

# Computing General Equilibrium Theories of Monopolistic Competition and Heterogeneous Firms

Edward J. Balistreri  
Colorado School of Mines

Thomas F. Rutherford  
ETH-Zürich

March 2011

Draft Chapter for the  
*Handbook of Computable General Equilibrium Modeling*  
edited by Peter B. Dixon and Dale W. Jorgenson

## 1 Introduction

Armington (1969) was the first to assume that goods might be differentiated by region of origin. Over the subsequent four decades, this assumption provided an effective framework for studying international trade policy. Once we consider that bilateral trade has an inherent idiosyncratic demand component, we can accommodate the observed pattern of international trade without taking a hard stance on the underlying motivations for trade. What matters for Computable General Equilibrium (CGE) modelers is not how the supply and demand functions got to their position but rather that we acknowledge their position and specify the empirically-based price responses. This approach to the study of international trade, however, has divided CGE analysis from much of the theoretic and econometric literature focused on production-side motivations for trade.

In this chapter we consider monopolistic competition theories as an alternative to the Armington assumption. We develop and apply a model with monopolistic competition among heterogeneous firms based on the Melitz (2003) theory. We look at two important policy issues:

economic integration and subglobal climate policy. The Melitz structure has the advantage that it is supported by empirical evidence on industrial organization and trade, and the structure is largely embraced by the theoretic community. We do find that the structure matters for CGE analysis. In our analysis of economic integration we find endogenous entry leading to important variety effects. We also find important productivity effects related to the competitive selection of more productive firms. In our examination of subglobal climate policy we see substantial trade diversion in the Melitz structure. This exacerbates the problem of carbon leakage and significantly impacts the emissions yields from carbon based tariffs. We aim to broaden the accessibility of these innovative theories within the CGE community, and we hope to foster the link between contemporary CGE analysis and contemporary trade theory.

A challenge for trade economists, going back to Leontief (1953) and his famous paradox, is how can we reconcile the data with the simple intuitive trade theories that we accept and promote? It turns out, the real world is complex. A gnawing issue in the 1960s and 1970s was the inability of our comparative advantage models to explain intra-industry trade. Why would a country both import and export the same good? Further compounding the problem was the fact that the volume of trade appeared most concentrated between industrialized countries that were relatively similar in their endowments and technologies. Burenstam Linder (1961) offered a narrative involving demand driven product development and subsequent export of these specialized goods to regions with similar demand idiosyncrasies. Armington (1969) directly assumed that goods from different regions were distinct in the import expenditure system.

Most theoretic work in international trade focuses on supply or production-side explanations. For many trade economists, maybe to their detriment, the Armington assumption feels like cheating. Under Armington the import expenditure system can be used to explain any trade pattern (that is feasible). The theory cannot be held in jeopardy with respect to cross-sectional trade flows. A feature advantageous for CGE modelers interested in calibrating to a point of ref-

erence, but a feature eschewed by the broader trade community. Theorists are often focused on more parsimonious descriptions of trade based on a set of primitives, while econometricians are often focused on testing the theory.

Unrest concerning traditional trade theory combined with innovations in the study of industrial organization, motivated Krugman (1980) to develop a formal theory of trade under Chamberlinian (large-group) monopolistic competition. The Krugman model offered a formal illustration of gains from trade in the absence of comparative advantage. We suddenly had a new theory that gave an intuitive explanation for intra-industry trade. The new trade theory, as it became known, generated a flurry of research on trade and industrial organization in the 1980s and 1990s. Some of the enthusiasm for the new models leaked into CGE analysis.<sup>1</sup>

In contrast to the Armington assumption, Krugman motivated his model with product differentiation at the firm level. Firm-level differentiation *feels* better founded than Armington's assumption, but as we will show in Section 4 the difference may not be material to the economics of the problem. Critical differences between Armington based perfect competition models and models of monopolistic competition arise primarily when there is some change in the number of varieties produced (entry or exit). Taken literally, the Krugman (1980) model does not include entry or exit of firms. Relative to autarky, trade allows consumers to enjoy new foreign varieties (the varieties previously only available to foreigners). By specifying CES preferences (which automatically reward variety) agents gain from trade.<sup>2</sup> Notice, however, that the same story could be told from an Armington perspective, where the new varieties are the new foreign regional goods. The gains from trade identified by Krugman are purely demand-side variety gains, and these gains are not dependent on the increasing returns to scale formulation.

---

<sup>1</sup>Some examples of CGE models that include an industrial organization treatment can be found in the 1992 special issue of *The World Economy* on the North American Free Trade Agreement (edited by Leonard Waverman). Consideration of industrial organization (and the new trade theories) can be found in Brown et al. (1992), Cox and Harris (1992), and Hunter et al. (1992), for example.

<sup>2</sup>The Dixit and Stiglitz (1977) formulation adopted by Krugman (1980), has a constant elasticity of substitution between 1 and  $\infty$ . Variety is rewarded in this framework as a unit of a new good is valued more than an additional unit of a currently consumed good.

In a popular theoretic context where there is no entry or exit and we only have iceberg trade costs, a Krugman type model is structurally equivalent to a similarly parameterized Armington model [Arkolakis et al. (forthcoming)].

So do the monopolistic competition based trade models out of the 1980s offer anything beyond Armington? The answer is yes. With a slight generalizations beyond Krugman (1980) a model with the same basic features will include endogenous entry. We illustrate this in Section 4. If firms respond to trade with net entry then the total number of varieties increases. In the trade literature this is referred to as the extensive margin, and there is evidence that links trade flows to the extensive margin [Hummels and Klenow (2005)]. Relative to an Armington model, a Krugman style model with trade induced entry will include gains from new varieties that did not exist in autarky. We caution, however, that trade induced entry is not guaranteed. It is relatively easy to formulate an example where reduced iceberg trade costs cause exit (for example, if traded and non-traded goods are complements). If trade induces exit the realized gains in the monopolistic competition setting will be lower than in an Armington setting.

The basic monopolistic competition models that enriched our understanding of trade and highlighted the importance of variety affects in the 1980s and 1990s began to run up against their own set of contradictory facts. It turns out, the real world is complex. Particularly relevant for our discussion, the assumptions that firms are small, symmetric, and there is a fixed markup over marginal cost contradict the data either directly or in their implications. Micro data suggests that there is a great deal of heterogeneity at the firm level. This is important for our study of trade because trade can affect the distribution of firms and generate gains purely due to the within industry reallocation of resources. In his pivotal paper Melitz (2003) formalized a model that included monopolistic competition and the competitive selection of heterogeneous firms. The model has many appealing features, and one of our main goals is to illustrate the operation of this new model in a CGE context.

Here we offer a quick review of some of the chief empirical findings that make the Melitz

(2003) trade structure appealing. A more complete review of this evidence can be found in Balistreri et al. (2011). Longitudinal micro data shows important and persistent dispersion in within industry firm-level productivities [see Bartelsman and Doms (2000)]. The few firms that select into export activities tend to be the most productive firms [see Bernard and Jensen (1999)]. Within industry reallocations among heterogeneous firms can drive productivity growth [Aw et al. (2001)], and trade liberalization can foster productivity growth consistent with eliminating marginal firms and favoring productive firms [Trefler (2004)].

Our approach is to start with the familiar and relatively transparent and build up to the empirical CGE application. First, we offer an introduction to the relevant trade theories and a set of corresponding computational maquettes in Sections 2 and 3, and we consider some illustrative computational experiments in Section 4. In Sections 5 and 6 we highlight some practicalities related to the calibration and computation of CGE models that include monopolistic competition. Finally, we present an applied model based on GTAP 7 data in Section 7. Our applications consider counterfactual simulations related to trade policy and subglobal climate-change policy. These applications highlight the innovations and their impact on policy considerations.

## **2 Trade Theories**

In this section we present the three basic theories of trade and industrial organization examined in this chapter. The presentation focuses on the trade equilibrium for a single good. The goal is to present the import demand and export supply formulations under the alternative assumptions about the nature of firm and product differentiation. The full general equilibrium, with endogenous incomes and intersectoral reallocations, is developed in section 3.

To begin we present a theory of trade based on the Armington (1969) assumption of differentiated regional products. The Armington formulation adopts the standard assumption of constant returns to scale and perfect competition. Firm-level products and technologies are

identical within a region, and firms sell their output at marginal cost. Relative to a formulation familiar to many CGE modelers, we introduce Samuelsonian *iceberg* transportation costs in the Armington structure. This change is made to maintain consistency with the standard monopolistic competition formulations and the geography-of-trade literature. The differentiated regional goods are aggregated by a Constant Elasticity of Substitution (CES) activity that yields a composite commodity available for consumption or intermediate use.

Next we present a Krugman (1980) based theory of trade under large-group monopolistic competition among symmetric firms. Each firm produces a unique product under the same increasing returns to scale technology. Specifically, the inputs used to produce an output level  $q$  equals  $f + q$ , where  $f$  is a fixed cost (measured in the input units). The differentiated firm-level goods are aggregated through a CES activity, where the composite commodity is available for consumption or intermediate use. The number of varieties can be endogenous as firms enter or exit. The CES aggregation is consistent with the Dixit and Stiglitz (1977) love-of-variety formulation, indicating industry-wide scale effects from new varieties.

The final theory we present is based on the Melitz (2003) heterogeneous-firms model. In the Melitz theory we maintain large-group monopolistic competition among firms producing differentiated products, but we also consider that firms face different technologies. Specifically, firms differ in their productivity, so the inputs required to produce output of  $q$  equals  $f + q/\varphi$ , where  $\varphi$  is a firm-specific measure of productivity. A firm with a higher  $\varphi$  has a lower marginal cost of production. Given a distribution of productivity levels, overall productivity can be affected by trade opportunities that reallocate resources between the different firms. The model is more elaborate in that we must track the competitive selection of firms.

To facilitate the presentation consider the following notation. Let  $i \in I$  indicate a commodity or industry, and let  $r \in R$  or  $s \in R$  indicate a region. Now decompose the set of commodities into Armington goods indexed by  $j \in J \subset I$ ; Krugman goods indexed by  $k \in K \subset I$ ; and Melitz goods indexed by  $h \in H \subset I$ . The variables that we track for each of the theories are presented in

Table 1: Notation and Variable Definitions

| <b>Variable</b>                    | <b>Armington</b> | <b>Krugman</b> | <b>Melitz</b>           |
|------------------------------------|------------------|----------------|-------------------------|
| Composite-commodity demand         | $Q_{jr}$         | $Q_{kr}$       | $Q_{hr}$                |
| Price index on composite commodity | $P_{jr}$         | $P_{kr}$       | $P_{hr}$                |
| Number of entered firms            |                  |                | $M_{hr}$                |
| Number of active firms             |                  | $N_{kr}$       | $N_{hrs}$               |
| Firm-level output                  |                  | $q_{krs}$      | $\tilde{q}_{hrs}$       |
| Firm-level price                   |                  | $p_{krs}$      | $\tilde{p}_{hrs}$       |
| Firm-level productivity            |                  |                | $\tilde{\varphi}_{hrs}$ |
| Composite-input unit cost          | $c_{jr}$         | $c_{kr}$       | $c_{hr}$                |
| Composite-input supply             | $Y_{jr}$         | $Y_{kr}$       | $Y_{hr}$                |

Table 1. Common across the models are the composite-commodity quantities and prices, and the composite-input quantities and prices.

In the first row of Table 1 we have regional demand for the sectoral composite commodity. Demand is determined in the general equilibrium and is, thus, taken as given in the initial presentation. To be clear let us approximate the general-equilibrium demand with a constant elasticity function of the composite price:

$$Q_{ir} = \bar{Q}_{ir} \left( \frac{\bar{P}_{ir}}{P_{ir}} \right)^\eta, \quad (1)$$

where symbols embellished with a *bar* indicate benchmark (calibrated) levels and  $\eta \geq 0$  is the price elasticity of demand.

Similarly, in the final row of Table 1 we have regional input supply to the sector, which is determined by upstream general-equilibrium conditions. We make the simplifying assumption that all factors and intermediate inputs are combined into a single composite input with a price  $c_{ir}$ . Again, the general-equilibrium link is brought into the presentation by specifying unit-

input supply as a constant elasticity function of the unit-input price:

$$Y_{ir} = \bar{Y}_{ir} \left( \frac{c_{ir}}{\bar{c}_{ir}} \right)^\mu, \quad (2)$$

where  $\mu \geq 0$  is the supply elasticity. Equations (1) and (2) establish our approximation of the general equilibrium allowing us to focus on the trade equilibrium and the industrial organization for each of the theories in isolation.<sup>3</sup>

## 2.1 Armington trade

As Armington (1969) observed, at any practical level of aggregation, products under a common classification (say,  $j = \{\text{machinery}\}$ ) sourced from different regions are not perfect substitutes. French machinery and Japanese machinery might be considered two different products. Observing that British firms use French, Japanese, domestic, and other machinery sourced from various regions (all at different unit values) is logically consistent if these different goods can be combined as imperfect substitutes. The machinery input to the British firm is the machinery composite of these regionally differentiated goods. This logical reconciliation of data on trade and the social accounts is the foundation for most CGE studies.

