \beta_3 C_t + v_{2t} \quad \beta_2 > 0 \quad \text{(supply)}
\]

where \( I_t \) = household sector income in \( t \)
\( C_t \) = production cost index in \( t \)
\( v_{1t} \) = demand shock in \( t \)
\( v_{2t} \) = supply shock in \( t \)

We assume that \( v_{1t} \) and \( v_{2t} \) are uncorrelated with one another. \( Q \) and \( P \) are jointly determined by this system given the \( v \)'s and the exogenous variables \( C \) and \( I \).

This a structural model –

- each equation has a distinct and natural economic interpretation.
- equations can include more than one endogenous variable
We wouldn’t directly estimate the supply and demand equations by OLS because of the endogenous r.h.s. variable in each of the two equations.

Instead we might estimate its reduced-form by OLS and use it to recover the structural parameter estimates:

\[
Q_t = a_1 + a_2 I_t + a_3 C_t + \varepsilon_{1t} \\
P_t = b_1 + b_2 I_t + b_3 C_t + \varepsilon_{2t}
\]

This a reduced model –

- explanatory variables are exogenous (or predetermined)
- shocks are functions of the structural shocks
- equations do not have natural economic interpretation.
It is natural to estimate the reduced form by OLS (why not SUR?) and then untangle the reduced form parameter estimates to recover estimates of the structural parameters. This is feasible provided that the structure is identified by the reduced form. In this example it is because of our exogeneity and exclusion restrictions. (Why would these particular restrictions seem implausible in a dynamic macro setting?)

How can we identify the structure from the reduced-form without using exogeneity and exclusion restrictions?
Consider a dynamic bivariate structural system without exogeneity or exclusion restrictions:

\[
\begin{align*}
    y_{1t} &= \beta_{12} y_{2t} + \gamma_{10} + \gamma_{11} y_{1,t-1} + \gamma_{12} y_{2,t-1} + v_{1t} \\
    y_{2t} &= \beta_{21} y_{1t} + \gamma_{20} + \gamma_{21} y_{1,t-1} + \gamma_{22} y_{2,t-1} + v_{2t}
\end{align*}
\]

where

\[v_t \sim \text{i.i.d.} \ (0, \Sigma), \ \Sigma \text{ a diagonal matrix.}\]

In this structural simultaneous equation model both variables are endogenous and appear in each equation; the lag structure is symmetric across equations.

What distinguishes the equations from one another (other than the arbitrary normalizations)? The uncorrelated \(v_t\)'s.
Example –

\[ y_{1t} = \log \text{real GDP} \]
\[ y_{2t} = \log \text{price level} \]

What kinds of orthogonal structural shocks would you expect to drive this system?

Aggregate Supply Shocks (technology shocks)
Aggregate Demand Shocks (monetary policy shocks)

Let

\[ v_{1t} = \text{supply shock} \]
\[ v_{2t} = \text{demand shock} \]

Then the first equation is the aggregate supply curve and the second equation is the aggregate demand curve.
The reduced-form for this structural system?

Rewrite the system as:

\[ B y_t = \Gamma_0 + \Gamma_1 y_{t-1} + v_t \]

where

\[ B = [B_{ij}], B_{11} = B_{22} = 1, B_{12} = -\beta_{12}, B_{21} = -\beta_{21} \]

This is sometimes called the structural form of the VAR(1).

Then, premultiply both sides of the equation by \( B^{-1} \):

\[ y_t = A_0 + A_1 y_{t-1} + \varepsilon_t \]

where \( A_0 = B^{-1} \Gamma_0, A_1 = B^{-1} \Gamma_1, \) and \( \varepsilon_t = B^{-1} v_t \).

This is the standard reduced-form VAR. We can fit this by OLS to estimate the A’s and then use the estimated A’s to see how the components of y respond over time to \( \varepsilon_1 \) and \( \varepsilon_2 \).
But this is not very interesting since 1) there is no interesting economic interpretation to give to either $\varepsilon$ (they are linear combinations of demand and supply shocks) and 2) the $\varepsilon$’s are correlated with one another so that, e.g., $dy_{1t+s}/d\varepsilon_{1t} \bigg| d\varepsilon_{2t}=0$ doesn’t make much sense.

Instead we would want to see how $y_{1t}$ and $y_{2t}$ respond over time to $v_{1t}$ and $v_{2t}$ shocks.

$$\varepsilon_t = B^{-1}v_t$$

So, let’s:

• introduce a one-time $v_1$ shock (holding $v_2$ fixed) [How large? $\sigma_1$]
• use inverse($B$-hat) to translate the $v_1$ shock into an $\varepsilon$ shock
• use the $A$-hat to simulate the response of the $y$’s to the $\varepsilon$ shock

Problem – $B$ is not identified! (There are 10 free structural parameters: $B$ (2), $\Gamma_0$ (2), $\Gamma_1$ (4), $\sigma_1^2$, $\sigma_2^2$. There are 9 free reduced-form parameters: $A_0$ (2), $A_1$ (4), $\Sigma_{\varepsilon \varepsilon}$ (3).)
Solution – We need to find at least one additional restriction on the structural model.