1. Consider the problem of solely owned fishery (say halibut), where the firm is assumed to be price taking in the broader market for halibut. Let \( p(t) \) denote the price of halibut at time \( t \) and \( h(t) \) denote the total harvest at time \( t \), with the stock of fish in the fishery given by \( X(t) \). Assume that the initial stock of fish is \( X_0 \) and that the fishery has the standard logistic natural growth pattern given by

\[
\dot{X}(t) = rX(t) \left[ 1 - \frac{X(t)}{K} \right].
\]

The total cost of fishing takes the form of \( C[X(t)]h(t) \), where \( C[X(t)] \) denotes the marginal cost of catching a fish. The discount rate is assumed to be constant at \( \delta \).

Finally, assume that the demand for the fish from this fishery can be represented by the demand function \( D[p(t)] \), where \( D'[p(t)] < 0 \).

a. Specify the fisher’s problem if their objective is to maximize the present value of their profits from owning the fishery.

b. Specify the corresponding first order conditions.

c. Show that the solution to the fisher’s problem can be characterized in terms of the differential equations system:

\[
\dot{X}(t) = F[X(t)] - D[p(t)]
\]

\[
\dot{p}(t) = \{\delta - F'[X(t)]\} \{p(t) - C[X(t)]\} + C'[X(t)]F[X(t)]
\]

d. Sketch the phase plane diagram associated with this problem assuming that \( C[X(t)] = \frac{1}{\theta X} \); \( D[p(t)] = \alpha - \beta p(t) \);

with \( r = 1, \theta = 0.25, \alpha = 1, \beta = 0.1, \delta = 0.05 \), and \( K = 1 \). In doing so, be sure to include the isoclines associated with \( \dot{X}(t) = 0 \) and \( \dot{p}(t) = 0 \) and the associated directions for the various isosectors. Also determine the value and nature of the steady state (i.e., is it a stable, unstable or a saddle point steady state).