Assuming that the aggregation of bilateral export quantities is CES (with substitution elasticity  $\sigma_j$ ) the Armington composite commodity is given by

$$Q_{js} = \left[ \sum_r \left( \frac{q_{jrs}}{\tau_{jrs}} \right)^{\frac{\sigma_j-1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j-1}}, \quad (3)$$

where  $q_{jrs}$  is the export quantity and  $\tau_{jrs} \geq 1$  is the *iceberg* trade cost factor. The import quantity is  $q_{jrs}/\tau_{jrs}$ . It is more convenient for us to present this technology in its dual form where we

---

<sup>3</sup>In Section 3 of this chapter we endogenize aggregate demand and input supply for a full general equilibrium treatment.

represent the composite-commodity price index as a function of the source-region prices and trade costs. The price index,  $P_{js}$ , is the minimized cost of one unit of the composite commodity available in region  $s$ . To proceed, note that goods sourced from region  $r$  sell at a net price of  $c_{jr}$ , given marginal cost pricing. The gross price paid in region  $s$  includes the bilateral trade costs factors  $\tau_{jrs}$ , where  $(\tau_{jrs} - 1)$  is interpreted as the ad valorem tariff equivalent. Solving the constrained optimization problem reveals the price index:

$$P_{js} = \left[ \sum_r (\tau_{jrs} c_{jr})^{1-\sigma_j} \right]^{1/(1-\sigma_j)}. \quad (4)$$

Equation (4) is convenient because it represents the aggregating technology and the optimizing behavior simultaneously. The product of (4) and  $Q_{js}$  gives us the cost function which can be used to derive the bilateral demand functions by applying the envelope theorem. Setting the sum of these bilateral demands equal to the supply quantity gives us the market clearance condition for the composite input:

$$Y_{jr} = \sum_s \tau_{jrs} Q_{j,s} \left( \frac{P_{js}}{\tau_{jrs} c_{jr}} \right)^{\sigma_j}. \quad (5)$$

Combining equations (1) and (2) with equation (4) and (5) we have a square system of  $[4 \times R \times J]$  equations in  $[4 \times R \times J]$  unknowns. The Armington trade equilibrium is fully specified. To illustrate the operation of the trade equilibrium in a numeric setting we provide the GAMS code in Appendix A, section A.1.

## 2.2 Krugman trade

Krugman (1980) proposed a trade model with monopolistic competition based on a Dixit and Stiglitz (1977) aggregation of firm-level varieties. Applying this model is an alternative method of dealing with the data challenges faced by Armington (1969). Intraindustry trade, for example,

is a natural feature of the Krugman structure. As in the Armington structure, varieties are aggregated at constant elasticity of substitution but we now need to track the number of firms in each region,  $N_{kr}$ , and note that there is a scale effect associated with increases in variety. Firms are assumed to be relatively small, symmetric, and produce under a simple linear increasing returns technology. Furthermore, we assume that entry is costless so profits are driven to zero as the product space becomes saturated with varieties.

Let  $p_{krs}$  be the gross (of trade cost) price set by a region- $r$  firm selling in market  $s$ , and let  $\sigma_k > 1$  indicate the elasticity of substitution. The dual Dixit-Stiglitz price index in region  $s$  is then given by

$$P_{ks} = \left[ \sum_r N_{kr} p_{krs}^{1-\sigma_k} \right]^{1/(1-\sigma_k)}, \quad (6)$$

and the corresponding bilateral firm-level demands are given by

$$q_{krs} = Q_{ks} \left( \frac{P_{ks}}{p_{krs}} \right)^{\sigma_k}. \quad (7)$$

Firms are assumed small enough such that their pricing decisions have negligible impacts on the  $P_{ks}$ , but they do have market power over their unique variety. Faced with a constant elasticity demand (where  $P_{ks}$  is assumed constant) firms maximize profits by charging their optimal markup over marginal cost (where nominal marginal cost is  $c_{kr}$ ):

$$p_{krs} = \frac{\tau_{krs} c_{kr}}{1 - 1/\sigma_k}. \quad (8)$$

We include the cost factor,  $\tau_{krs}$ , in the numerator to be consistent with our definition of  $p_{krs}$  as gross of trade costs. In addition to marginal cost, firms incur a fixed cost, denoted  $f_k$  (measured in composite input units). The free-entry assumption indicates that the number of firms will

adjusts such that nominal fixed cost payments equal profits:

$$c_{kr}f_k = \sum_s \frac{p_{krs}q_{krs}}{\sigma_k}. \quad (9)$$

With the industrial organization well specified we proceed with a condition for market clearance for the composite input.

$$Y_{kr} = N_{kr} \left( f_k + \sum_s \tau_{krs} q_{krs} \right). \quad (10)$$

Again the  $\tau_{krs}$  term reflects the real resource cost of transport. Combining the downstream demand equation (1) and the upstream supply equation(2) with the Krugman specific equations (6 – 10) we have a square system of dimension  $[(5 \times R \times K) + (2 \times R \times R \times K)]$ . The partial equilibrium Krugman trade equilibrium is fully specified. To illustrate the operation of the trade equilibrium in a numeric setting we provide the GAMS code in Appendix A, section A.2. .

### 2.3 Melitz trade

Trade under the Melitz (2003) theory is more complex in that it extends the monopolistic competition model by incorporating firm heterogeneity. Firms have different, although well specified, productivities and they select themselves into profitable markets. Trade can impact the selection of firms and therefore can impact industry-wide productivity. Adopting Melitz’s representation of the representative (or average) firm operating in each bilateral market greatly simplifies the model.

The basic narrative that accompanies the Melitz model is as follows. Firms can choose to incur a sunk cost, which pays for a productivity draw (a “blueprint”). Once the productivity is realized the firm chooses to operate in those markets that are profitable. The firms face a market specific fixed cost and marginal cost is determined by the productivity draw. Some firms, with

sufficiently low productivity draws, will choose not to operate in any market. Other firms with high productivity draws may operate in multiple markets. With larger fixed costs associated with foreign markets we observe that export firms are among the largest and most productive. Further, trade liberalization induces the exit of low-productivity domestic firms through import competition, while inducing some relatively productive firms to enter external markets. Relative to autarky productivity increases through an intraindustry reallocation of resources toward the more productive firms.

Similar to the Krugman formulation we have a Dixit-Stiglitz price index. The firm-level prices are not the same, however, so we first consider the price index as a function of the continuum of prices. Let  $\omega_{hrs} \in \Omega_{hr}$  index the differentiated products sourced from region  $r$  shipped into region  $s$ , and let  $\sigma_h$  be the constant elasticity of substitution. The price index is given by

$$P_{hs} = \left[ \sum_r \int_{\omega_{hrs}} p_{hrs}(\omega_{hrs})^{1-\sigma_h} d\omega_{hrs} \right]^{\frac{1}{1-\sigma_h}}. \quad (11)$$

Simplifying this equation using the representative (or average) firm's price,  $\tilde{p}_{hrs}$ , and a measure of the number of firms operating,  $N_{hrs}$ , we have

$$P_{hs} = \left[ \sum_r N_{hrs} \tilde{p}_{hrs}^{1-\sigma_h} \right]^{1/(1-\sigma_h)}. \quad (12)$$

Melitz (2003) obtains this simplification by defining  $\tilde{p}_{hrs}$  as the price set on the variety from the firm with CES-weighted average productivity operating on the  $r$  to  $s$  link. Demand for the average variety is

$$\tilde{q}_{hrs} = Q_{hs} \left( \frac{P_s}{\tilde{p}_{rs}} \right)^{\sigma_h}, \quad (13)$$

where the average price,  $\tilde{p}_{hrs}$ , is defined as gross of trade costs.

Let  $\tilde{\varphi}_{hrs}$  indicate the productivity of the average firm (such that the nominal marginal cost

is  $c_{hr}/\tilde{\varphi}_{hrs}$ ). Faced with a constant demand elasticity of  $\sigma_h$  the firm optimally chooses a price

$$\tilde{p}_{hrs} = \frac{c_{hr}\tau_{hrs}}{\tilde{\varphi}_{hrs}(1-1/\sigma_h)}. \quad (14)$$

Again, we are assuming the firm is relatively small; the firm chooses a price without considering any impact of its decision on  $P_{hs}$ .<sup>4</sup>

We now have to determine which firms operate in a given bilateral market. We need to adopt a specific distribution for the productivity draws and link the marginal firm (earning zero profits) in a given bilateral market to the representative firm earning positive profits. We assume that each of the  $M_{hr}$  firms choosing to incur the entry cost receives their firm-specific productivity draw from a Pareto distribution with probability density

$$g(\varphi) = \frac{a}{\varphi} \left( \frac{b}{\varphi} \right)^a; \quad (15)$$

and cumulative distribution

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^a, \quad (16)$$

where  $a$  is the shape parameter and  $b$  is the minimum productivity.

For this continuous distribution there will be some level of productivity  $\varphi_{hrs}^*$ , at which operating profits for a firm drawing  $\varphi_{hrs}^*$  are zero. This is determined by the fixed cost of operating,  $f_{hrs}$ , on the  $r$ - $s$  link. All firms drawing a  $\varphi$  above  $\varphi_{hrs}^*$  will serve the  $s$  market, and firms drawing a  $\varphi$  below  $\varphi_{hrs}^*$  will not. A firm drawing  $\varphi_{hrs}^*$  is the marginal firm from  $r$  supplying region  $s$ . This leads us to the condition that determines which firms operate in a given market. Let  $r(\varphi) = p(\varphi)q(\varphi)$  indicate the gross of trade cost firm-level revenues as a function of the draw  $\varphi$ .

---

<sup>4</sup>This is an uncomfortable assumption given that the most productive firms must, in fact, be large.

Zero profits for the marginal firm requires

$$c_{hr} f_{hrs} = \frac{r(\varphi_{hrs}^*)}{\sigma_h}. \quad (17)$$

As we are not solving for the revenues of the marginal firm, we would like to define this condition in terms of the representative firm. We need to link the representative firm's productivity and revenue to the marginal firm through the Pareto distribution.

The probability that a firm will operate is  $1 - G(\varphi_{hrs}^*)$ , so we find the CES weighted average productivity:

$$\tilde{\varphi}_{hrs} = \left[ \frac{1}{1 - G(\varphi_{hrs}^*)} \int_{\varphi_{hrs}^*}^{\infty} \varphi^{\sigma_h - 1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma_h - 1}}. \quad (18)$$

Applying the Pareto distribution this becomes

$$\tilde{\varphi}_{hrs} = \left[ \frac{a}{a + 1 - \sigma_h} \right]^{\frac{1}{\sigma_h - 1}} \varphi_{hrs}^*. \quad (19)$$

Again, following Melitz (2003) we use optimal firm pricing and the input technology ( $f_{hrs} + q/\varphi$ ) to establish the relationship between the revenues of firms with different productivity draws:

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left( \frac{\varphi_1}{\varphi_2} \right)^{\sigma_h - 1}. \quad (20)$$

Using (19) and (20) to simplify (17) we derive the zero cutoff profit condition in terms of average-firm revenues and the parameters:

$$c_{hr} f_{hrs} = \tilde{p}_{hrs} \tilde{q}_{hrs} \frac{(a + 1 - \sigma_h)}{a \sigma_h}. \quad (21)$$

Next we turn to the entry condition which determines the mass of firms,  $M_{hr}$ , that take a productivity draw. A productivity draw costs a firm a one-time entry payment of  $f_{hr}^s$  input units. Entered firms then face a probability  $\delta$  in each future period of a shock that forces exit. In

a steady-state equilibrium  $\delta M_{hr}$  firms are lost in a given period so total nominal entry payments in that period must be  $c_{hr} \delta f_{hr}^s M_{hr}$ . From an individual firm's perspective the *annualized* flow of entry payments is  $c_{hr} \delta f_{hr}^s$ .

