1. Robin Crusoe has been trapped on an island with a limited supply of fresh water $(W_0)$, but knows that she will be rescued at time $T$. Her utility from consuming $q(t)$ water as time $t$ is given by
\[ u(t) = a q(t) - \frac{b}{2} q(t)^2, \]
where $a > \frac{bW_0}{2}$. Her discount factor takes the standard exponential form of $\exp(-\delta t)$.

a) Derive Robin’s optimal time path for her water consumption as a function of $a$, $b$, $W_0$ and $T$.

b) How do changes in $a$, $b$, $W_0$ and $T$ alter this extraction path? Be explicit.

2. You have been given the task of managing fishery over the next three years. You are provided with the following information:

- The initial stock of fish is given by $R_0 = 1000$.
- Fish are allowed to reproduce at the beginning of the time period, with the natural growth to the fish stock during time period $t$ equal to $\alpha \ln (K/R_t)$.
- Fish are harvested at the end of the time period, with the cost of harvesting fish in period $t$ given by $c Y_t^2$, where $Y_t$ denotes the amount of fish harvested in period $t$. In harvesting the fish, the constraint on the fish caught in that period is $Y_t \leq R_t$; i.e., you cannot harvest the young (baby) fish.
- The price of fish is constant over time at $P = 1$. You can only sell fish that you catch in periods 0 and 1.
- At the end of the time period ($T=2$), you have the option of selling the fishery at a price of $sR_2$.

a) Set up your optimization problem, so as to maximize your stream of returns from the fishery. Be sure to specify all of the necessary constraints.

b) Specify your first order necessary conditions.

c) Determine implicit equations to solve the problem using dynamic programming, providing an optimal harvesting plan; i.e., $R_t$ (t=1,2) and $Y_t$ (t=0,1,2). Develop specific solutions for the following parameters: $\alpha = 0.2, K = 2000, s = 0.25, c = 0.001$. In doing so, keep in mind that you may need to use numerical procedures to solve the problem. Specifically, I would suggest setting up the problem in EXCEL and using the TOOLS:SOLVER capabilities to find $Y_0$. The rest of the values should follow from this value.

d) Describe how changes in the values of each of the four parameter impact your solution and interpret the results.