Assuming risk neutrality and no discounting, firms enter to the point that expected operating profits equal the entry payment. A firm from  $r$  operating in market  $s$  can expect to earn the average profit in that market:

$$\tilde{\pi}_{hrs} = \frac{\tilde{p}_{hrs} \tilde{q}_{hrs}}{\sigma_h} - c_{hr} f_{hrs}. \quad (22)$$

Using the zero cutoff profit condition to substitute out the operating fixed cost this reduces to

$$\tilde{\pi}_{hrs} = \tilde{p}_{hrs} \tilde{q}_{hrs} \frac{(\sigma_h - 1)}{a \sigma_h}. \quad (23)$$

The probability that a member of  $M_{hr}$  will operate in the  $s$  market is simply given by the ratio of  $N_{hrs}/M_{hr}$ . Setting the firm-level entry-payment flow equal to the expected profits from each potential market gives us the free entry condition

$$c_{hr} \delta f_{hr}^s = \sum_s \tilde{p}_{hrs} \tilde{q}_{hrs} \frac{(\sigma_h - 1) N_{hrs}}{a \sigma_h M_{hr}}, \quad (24)$$

which determines the mass of firms,  $M_{hr}$ .

We can now recover the productivities as a function of the fraction of operating firms from  $1 - G(\varphi_{hrs}^*) = N_{hrs}/M_{hr}$ . Applying the Pareto distribution and substituting  $\varphi_{hrs}^*$  out of the system using (19) we have an equation for the productivity of the representative firm;

$$\tilde{\varphi}_{hrs} = b \left( \frac{a}{a + 1 - \sigma_h} \right)^{1/(\sigma_h - 1)} \left( \frac{N_{hrs}}{M_{hr}} \right)^{-1/a}. \quad (25)$$

Finally, we need to close the model by specifying market clearance in inputs. Supply is  $Y_{hr}$ , and demand has three components: inputs used in sunk costs, inputs used in operating fixed

costs, and operating inputs;

$$Y_{hr} = \delta f_{hr}^s M_{hr} + \sum_s N_{hrs} \left( f_{hrs} + \frac{\tau_{hrs} q_{hrs}}{\bar{\varphi}_{hrs}} \right). \quad (26)$$

This completes our description of the Melitz trade equilibrium. Equations (1), (2), (12), (13), (14), (21), (24), (25), and (26) form a square system of dimension  $[(5 \times R \times H) + (4 \times R \times R \times H)]$ . To illustrate the operation of the trade equilibrium in a numeric setting we provide the GAMS code in Appendix A, section A.3.

### 3 General equilibrium formulation

In the previous section we approximated general equilibrium impacts on trade by specifying constant elasticity aggregate-demand and input-supply functions. In this section we formalize the general equilibrium in a model that accommodates all three theories of trade. The goal is to develop a relatively transparent framework for illustrating model responses and for comparing the three formulations.

The first step in endogenizing the general equilibrium is to fully specify the demand system as derived from preferences. We assume that consumers derive utility through CES preferences over the different composite goods (indexed by  $i$ ). Again it is convenient to represent this in its dual form (which simultaneously represents preferences and the optimizing behavior). Preferences in region  $r$  are indicated by the unit expenditure function,

$$E_r = \left[ \sum_i \beta_{ir}^\alpha P_{ir}^{1-\alpha} \right]^{1/(1-\alpha)}, \quad (27)$$

where  $E_r$  is the minimized expenditures needed to generate one unit of utility.  $E_r$  is the ideal or true cost-of-living price index. The parameters  $\alpha$  and  $\beta_{ir}$  indicate the elasticity of substitution and relative preference weights across the goods. Welfare in region  $r$  is simply measured as

nominal income deflated by the price index,

$$U_r = \frac{GDP_r}{E_r}, \quad (28)$$

where  $GDP_r$  indicates income. Applying Shephard's Lemma to the expenditure function we recover the compensated demand functions for each aggregated good:

$$Q_{ir} = U_r \left( \frac{\beta_{ir} E_r}{P_{ir}} \right)^\alpha. \quad (29)$$

Where equation (29) now replaces its partial equilibrium counterpart [equation (1)].

Moving upstream of the trade equilibrium we now consider input supply. Assume that the composite input selling for  $c_{ir}$  is produced according to a Cobb-Douglas technology using various primary inputs. Let  $f \in F$  index the primary factors with corresponding prices  $w_{fr}$ , and denote the value-share parameters  $\gamma_{fir}$  (where  $\sum_f \gamma_{fir} = 1$ ). The unit cost function for sector  $i$  in region  $r$  is given by

$$c_{ir} = \prod_f (w_{fr})^{\gamma_{fir}}. \quad (30)$$

With fixed factor endowments equal to  $\bar{L}_{fr}$  (and again applying Shephard's Lemma, this time to the cost function) we derive the market clearance conditions for primary factors:

$$\bar{L}_{fr} = \sum_i \frac{\gamma_{fir} Y_{ir} c_{ir}}{w_{fr}}. \quad (31)$$

The remaining condition needed to close the general equilibrium is the calculation of nominal income:

$$GDP_r = \sum_f w_{fr} \bar{L}_{fr}. \quad (32)$$

Combining equations (27) – (32) with the specific trade equations from the previous section yields our illustrative computable general equilibrium. We summarize the full set of conditions

Table 2: Multiregion General Equilibrium with Alternative Trade Theories

| Equation<br>Description   | Associated<br>Variable                       | Equation Number |           |         |        | Dimensions                  |
|---------------------------|--|-----------------|-----------|---------|--------|-----------------------------|
|                           |  | General         | Armington | Krugman | Melitz |                             |
| Unit Expenditure Function | $U_r$ : Welfare                              | (27)            |           |         |        | $R$                         |
| Final Demand              | $E_r$ : Consumer Price Index                 | (28)            |           |         |        | $R$                         |
| Demand by Sector          | $P_{ir}$ : Good Price                        | (29)            |           |         |        | $I \times R$                |
| Composite Price Index     | $Q_{ir}$ : Aggregate Quantity                |                 | (4)       | (6)     | (12)   | $I \times R$                |
| Free Entry                | $N_{kr}$ or $M_{hr}$ : Entered Firms         |                 |           | (9)     | (24)   | $(K + H) \times R$          |
| Zero Cutoff Profits       | $N_{hrs}$ : Operating Firms                  |                 |           |         | (21)   | $H \times R \times R$       |
| Firm-level Demand         | $p_{krs}$ or $\tilde{p}_{hrs}$ : Firm Price  |                 |           |         | (7)    | $(K + H) \times R \times R$ |
| Firm-level Markup         | $q_{krs}$ or $\tilde{q}_{hrs}$ : Firm Output |                 |           |         | (8)    | $(K + H) \times R \times R$ |
| Firm-level Productivity   | $\tilde{\varphi}_{hrs}$ : Productivity       |                 |           |         | (18)   | $H \times R \times R$       |
| Composite-input Markets   | $c_{ir}$ : Unit-cost Index                   |                 | (5)       | (10)    | (26)   | $I \times R$                |
| Unit-cost Function        | $Y_{ir}$ : Upstream Output                   | (30)            |           |         |        | $I \times R$                |
| Primary-factor Markets    | $w_{fr}$ : Factor price                      | (31)            |           |         |        | $F \times R$                |
| Income                    | $GDP_r$ : Income                             | (32)            |           |         |        | $R$                         |

in Table 2. In addition, the GAMS code for this model is made available in Appendix B. The model is capable of incorporating various combinations of Armington, Krugman, and Melitz structures by applying various definitions of the subsets  $J$ ,  $K$ , and  $H$ .

## 4 Computation as a companion to theory

There is an expansive literature on the trade theories outlined above. One of the common threads is that all three support the equally expansive econometric work on the *new* geography of trade. With restrictions, these theories readily produce a fairly simple gravity equation. This is so common that many theoretic exercises actually impose gravity as a precursor to an analytical studying of what are consider *relevant* versions of the more general theories. Examples include “trade separability” as imposed by Anderson and van Wincoop (2004), or the “CES import demand system” imposed by Arkolakis et al. (forthcoming) (which is much more restrictive than imposing CES preferences). Unlike many theoretic studies, our computational exploration of the theory is not restricted to sterilized versions of the models. We feel the computational platform can contribute as a companion to our understanding of these models by demonstrating the impact of parametric and structural assumptions.

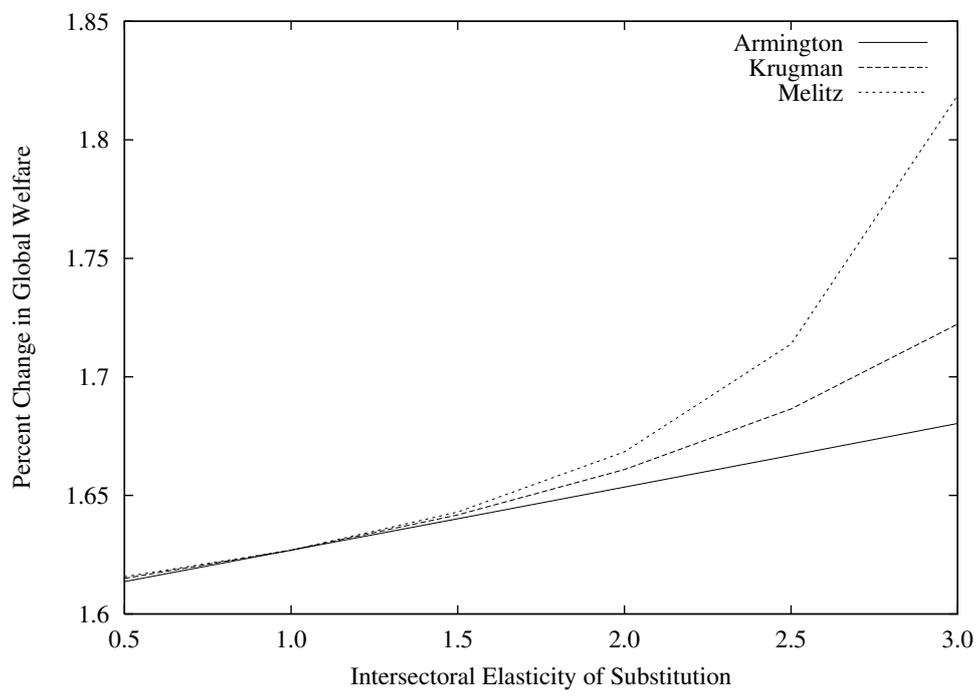
As a first example consider the strong equivalence result found by Arkolakis et al. (2008). In their paper they contend that the Melitz and Krugman [and Eaton and Kortum (2002)] models are equivalent in their welfare predictions. This is true (and in fact these models are equivalent to an Armington based model) in one-good one-factor environments. In Figure 1 we use the computational model presented in section 3 to illustrate the fragility of the equivalence in a model that includes multiple sectors. This is similar to the exercise conducted by Balistreri et al. (2010), although here we add the Krugman simulations. In the models we include three regions, three sectors, and three factors of production, while alternatively formulating trade as Armington, Krugman, or Melitz. We calibrate the models to a symmetric equilibrium with iceberg transport costs and compute an experiment where we reduce transport costs on bilateral trade in one of the goods. The figure plots the sum of the changes in welfare (utilitarian %EV) as a function of the top-level elasticity of substitution,  $\alpha$  from equation (27). In a multisector model, and with  $\alpha \neq 1$ , factors will reallocate across sectors leading to different outcomes across the different structures. Arkolakis et al. (forthcoming) state the additional assumptions necessary for the equivalence we observe at  $\alpha = 1$  (e.g., no tradeable intermediates). Most of these restrictions are not reasonable in an empirical setting, which suggests to us that computational models are the preferred approach to the data.

An important question is why the models differ? Feenstra (2010) is a very good guide to answering this question. Utilizing the simplified framework where we only have one sector and one factor of production, Feenstra examines the gains from trade in the Krugman and Melitz frameworks. In this environment an important feature is that there is no entry or exit, because the factor is inelastically supplied.<sup>5</sup> Feenstra explains that in the Krugman model, relative to autarky, agents enjoy *import* variety gains. The set of goods available to consumers expands by the number of foreign varieties. In the one sector one factor Melitz structure the nature

---

<sup>5</sup>Balistreri et al. (2010) show how setting the top-level elasticity of substitution equal to one in a simplified multisector model also indicates perfectly inelastic factor supply.

Figure 1: Welfare Impacts Across Structures

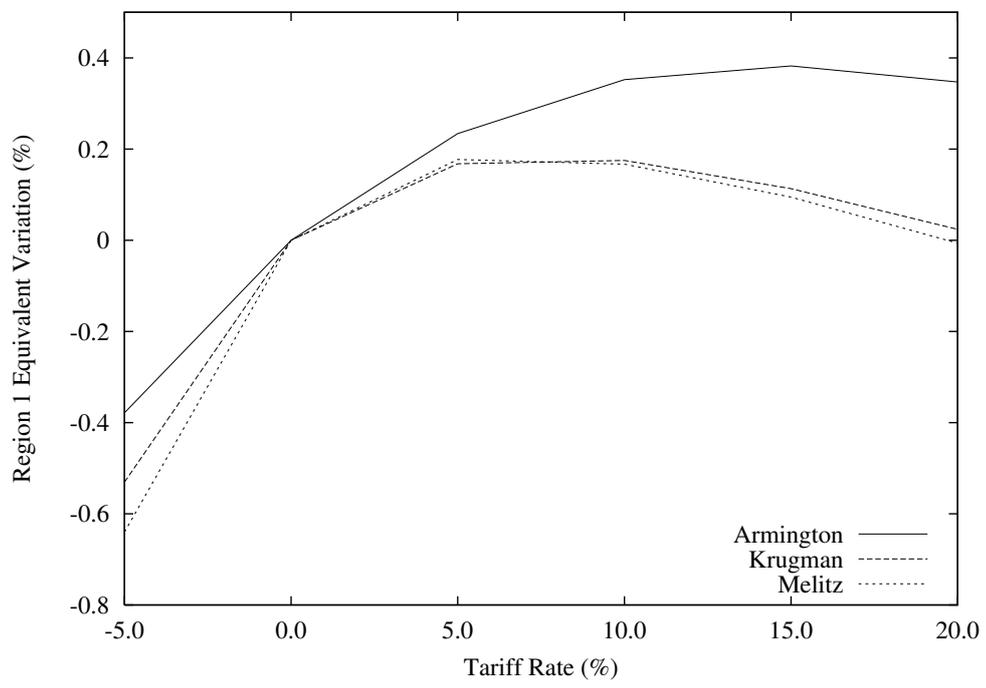


of the gains are different. Relative to autarky, trade allows profitable firms to duplicate their technology and service export markets. Feenstra terms the resulting gains *export* variety gains. Feenstra shows that, although the Melitz model indicates gains from import varieties, the net welfare impact is exactly zero because of lost domestic varieties. Noting that quantitatively the *import* variety gains in the Krugman model are the same as the *export* variety gains in the Melitz model (given equivalent trade responses to variable trade costs) we have a clean explanation of the Arkolakis et al. (2008) equivalence. This can be augmented to include the Armington structure by noting that a Krugman model without entry is effectively identical to Armington.

Extending Feenstra's description to an economy where there are factor supply responses (e.g., due to intersectoral reallocations), entry becomes important. If trade induces net entry the Krugman model will indicate larger gains, relative to the Armington model, because the *import* variety gains will include the new varieties as well as the varieties that were only available to foreigners in autarky. Further, additional gains will be realized in the Melitz structure as gross *import* variety gains dominate lost domestic varieties. Of course, the ordering of the gains is reversed if trade induces exit. This gives us a useful and intuitive explanation of the ordering of effects in Figure 1. When liberalized goods are net substitutes for the non-liberalized goods we observe entry and compounding demand and production side gains in the Melitz structure.

Another area that we can explore in our transparent computational model involves tariffs. Trade distortions that have revenue implications (tariffs and other trade taxes and subsidies) have been purged from much of the theoretic literature. Iceberg trade costs have convenient analytical properties, which explains their use in contemporary theory, but one cannot consider them equivalent to tariffs. We provide a simple demonstration of this in Figure 2. In our symmetric three-region three-good illustrative model we consider Region 1's unilateral incentive to impose a tariff on imports of good 1. We set  $\alpha = 1$  and  $\sigma_j = \sigma_k = a + 1$ , so there would be no difference between the models if we were changing iceberg costs. In each case there is a positive optimal tariff. Consistent with Balistreri and Markusen (2009) we find a lower optimal

Figure 2: Optimal Tariff Across Structures



tariff (between 5% and 10%) in the monopolistic competition models relative to the Armington model (about a 15% optimal tariff). In the monopolistic competition models firms are pricing at average cost indicating less room for the policy authority to leverage the terms of trade. In the applications below we find a similar pattern (lower optimal tariffs under the Melitz structure), but this is not always true.<sup>6</sup>

## 5 Calibration

### 5.1 The accounts and unit choices

In the previous sections we develop the basic trade theories and some computational maquettes that illustrate responses in an intentionally simplified setting. Informing policy in an empirical context requires a procedure for fitting the structure to a set of benchmark observables. In this section we consider the basic mechanics of calibrating a computational model with monopolistic competition and heterogeneous firms. The goal is to accommodate the data in a way that allows for a “replication check.” Rule 1 of CGE modeling: make sure your model can replicate a micro-consistent data set. The primary identities come from a set of social accounts (like the GTAP accounts), which are assumed to represent an equilibrium. We denote the value that a particular variable takes on at the benchmark by embellishing it with a superscript “0” (e.g.,  $Q_{ir}^0$  is the benchmark demand of commodity  $i$  in region  $r$ ). In addition to the social accounts we will discuss additional parameter choices and other evidence on the calibration that might be informed by other branches of empirical economics. In the initial subsections we tackle a static reconciliation of the theories and accounts. In the final subsection we consider response parameters including elasticities and the Pareto shape parameter, which plays a critical role in

---

<sup>6</sup>The optimal tariff in increasing returns models will depend on the specifics. The level of the optimal tariff is an empirical question. There can be compounding scale effects resulting in large gains from diverting production to home firms, but there may also be specialized intermediate inputs which could drive the optimal tariff negative [Markusen (1990)].

Melitz trade responses.

In a standard CGE exercise we can rely on the following relevant observables (for commodity  $i$ ) from a set of social accounts:

- $vafm_{is}$  The value of demand for commodity  $i$  in region  $s$ .
- $vxmd_{irs}$  The value of f.o.b. exports in commodity  $i$  (including  $r = s$ ).
- $vtwr_{igrs}$  Transport payments to sector  $g$  associated with  $vxmd_{irs}$ .
- $tx_{irs}$  Taxes associated with  $vxmd_{irs}$ .
- $vom_{ir}$  The value of output of commodity  $i$  in  $r$ .
- $vfm_{fir}$  The value of factor  $f$  inputs to  $i$  in  $r$ .
- $vifm_{gir}$  The value of intermediate  $g$  inputs to  $i$  in  $r$ .

The social accounts will also include additional information on the nature of final demand by consumers and governments, and will include a reconciliation of factor returns and tax revenues with regional income. These accounts are important for the general equilibrium calibration, but are not discussed here as we focus on calibrating the introduced Melitz (2003) trade theory.

Note that these accounts restrict the calibration on the composite-commodity demand and composite-input supply sides of the trade equilibrium. The following identities must hold if the accounts represent an equilibrium:

$$P_{is}^0 Q_{is}^0 \equiv vafm_{is}; \quad (33)$$

$$c_{ir}^0 Y_{ir}^0 \equiv vom_{ir}. \quad (34)$$

Choosing units such that  $P_{is}^0 = c_{ir}^0 = 1$  the quantities demanded and the quantities of composite-inputs supplied are locked down.

Consider the calibration of the upstream production technologies, which will be familiar to CGE modelers. Proper balancing of the accounts ensures that all revenues are assigned. We

have the identity

$$vom_{ir} \equiv \sum_f vfm_{fir} + \sum_g vifm_{gir}, \quad (35)$$

and the value shares are simply calculated as  $\gamma_{fir} = vfm_{fir}/vom_{ir}$  or  $\gamma_{gir} = vifm_{gir}/vom_{ir}$ . With the value shares well specified, calibration of the unit cost function for each industry in each region is relatively transparent. Of course, equation (30) would need to be elaborated to include intermediate inputs. In the applications section of this chapter we move to a more general nested CES form of the production technology which accommodates a more realistic representation of energy demand. The unit-cost calibration still uses the value shares (and a series of elasticities of substitutions), but these added features are not directly related to the calibration of the new trade theories.

To facilitate our discussion of the trade calibration, and to bring the discussion closer to standard practice in CGE modeling, let us make some additional modifications to the theory. First we need to accommodate the tariffs and other trade distortions. We also need to dispense with the notion of iceberg transport costs, so that the payments can be allocated appropriately. Let the single tax instrument  $t_{irs}$  indicate the ad valorem trade and transport margin. Where the revenues generated by  $t_{irs}$  are allocated in the correct proportions to the transport sector, the importing country (tariff revenues), and to the exporting country (in the case of export taxes). Let us, also, expand the theory to consider the possibility of bilateral preference weights. As we will see, this is not necessary and a modeler may choose to set these weights at one, but for now let us introduce the notation. Elaborating the price indexes with bilateral preference weights,  $\lambda_{irs}$ , for each trade formulation we have

$$P_{js} = \left[ \sum_r \lambda_{jrs} \left( (1 + t_{jrs}) c_{jr} \right)^{1-\sigma_j} \right]^{1/(1-\sigma_j)}, \quad (36)$$

$$P_{ks} = \left[ \sum_r \lambda_{krs} N_{kr} p_{krs}^{1-\sigma_k} \right]^{1/(1-\sigma_k)}, \text{ and} \quad (37)$$

$$P_{hs} = \left[ \sum_r \lambda_{hrs} N_{hrs} \tilde{p}_{hrs}^{1-\sigma_h} \right]^{1/(1-\sigma_h)}. \quad (38)$$

Relative to the above formulation the Armington price index no longer includes  $\tau_{jrs}$ , which is replaced by the tax markup. The monopolistic competition indexes do not include the tax because this is embedded in the gross prices ( $p_{krs}$  and  $\tilde{p}_{hrs}$ ). Each equation includes the  $\lambda_{irs}$  parameters, which has the immediate advantage of decoupling the scale of composite and firm level goods. We are free to choose these units independently, which only affects the scale of  $\lambda_{irs}$ , which are free parameters.

## 5.2 Armington Calibration

Calibrating Armington trade is rather straightforward and familiar to CGE modelers. With our choice of units (such that  $P_{is}^0 = c_{ir}^0 = 1$ ) and the elasticity of substitution ( $\sigma_j$ ) we can recover the values of  $\lambda_{jrs}$  by setting the bilateral demand functions equal to bilateral trade and inverting:

$$\lambda_{jrs} = (1 + t_{jrs}^0)^\sigma \left( \frac{vxmd_{jrs}}{vafm_{js}} \right). \quad (39)$$

An important thing to notice in this relatively transparent setting is that we could have accommodated the trade equilibrium in a different way. Consider setting all of the  $\lambda_{jrs}$  equal to some arbitrary constant,  $\bar{\lambda}$ , such that there are no taste biases, but also consider that the measured  $t_{jrs}^0$  could be missing something important — unobserved iceberg trade costs. Including both iceberg cost and tariffs in the bilateral demand equations we can calculate the implied iceberg costs

$$\tau_{jrs} = \frac{\bar{\lambda} vafm_{js}}{vxmd_{jrs}} \left( \frac{1}{1 + t_{jrs}^0} \right)^{\sigma_j}. \quad (40)$$

Attempts to measure unobserved trade costs from bilateral trade flows [e.g., Anderson and van Wincoop (2003)] approach the data from a perspective consistent with (40), no taste bias and unobserved iceberg costs. A gravity regression can be specified where  $vxmd_{jrs}$  is assumed

to be measured with, well behaved, stochastic error. In this literature, structure is added to  $\tau_{jrs}$  (such that it is symmetric and changes parametrically with borders and distance). The trade flows will not be replicated in the model without adding a structural bilateral residual (like  $\lambda_{jrs}$ ). Accommodating the trade pattern through the  $\lambda_{jrs}$  or the unobserved  $\tau_{jrs}$  is irrelevant for the CGE modeler, unless the counterfactual of interest involves directly looking at changes in  $\tau_{jrs}$  [see Balistreri and Hillberry (2008)]. Even in that case there is always a set of equivalent experiments that adjust the  $\lambda_{jrs}$ . We highlight this latitude in calibration choices because in the monopolistic competition calibrations that follow there will be similar choices. We argue along these lines that our insertion of the taste parameters  $\lambda_{irs}$  is out of convenience and does not affect outcomes, unless the taste bias is a proxy for a potential policy instrument.

### 5.3 Krugman Calibration

Consider calibrating Krugman style trade given the same information from the social accounts. We have the following identity for nominal trade

$$p_{krs}^0 q_{krs}^0 N_{kr}^0 \equiv [(1 + t_{krs}^0) v x m d_{krs}]. \quad (41)$$

Solving for gross firm-level revenues and substituting this into the free-entry equation (9) at  $c_{kr}^0 = 1$  we see that

$$f_k N_{kr}^0 = \frac{\sum_s (1 + t_{krs}^0) v x m d_{krs}}{\sigma_k}. \quad (42)$$

If  $f_k$  is measured then  $N_{kr}^0$  is given. In most cases, however, it is equivalent to set the number of firms at an arbitrary value and calculate a consistent  $f_k$ . The only case where the absolute size of  $f_k$  matters is when we intend to manipulate  $f_k$  as an instrument in counterfactual simulations.

Benchmark firm-level pricing, at  $c_{kr}^0 = 1$ , is determined by the markup equation

$$p_{krs}^0 = \frac{(1 + t_{krs}^0)}{1 - 1/\sigma_k}; \quad (43)$$

and given  $N_{kr}^0$  we can calculate the benchmark firm quantity from (41)

$$q_{krs}^0 = \frac{(1 + t_{krs}^0)vxmd_{krs}}{p_{krs}^0 N_{kr}^0}. \quad (44)$$

The only remaining parameter to be calibrated is  $\lambda_{krs}$  which can be solved by inverting the firm-level demand functions at the benchmark ( $P_{ks} = 1$  and  $Q_{ks} = vafm_{ks}$ );

$$\lambda_{krs} = \frac{q_{krs}^0 (p_{krs}^0)^{\sigma_k}}{vafm_{ks}}. \quad (45)$$

There are other, largely equivalent, calibration procedures that one may employ. For example we could set the  $\lambda_{krs}$  equal to a constant and back out the unobserved trade costs that need to be included for consistency. In general, if we choose to lock in one parameter there must be a compensating change in another parameter such that the benchmark equilibrium is achieved.

## 5.4 Melitz Calibration

The Melitz model calibration, although expanded by the added parameters, follows along the same steps as above. In addition to the elasticity of substitution ( $\sigma_h$ ), we will assume that information on the Pareto parameters ( $a$  and  $b$ ), the bilateral fixed costs ( $f_{hrs}$ ), and the ratio of operating domestic firms to the total mass of firms ( $N_{hr}^0/M_{hr}^0$ ) are given. Benchmark firm-level revenues will be consistent with the zero-cutoff-profit condition [equation (21)]

$$\tilde{p}_{hrs}^0 \tilde{q}_{hrs}^0 = \frac{f_{hrs}(a + 1 - \sigma_h)}{a\sigma_h}, \quad (46)$$

where again we choose the units for inputs such that  $c_{hr} = 1$ . Combining this relationship with the trade identity,  $\tilde{p}_{hrs}^0 \tilde{q}_{hrs}^0 N_{hrs}^0 \equiv [(1+t_{hrs}^0)vxmd_{hrs}]$ , we establish the number of operating firms on each bilateral link;

$$N_{hrs}^0 = [(1+t_{hrs}^0)vxmd_{hrs}] \frac{a\sigma_h}{f_{hrs}(a+1-\sigma_h)}. \quad (47)$$

As we had with the Krugman calibration, if the fixed costs are not measured, we could calibrate the bilateral fixed costs given a measure of the number of firms. In the applications that follow (and in Balistreri et al. (2011)) we run counterfactual experiments that change the fixed costs (as a potential instrument of economic integration). The bilateral shocks are dependent on the pattern of  $f_{hrs}$ , and so we calibrate the implied  $N_{hrs}^0$  based on our measures of the fixed costs.

With the  $N_{hrs}^0$  established and given  $N_{hrs}^0/M_{hr}^0$  we have  $M_{hr}^0$ . Now we calibrate the sunk cost payments using the free-entry condition [equation (24)];

$$\delta f_{hr}^s = \sum_s \tilde{p}_{hrs}^0 \tilde{q}_{hrs}^0 \frac{N_{hrs}^0}{M_{hr}^0} \frac{\sigma_h - 1}{a\sigma_h} \quad (48)$$

It is not necessary in our static model to consider  $\delta f_{hr}^s$  as two separate parameters.

We can use the ratio of operating to entered firms to calculate benchmark productivities,

$$\tilde{\varphi}_{hrs}^0 = b \left( \frac{a}{a+1-\sigma_h} \right)^{1/(\sigma_h-1)} \left( \frac{N_{hrs}^0}{M_{hr}^0} \right)^{-1/a}; \quad (49)$$

and this allows us to calculate the benchmark prices according to the optimal markup (and  $c_{hr} = 1$ ),

$$\tilde{p}_{hrs}^0 = \frac{1+t_{hrs}^0}{\tilde{\varphi}_{hrs}^0(1-1/\sigma_h)}. \quad (50)$$

The firm level quantity must be consistent with bilateral trade volumes;

$$\tilde{q}_{hrs}^0 = \frac{(1 + t_{hrs}^0)vxmd_{hrs}}{\tilde{p}_{hrs}N_{hrs}^0}. \quad (51)$$

The only remaining calibration parameters are the  $\lambda_{hrs}$ , and these are recovered by inverting the demand functions;

$$\lambda_{hrs} = \frac{\tilde{q}_{hrs}^0 (\tilde{p}_{hrs}^0)^{\sigma_h}}{vafm_{hs}}. \quad (52)$$

The mechanical process of calibrating the Melitz structure is complete. Again, we could change the order of determining parameters if alternative information is considered. For example, in Balistreri et al. (2011) we estimate a set of bilateral fixed costs which allow us to set all of the  $\lambda_{hrs}$  equal to one.

## 5.5 Deeper Calibration Issues

While the mechanics of matching the social accounts is necessary (and often tedious), CGE modelers must also consider carefully the response parameters. Most CGE modelers are familiar with the never-ending debate over Armington elasticities ( $\sigma_j$  in our example). Trade responses to policy are critically dependent on the elasticity choice, and modelers often worry about the quality of information provided by our econometrician friends. While others contributing to this volume are in a better position to comment on the econometric difficulties, we will note here that structure and interpretation matter. To the extent that the econometric and CGE models adopt different structures the interpretation is almost always strained and problematic.

Arkolakis et al. (forthcoming) argue that we should interpret the trade elasticities generated from gravity models as  $(1 - \sigma_j)$  or  $(1 - \sigma_k)$  for Armington and Krugman structures and  $(-a)$  for the Melitz structure. This applies for a class of models that they, rather unfortunately, call

*quantitative trade models*. We accept this as the proper interpretation, but the class of models that it applies to is so narrow that the advice is practically useless—at least for anything that we would call a *quantitative* assessment of policy. Using the simple toy model presented by Balistreri et al. (2010), or the one presented above in Section 3, it is relatively easy to show that (once we allow for intersectoral reallocation of resources) the Armington and Melitz models generate different marginal trade responses regardless of how we set the elasticities.

One area of promising research involves extensive-form structural estimation. Structural estimation binds the econometric and economic models in a way that eliminates interpretation errors. The idea is to estimate a set of parameters subject to the nonlinear structure in which the parameters will be used. Advanced non-linear optimization solvers allow us to estimate without reducing the form of the intended economic model. Applications of this technique include Balistreri et al. (2011), where we estimate the shape parameter  $a$  and a set of source and destination fixed trade costs subject to the relevant (Melitz based) trade equations from our CGE model. This offers an opportunity to measure  $a$  in the context of the structure (including the assumed value of the other key response parameter  $\sigma_h$ ) that is used for the counterfactual welfare analysis. Further, it gives us a set of fixed-cost instruments to consider in our welfare analysis of economic integration.

## **6 A decomposition strategy for computation of large models**

This section of the chapter outlines a general strategy for computing large scale applied models that include scale economies. CGE modelers have experienced enormous advances in computing power over the past decades. Computing speed and the performance of *off-the-shelf* algorithms are remarkable. We now routinely solve very large non-linear general equilibrium problems. Part of this success is attributable to the constant-returns-to-scale class of problems that we typically solve. The advanced theories considered in this chapter, however, can be par-

ticularly problematic in applied numeric models. We present a decomposition algorithm that has proven successful for a number of our applications.

Attention to dimensionality and potential non-convexities is the rule in computing challenging equilibrium problems. Even the very robust algorithms available today cannot guarantee convergence once the dimensions of the problem become large. The inherent non-convexities associated with income effects in GE models [Mathiesen (1987)] when coupled with excessive dimensions can lead to a failure of the algorithm.

Examining Table 2 we can see that the Melitz theory is potentially problematic in application, relative to a comparable Armington model, because the dimensions of the problem are much larger. There are four bilateral equilibrium conditions for each Melitz good. (The Krugman model is slightly better in that there are only two bilateral conditions per good.) Cleverly, the Armington formulation includes *no* bilateral conditions. We sum across bilateral import demands in the market clearance conditions, and only recover the bilateral trade flows as a post-solve artifact of the Armington equilibrium. Even worse, for attempting to solving large-scale models with Melitz trade, is the fact that there are new sources of non-convexities. There are Dixit-Stiglitz scale effects, and endogenous productivity effects associated with the competitive selection of firms in each bilateral market.<sup>7</sup>

Faced with these challenges, consider that we can recalibrate a purely Armington general equilibrium to represent any realized counterfactual solution to the true model that includes Melitz (and Krugman) goods. The recalibration involves equilibrium-specific adjustments in the bilateral CES distribution parameters, the  $\lambda_{irs}$ , so they reflect scale and productivity adjustments relative to the benchmark. In essence, the productivity of the factor content of trade

---

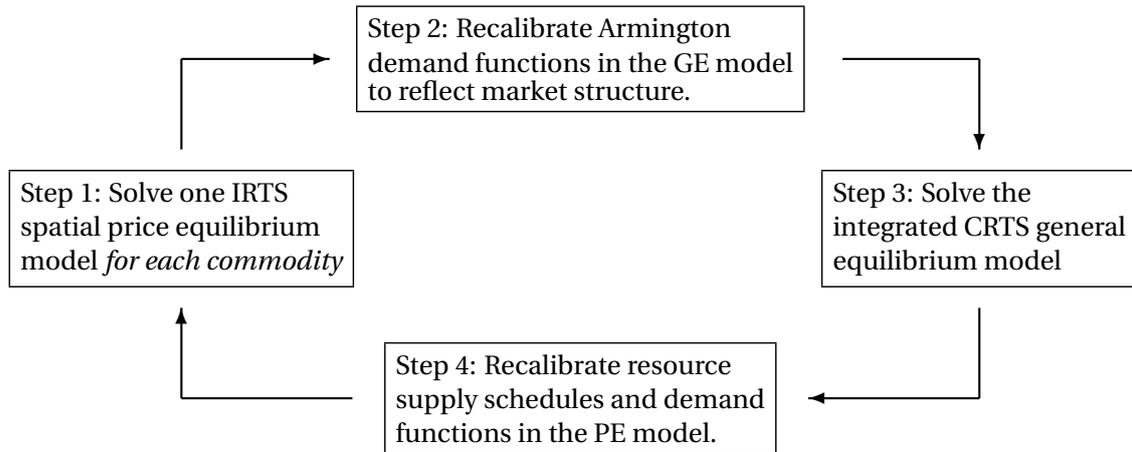
<sup>7</sup>Compounding scale effect in industries that have a large share of intermediate use of their own output is a known problem in CGE applications with scale economies. An industry that has a compounding scale effect will be favored to grow very large. In our applications of the Melitz formulation we have not encountered this problem, either as a computational issue or as an oddity in the resulting equilibrium. This may, however, be do to the fact that we are currently working with fairly coarse commodity aggregations that probably mask exceptionally large own-use coefficients in particular sectors.

must be adjusted to accurately reflect any changes in the industrial organization. The problem, of course, is that we do not know what the productivity adjustments (the adjustments in the  $\lambda_{irs}$ ) are without solving the true general equilibrium. Our strategy is to find the appropriate (solution) adjustments to the  $\lambda_{irs}$  by iterating between a partial equilibrium model which captures the heterogeneous-firms industrial organization and the purely Armington general equilibrium which establishes aggregate demand and input supply for the increasing-returns sectors.

As a first step consider a policy simulation that affects a Melitz good  $h$ . Let us solve the partial equilibrium trade model presented in section 2.3 as an approximation. By isolating good  $h$ , we have a relatively small numeric problem that does not include the troublesome general-equilibrium income effects. The approximation indicates new values for  $P_{hr}, Q_{hr}, c_{hr}$ , and the full set of bilateral trade flows for commodity  $h$ . Given this information one can recalibrate the Armington technology to accommodate the new productivity and variety effects. The recalibration recovers a new set of implied  $\lambda_{irs}$ . Manipulating the  $\lambda_{irs}$  in the benchmark Armington CGE model, however, will lead to an imbalance in the equilibrium (as the relative and absolute demands on specific bilateral links are altered). Solving the general equilibrium at this new point indicates a changed set of equilibrium quantities and prices, including new values for the following variables related to the Melitz sector:  $Q_{hr}, P_{hr}, Y_{hr}$ , and  $c_{hr}$ . These can be fed into the partial equilibrium model as  $\bar{Q}_{hr}, \bar{P}_{hr}, \bar{Y}_{hr}$ , and  $\bar{c}_{hr}$  as they appear in equations (1) and (2). The partial equilibrium demand and supply functions are recentered at the new general equilibrium solution point, which likely improves the accuracy of the partial equilibrium approximation in the subsequent solve. Continuing this procedure iteratively until the partial and general equilibrium models are mutually consistent reveals the numeric solution to the intended general equilibrium. The four steps involved in the solution algorithm are depicted in Figure 3.

Figure 4 illustrates the algorithm in action. In section 7.2 of this chapter we present results from a number of trade scenarios, and here we show the convergence report from one of these

Figure 3: A Decomposition Algorithm



scenarios (the scenario where we have Melitz trade in manufactured, MAN, and energy intensive, EIT, goods and there is a world-wide 50% cut in tariffs on these goods). Figure 4 plots the numeric proximity to the general-equilibrium solution as a function of the number of iterations. We measure proximity in a given iteration by first identifying the largest imbalances in the regional input and composite commodity markets [equations (1) and (2)] as the GE solution values are handed over to the partial equilibrium model. We square these imbalances and add the numbers together to form a proximity index (or norm) for each heterogeneous-firms good. Once the indexes across all goods are approximately zero the partial equilibrium model will only add trivial adjustments to the general equilibrium. That is, once the indexes simultaneously fall below a predetermined numeric tolerance level the full solution is realized. (The tolerance level can be adjusted to balance speed versus accuracy.) In application we find the convergence properties to be relatively rapid and robust, although considerably dependent on the choice of the partial equilibrium elasticities ( $\eta$  and  $\mu$ ), and the tolerance level.

Figure 4: Example of Convergence in Application

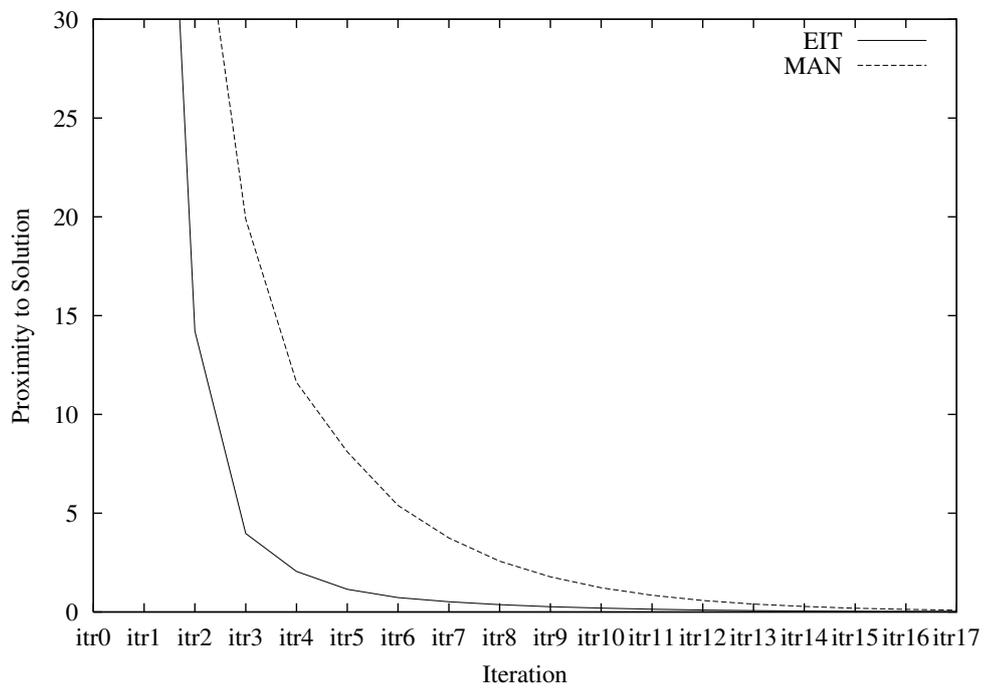


Table 3: Scope of the Empirical Model

| <b>Regions:</b> |                              | <b>Goods:</b> |                      | <b>Factors:</b> |                   |
|-----------------|------------------------------|---------------|----------------------|-----------------|-------------------|
| EUR             | Europe                       | OIL           | Refined oil products | LAB             | Unskilled Labor   |
| USA             | United States                | GAS           | Natural Gas          | SKL             | Skilled Labor     |
| RUS             | Russia                       | ELE           | Electricity          | CAP             | Capital           |
| RA1             | Rest of Annex 1              | COL           | Coal                 | RES             | Natural Resources |
| CHN             | China                        | CRU           | Crude Oil            | LND             | Land              |
| IND             | India                        | EIT           | Energy Intensive     |                 |                   |
| EEX             | Energy Exporting             | MAN           | Manufacturing        |                 |                   |
| MIC             | Middle-High Income, n.e.c.   | TRN           | Transportation       |                 |                   |
| LIC             | Low Income Countries, n.e.c. | AOG           | All other goods      |                 |                   |

## 7 Applications

### 7.1 Introduction

In this section of the chapter we present policy analysis applications in a CGE model that includes Melitz (2003) style industrial organization. The policies instruments that we examine include tariffs and other trade costs, as well as restrictions on carbon emissions. The model is based on GTAP 7 data. The model extends Balistreri et al. (2011) to include industry level input-output data and details on energy supply and demand. The core structure is most closely related to Rutherford (2010b), which has an Armington trade structure. The Rutherford (2010b) model is extended to include the option of a Melitz treatment of non-energy sectors. A technical description of the model is available in Appendix C.

For the exercises in this chapter the GTAP 7 data are aggregated to include nine regions and nine production sectors. Table 3 shows the regions and sectors included. The first six regions are important players in the formation of carbon policy. For clarification the rest of Annex 1 aggregate region includes Canada, Japan, Australia, and New Zealand. The energy-exporting region (EEX) includes the oil rich Middle Eastern and African countries. The remainder of the world is divided into two aggregates based on World Bank income classifications.

The production sector aggregation reflects our desire to consider climate policy. We include three fuels (OIL, GAS, and COL)<sup>8</sup> The crude oil sector (CRU) is tracked, which provides the input for the oil refining sector. The final energy good is electricity (ELE). We also include the transportation sector because of its emissions intensity and important role in international trade. We aggregate the manufacturing sectors in the GTAP data into two subaggregates. Energy intensive production (EIT) includes ferrous and non-ferrous metals, non-metallic minerals production, chemicals, rubber, and plastics. The remainder of manufacturing is captured in the MAN sector. The Melitz heterogeneous firms structure is applied to EIT and MAN. The final sector is the catch all AOG sector which includes agriculture and services.

Table 3 also shows that we maintain the five GTAP factors of production. Key to our analysis of climate policy is the resource factor (RES). This factor is used in the primary energy sectors GAS, COL, and CRU. The RES input is assumed to be sector specific, which allows us to calibrate the supply elasticities for these sectors by choosing the elasticity of substitution between RES and the other inputs. The upstream energy price responses to climate policy depend critically on these elasticities. Models that assume primary energy extraction is a constant returns activity using mobile factors tend to understate price responses, relative to our formulation of calibrated upward sloping supply. More details are offered in Appendix C.

## 7.2 Trade Policy Applications

The first set of experiments that we consider are similar to those that appear in Balistreri et al. (2011). We assume monopolistic competition among heterogeneous firms for the manufacturing and (MAN) energy intensive (EIT) sectors. The scenarios include changes in measured tariffs and fixed trade costs for these sectors. Table 4 shows the welfare impacts across regions and scenarios. The policy shocks are a fifty percent reduction in tariffs, a fifty percent reduction in fixed trade costs, or a combined fifty percent reduction in tariffs and fixed trade costs. The

---

<sup>8</sup>The purchase of a fuel indicates emissions of CO<sub>2</sub> based on the carbon content of the fuel.

Table 4: Regional welfare impacts of trade-cost reductions (%EV)

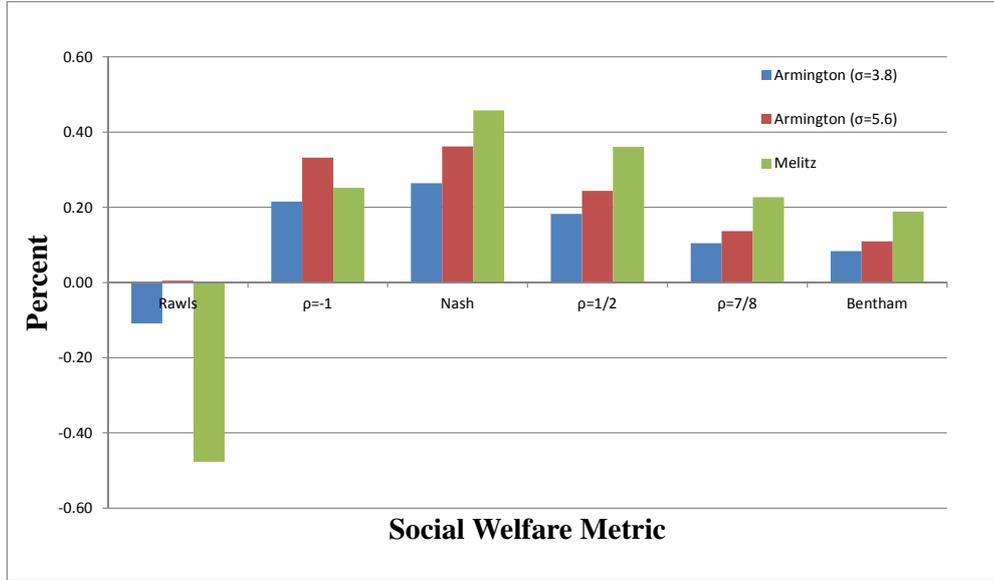
| <b>Region</b> | <b>Armington</b>               |                                | <b>Melitz</b> |            |                        |
|---------------|--------------------------------|--------------------------------|---------------|------------|------------------------|
|               | tariff<br>( $\sigma_j = 3.8$ ) | tariff<br>( $\sigma_j = 5.6$ ) | tariff        | fixed-cost | tariff &<br>fixed-cost |
| EUR           | 0.09                           | 0.10                           | 0.23          | 0.63       | 0.77                   |
| USA           | -0.08                          | -0.09                          | -0.12         | 0.96       | 0.84                   |
| RUS           | 0.25                           | 0.46                           | -0.43         | 4.35       | 4.73                   |
| RA1           | 0.15                           | 0.17                           | 0.19          | 1.81       | 2.04                   |
| CHN           | 1.11                           | 1.31                           | 1.86          | 7.25       | 9.28                   |
| IND           | -0.13                          | -0.06                          | -0.34         | 1.36       | 1.03                   |
| EEX           | 0.12                           | 0.20                           | 0.00          | 5.40       | 6.72                   |
| MIC           | 0.19                           | 0.27                           | 0.71          | 4.26       | 4.79                   |
| LIC           | -0.11                          | 0.01                           | -0.48         | 3.47       | 3.57                   |

general findings from our earlier paper are maintained. The Melitz structure indicates larger average welfare gains from tariff liberalization. In addition the same proportional reduction in fixed trade costs generates substantially larger gains.

In the second column of Table 4 we increase the Armington elasticity of substitution to  $a + 1 = 5.6$  based on the arguments in Arkolakis et al. (forthcoming) that this is the appropriate elasticity for comparison with a Melitz structure, where  $a = 4.6$ . Although this reduces the relative difference between the Armington and Melitz structure, it does not indicate a significant match across the structures. The strong equivalence results suggested by Arkolakis et al. (2008) and by Arkolakis et al. (forthcoming) are not supported in our empirical model. For us, this indicates that the real-world complexities accommodated in CGE models are, indeed, important. The significant differences that we show across structures are likely missed in empirical exercises that rely on aggregate gravity regressions.

We continue our comparison of the tariff scenarios in Figure 5. In the figure we consider alternative measures of global welfare based on an aggregation of money-metric per-capita utility. Let  $W(\rho)$  indicate social welfare as a function of the equity parameter  $\rho \in [-\infty, 1]$ , and let  $u_r$  indicate the money-metric per-capita utility level in region  $r$ . In general, global social welfare

Figure 5: Global welfare under a 50% cut in manufacturing tariffs (% change)



can be defined as

$$W(\rho) = \left( \sum_r u_r^\rho \right)^{1/\rho}. \quad (53)$$

In the limit, as  $\rho \rightarrow -\infty$  we have a Rawlsian (maximin) social welfare function, where the welfare of the poorest region (LIC in our application) is all that matters. For  $\rho \rightarrow 0$  we have a Nash (multiplicative) social welfare function, and for  $\rho = 1$  we have Bentham's standard additive utilitarian social welfare function. Figure 5 is interesting in that it shows significant differences between the Armington and Melitz treatments. We also see that tariff liberalization has an important equity component. Under a Nash social welfare metric the gains from tariff liberalization are more than twice that when we only consider efficiency (the Bentham metric).

Our CGE model with a Melitz treatment of energy intensive and other manufactured goods includes two channels by which outcomes are affected differently than in a constant-returns model. First, there are productivity impacts. Liberalization induces a within industry reallo-

cation of resources towards marginal exporters, while inducing exit of the least efficient firms. This changes a sector's productivity. Feenstra (2010) interprets these reallocations as *export* variety gains. The other channel, by which model outcomes are different, is through changes in the extensive margin of trade, or in Feenstra's words *import* variety gains. That is, to the extent that liberalization induces entry, new varieties will be produced and consumed. In our multidimensional application of the Melitz structure we find evidence that both of these effects are important. This is in contrast to much of the theoretic work, which focuses on one-factor one-sector models [e.g., Arkolakis et al. (2008) and Feenstra (2010)]. In a one-factor one-sector Melitz model the *import* variety gains are exactly zero! [see Feenstra (2010) and Balistreri et al. (2010)].

In Table 5 we show the productivity impacts by directly examining changes in  $\tilde{\varphi}_{hrs}$ . Focusing on the 50% tariff cut scenario we present two measures of productivity for each of the heterogeneous-firms industries. In the "Domestic" columns we report percent changes in  $\tilde{\varphi}_{hrs}$ , and in the "Industry" columns we present the percent changes in the weighted average  $\tilde{\varphi}_{hrs}$ , where the weights are determined by the average firm quantity and the number of firms in each bilateral market ( $\tilde{q}_{hrs}N_{hrs}$ ). The weights reflect the relative resources allocated to marginal costs. Notice that productivity gains in the broader industry are generally higher than for the domestic market, but this is not always the case. There are two competing effects in an export market. Relative to the benchmark the weights associated with these relatively productive markets are increasing, but the average firm productivity is falling as marginal firms begin exporting. It is possible, as in the case of the energy intensive industry in the MIC region, that industry wide productivity can fall with liberalization. In this case the benchmark exporting firms are very productive and the entry and expanded production of the marginal firms dominates the measure of average productivity.

Examining the extensive margin in the Melitz structure is more complicated than just counting varieties, because the varieties enter the expenditure system at different prices. Further-

Table 5: Productivity impacts from Tariff Cuts (%change)

| <b>Region</b> | <b>Energy Intensive (EIT)</b> |          | <b>Manufacturing (MAN)</b> |          |
|---------------|-------------------------------|----------|----------------------------|----------|
|               | Domestic                      | Industry | Domestic                   | Industry |
| EUR           | 0.4                           | 1.7      | 0.5                        | 3.3      |
| USA           | 0.7                           | 1.7      | 0.6                        | 1.9      |
| RUS           | 4.8                           | 1.7      | 1.3                        | 2.6      |
| RA1           | 0.6                           | 1.9      | 1.0                        | 2.7      |
| CHN           | 0.1                           | 1.1      | 1.4                        | 2.2      |
| IND           | 0.8                           | 2.3      | 1.7                        | 3.2      |
| EEX           | 3.0                           | 1.3      | 1.8                        | 4.6      |
| MIC           | 0.5                           | -0.2     | 1.7                        | 2.2      |
| LIC           | 2.2                           | 2.0      | 3.8                        | 4.9      |

more, liberalization causes domestic varieties to vanish, offsetting the standard gain in foreign varieties. Feenstra (2010) sorts out these various effects showing that the variety gains can be tracked across equilibria by deviations in the ratio

$$\left( \frac{\Lambda_{hr}(\text{scenario})}{\Lambda_{hr}(\text{benchmark})} \right)^{-1/(\sigma_h-1)},$$

where  $\Lambda_{hr}(z)$  is region- $r$ 's shares of expenditure on good- $h$  varieties that are available in both equilibria to the total expenditures on good- $h$  varieties at  $z$ . In Table 6 we present the percentage changes in this ratio for the 50% tariff cut scenario. There are many instances where we observe losses from liberalization-induced changes in the number of varieties. For a given level of import penetration more domestic varieties are lost relative to the new import varieties [Baldwin and Forslid (2010)], but because the lost domestic varieties have relatively high prices and low quantities (low productivity goods), the net impact is ambiguous.

In addition to the multilateral scenarios, we examine the impacts of unilateral policy. We find that for some regions the differences across the Armington and Melitz structures are striking. We consider how China's welfare is impacted by changes in (EIT) protection. In the benchmark the weighted average rate of protection on China's imports of EIT is 7.3%. Maintaining

Table 6: Variety Impacts (%change in Feenstra Ratio)

| <b>Region</b> | <b>Energy Intensive (EIT)</b> | <b>Manufacturing (MAN)</b> |
|---------------|-------------------------------|----------------------------|
| EUR           | 0.06                          | 0.20                       |
| USA           | -0.04                         | -0.02                      |
| RUS           | -0.38                         | -0.26                      |
| RA1           | 0.04                          | 0.04                       |
| CHN           | 0.02                          | 0.30                       |
| IND           | 0.02                          | 0.06                       |
| EEX           | -0.01                         | 0.02                       |
| MIC           | 0.09                          | 0.51                       |
| LIC           | -0.04                         | 0.13                       |

the distribution of the tariffs across its trade partners we proportionally change this tariff rate from -50% to 400%. Figure 6 plots the impact of the tariff changes on China's welfare under the Armington and Melitz structures. The Armington structure indicates a relatively high optimal tariff (at an average tariff rate of about 25.7%). In contrast, if the EIT (and the MAN) sectors are characterized by Melitz trade the optimal tariff is at about half of the benchmark level of protection, or 3.7%. We run the same set of unilateral experiments for the United States. The results are plotted in Figure 7. For the United States we do not see the dramatic difference, although this is partially due to a relatively low benchmark rate of protections (2.8%).

### 7.3 Heterogeneous firms and carbon policy

In this section we explore carbon policy and the particular problems associated with subglobal action on climate change. We explore the impact of coalition size on the cost of carbon policy and on carbon leakage rates. The emissions target is a global reduction equal to 20% of Europe's benchmark emissions. The coalitions that we consider include Europe alone, an OECD coalition (Annex 1 except Russia), the OECD plus China, and a full global coalition. As the common emissions goal is spread across broader coalitions the marginal abatement cost falls and carbon leakage rates fall. We also consider carbon based tariffs that the coalition may place on energy

Figure 6: Chinese Welfare given a Unilateral Change in its EIT Tariffs

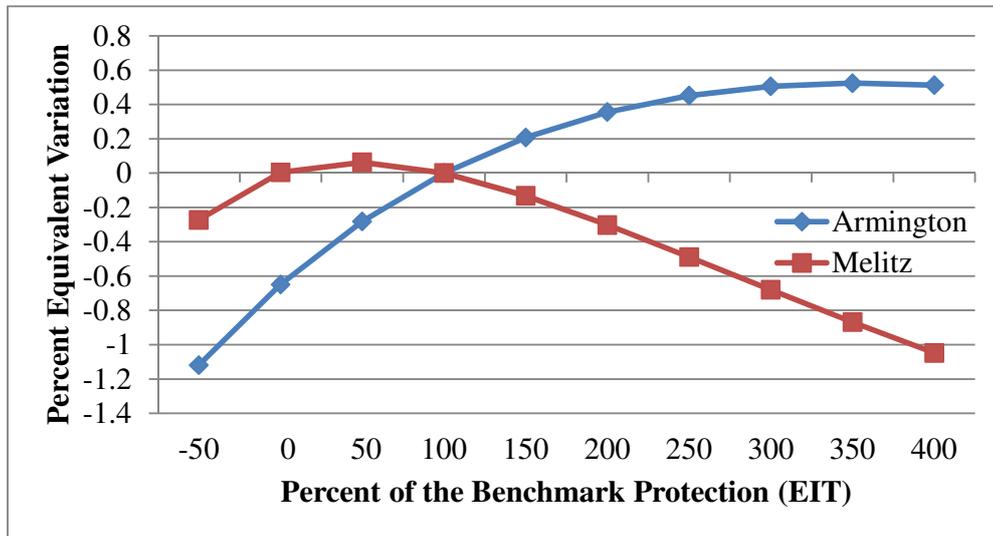


Figure 7: US Welfare given a Unilateral Change in its EIT Tariffs

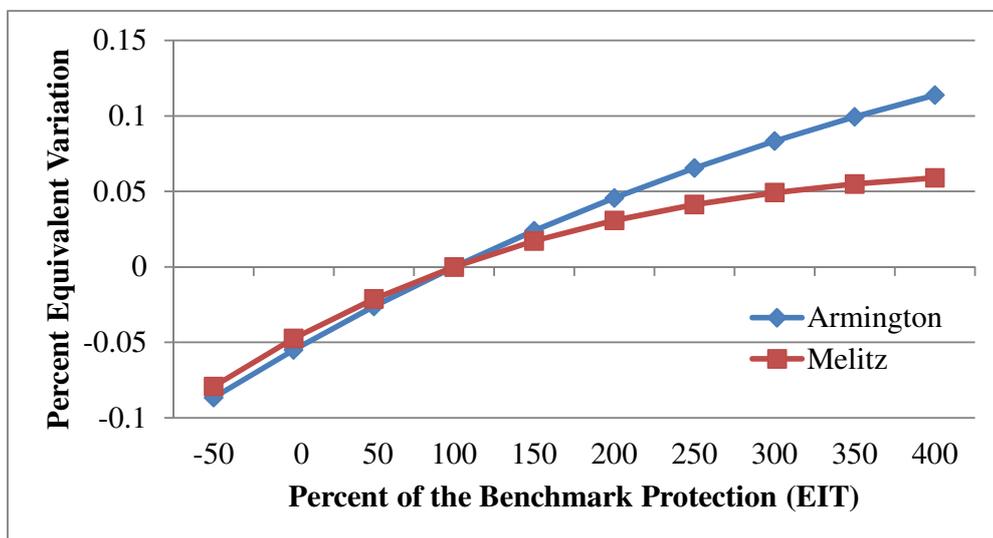


Table 7: CO<sub>2</sub> Emissions Under Carbon Coalitions (MMt)

| Policies and Coalitions: | Benchmark | Carbon Cap (No Border Adjustments) |       |          |        | Carbon Cap with Carbon Based Tariffs |       |          |        |
|--------------------------|-----------|------------------------------------|-------|----------|--------|--------------------------------------|-------|----------|--------|
|                          |           | Europe                             | OECD  | OECD+CHN | Global | Europe                               | OECD  | OECD+CHN | Global |
| EUR                      | 4153      | 2878                               | 3854  | 4022     | 4097   | 3029                                 | 3880  | 4032     | 4097   |
| USA                      | 6069      | 6128                               | 5505  | 5824     | 5926   | 6134                                 | 5533  | 5830     | 5926   |
| RUS                      | 1542      | 1624                               | 1572  | 1557     | 1496   | 1558                                 | 1551  | 1548     | 1496   |
| RA1                      | 2033      | 2058                               | 1859  | 1958     | 1994   | 2067                                 | 1871  | 1961     | 1994   |
| CHN                      | 4305      | 4336                               | 4337  | 3822     | 3986   | 4327                                 | 4328  | 3831     | 3986   |
| IND                      | 1061      | 1069                               | 1073  | 1071     | 982    | 1067                                 | 1072  | 1070     | 982    |
| EEX                      | 1987      | 2100                               | 2037  | 2010     | 1937   | 2037                                 | 2015  | 2000     | 1937   |
| MIC                      | 4412      | 4536                               | 4496  | 4470     | 4316   | 4512                                 | 4482  | 4460     | 4316   |
| LIC                      | 161       | 165                                | 162   | 161      | 159    | 162                                  | 162   | 161      | 159    |
| Total                    | 25725     | 24894                              | 24894 | 24894    | 24894  | 24894                                | 24894 | 24894    | 24894  |

intensive (EIT) imports from non-coalition countries. The direct plus indirect carbon content of EIT trade is determined using an input-output calculation.<sup>9</sup>

Table 7 shows the distribution of global emissions across these scenarios when we model the EIT and MAN sectors using the heterogeneous-firms formulation. The various possible coalitions indicate a significant reallocation of global emissions and the costs of policy. In Table 8 we show the marginal abatement costs across the same set of policies. The price of CO<sub>2</sub> permits falls dramatically as the coalition expands. In Figure 8 we show this and include the results from a comparable set of runs in the pure Armington trade model. Under the Melitz formulation the price of permits is higher (although only significantly in the Europe only case). With Melitz trade in energy intensive and other manufactured goods the trade responses to sub-global carbon policy are larger. Larger leakage rates indicate that the coalition must impose a more restrictive cap in order to meet the global goal.

In Figure 9 we directly consider the leakage rates. Carbon leakage is defined as the ratio of non-coalition emissions increases over the total emissions reductions by the coalition. With small coalitions (Europe alone) and with the assumed Melitz structure we have the highest carbon leakage. Border tariffs have a larger absolute impact on leakage under the Melitz formu-

<sup>9</sup>See Rutherford (2010b) for an explanation and example GAMS code for application to the GTAP accounts. Other applications of the input-output measurement of embodied emissions include Peters and Hertwich (2008), Peters (2008), and Wyckoff and Roop (1994).

Table 8: CO<sub>2</sub> Prices Under Carbon Coalitions (USD/tonne)

| Policies and Coalitions: | Benchmark | Carbon Cap (No Border Adjustments) |       |          |        | Carbon Cap with Carbon Based Tariffs |       |          |        |
|--------------------------|-----------|------------------------------------|-------|----------|--------|--------------------------------------|-------|----------|--------|
|                          |           | Europe                             | OECD  | OECD+CHN | Global | Europe                               | OECD  | OECD+CHN | Global |
| EUR                      |           | 85.39                              | 10.71 | 4.37     | 2.87   | 72.18                                | 10.37 | 4.36     | 2.87   |
| USA                      |           |                                    | 10.71 | 4.37     | 2.87   |                                      | 10.37 | 4.36     | 2.87   |
| RUS                      |           |                                    |       |          | 2.87   |                                      |       |          | 2.87   |
| RA1                      |           |                                    | 10.71 | 4.37     | 2.87   |                                      | 10.37 | 4.36     | 2.87   |
| CHN                      |           |                                    |       |          | 4.37   |                                      |       | 4.36     | 2.87   |
| IND                      |           |                                    |       |          | 2.87   |                                      |       |          | 2.87   |
| EEX                      |           |                                    |       |          | 2.87   |                                      |       |          | 2.87   |
| MIC                      |           |                                    |       |          | 2.87   |                                      |       |          | 2.87   |
| LIC                      |           |                                    |       |          | 2.87   |                                      |       |          | 2.87   |

Figure 8: CO<sub>2</sub> Prices and Coalition Expansion

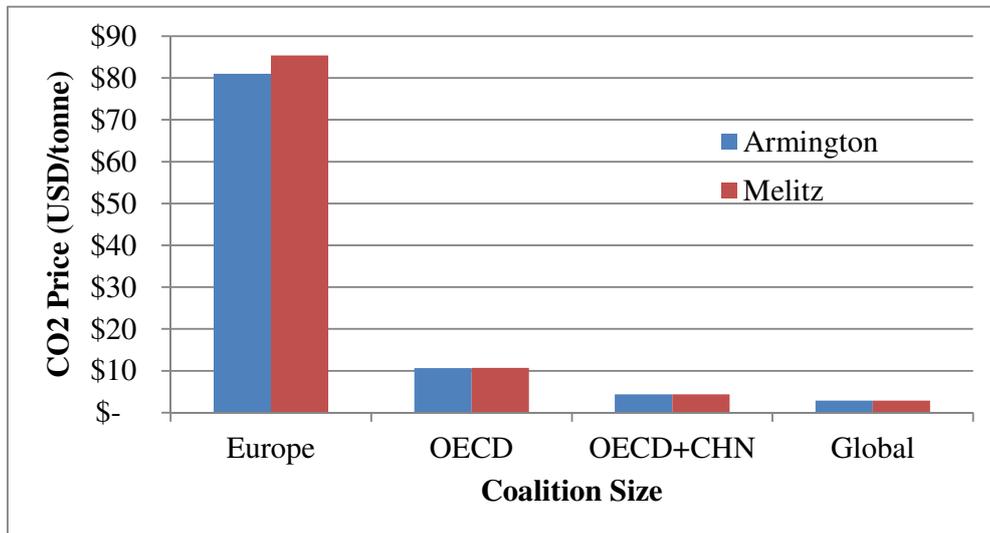
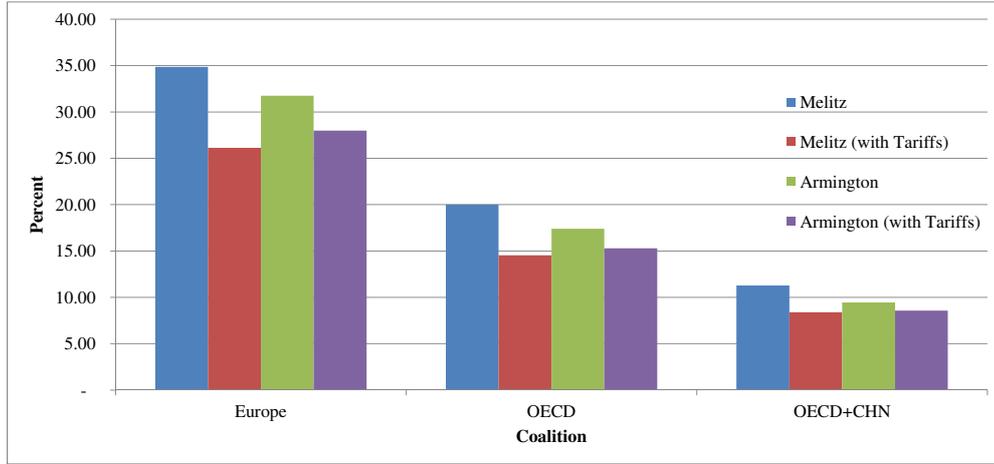


Figure 9: CO<sub>2</sub> Leakage Rates (%)



lation. Notice, however, that the average leakage rate with and without tariffs is higher under the Melitz structure. This indicates that the relative effectiveness of the border adjustments is lower conditional on the trade response. This is because we observe greater trade diversion in the Melitz model.

An interesting question to ask concerning the carbon based tariffs is how much emissions reduction is achieved relative to the trade reduction. We define the border tariff yield as follows. Let the carbon coefficient per unit of bilateral EIT trade be denoted  $\nu_{rs}$  then we calculate the yield of a tariff imposed by the coalition  $\{S\}$  on  $r$  as

$$\text{Yield}_r = \frac{\Delta \text{Emissions}_r}{\sum_{s \in S} \nu_{rs} \Delta \text{Trade}_{rs}}. \quad (54)$$

If the world operated in an input-output fashion then the yields would always be 100%, but because agents respond to the carbon tariffs we get trade diversion (including diversion into non-coalition domestic markets). Table 9 reports the yields across our scenarios. The first thing to notice is that the yield rates can be quite low. A European unilateral carbon cap combined with border adjustments diverts a great deal of trade within the Melitz framework. Emissions

Table 9: Carbon Border Tariff Yield Rates (%)

| <b>Coalitions<br/>Region</b> | <b>Europe</b> |        | <b>OECD</b> |        | <b>OECD+CHN</b> |        |
|------------------------------|---------------|--------|-------------|--------|-----------------|--------|
|                              | Armington     | Melitz | Armington   | Melitz | Armington       | Melitz |
| EUR                          |               |        |             |        |                 |        |
| USA                          | 1             | -2     |             |        |                 |        |
| RUS                          | 45            | 60     | 45          | 52     | 44              | 47     |
| RA1                          | -44           | -88    |             |        |                 |        |
| CHN                          | 34            | 17     | 34          | 25     |                 |        |
| IND                          | 43            | 15     | 50          | 26     | 50              | 23     |
| EEX                          | 51            | 101    | 50          | 82     | 47              | 71     |
| MIC                          | 27            | 38     | 32          | 42     | 31              | 42     |
| LIC                          | 35            | 64     | 26          | 22     | 20              | -2     |

in the USA and the rest of Annex 1 actually rise due to trade diversion. It is also the case that the trade taxes hit the efficient (less energy intensive) exporting firms disproportionately. This works against the yields in the large trade exposed economies (CHN and IND). In these countries, although emissions are lower under the Melitz structure, the quantity of trade reduced is even higher (lowering the yield rates). In contrast we see much higher yield rates in Russia and the Energy Exporting countries as emissions from their EIT sectors go down substantially relative to the reduction in exports.

To illustrate, Table 10 shows the changes in sectoral emissions in CHN and RUS under the OECD coalition policy. The tables show the impact the tariffs have on emissions from energy intensive production. In China the indirect emissions reductions in electricity are less than the direct reduction, and we see emissions increases in the broader manufacturing and AOG sectors. These sectors expand in response to the EIT tariffs. In Russia we see much larger indirect emissions reductions.

We conclude our look at carbon abatement scenarios by reporting the global welfare costs. Each scenario embodies the same level of emissions to give us a fair comparison of costs by holding the benefit of action unspecified but fixed. Figures 10 and 11 report the welfare costs under the social welfare metrics introduced in our analysis of trade policy. Using the Bentham metric we see the usual pattern; larger coalitions reduce the cost of action. Considering eq-

Table 10: OECD Carbon Border Tariff Induced Changes in Emissions (MMt)

| Sector   | China (CHN) |        | Russia (RUS) |        |
|--|-------------|--------|--------------|--------|
|  | Armington   | Melitz | Armington    | Melitz |
| OIL  | 0.00        | 0.00   | -0.08        | -0.24  |
| GAS  | 0.00        | -0.01  | -0.02        | -0.07  |
| ELE  | -3.12       | -4.36  | -4.73        | -11.28 |
| COL  | -0.11       | -0.17  | 0.00         | 0.01   |
| CRU  | 0.00        | -0.01  | -0.02        | 0.04   |
| EIT  | -4.95       | -6.82  | -3.90        | -10.71 |
| MAN  | 0.48        | 0.96   | 0.12         | 0.77   |
| TRN  | 0.11        | 0.19   | 0.50         | 1.00   |
| AOG  | 0.26        | 0.44   | 0.03         | -0.03  |
| Total  | -7.34       | -9.77  | -8.06        | -20.51 |
| Change in Implied CO <sub>2</sub> Embodied in Exports: |             |        |              |        |
| EIT  | -21.60      | -39.83 | -17.72       | -39.39 |

uity, however, we see an interesting pattern for the Melitz structure. Abatement by an OECD coalition harms the poorest countries, but it offers a competitive advantage to China and the MIC countries (and their welfare goes up). This reallocation indicates approximately zero policy costs under the Nash metric. We do not see the same result in the Armington model, as the welfare gains for CHN and MIC are positive but not nearly as large.

## 8 Conclusion

The processors are still warm from our first set of runs. Its time to start warming up the neurons. Successful modelers know that having a solid computational model does not convert into solid economic analysis without considerable reflection and experimentation. We offer, in this chapter, a dose of methodology and hopefully enough demonstration to keep the reader interested. The economic analyses of our applications are, however, at this point admittedly superficial. We are committed to understanding how the new heterogeneous firms theory might change our view of climate-linked trade policies. Having demonstrated that we can numerically consider carbon policy and the heterogeneous firm theory in a CGE model, we look forward to

Figure 10: Melitz Model Global Welfare Cost of Abatement (%)

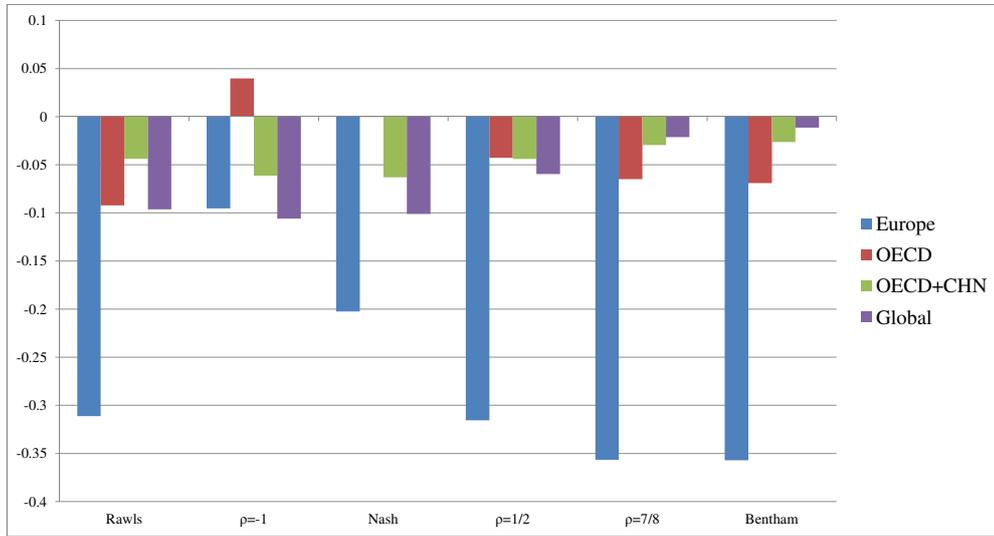
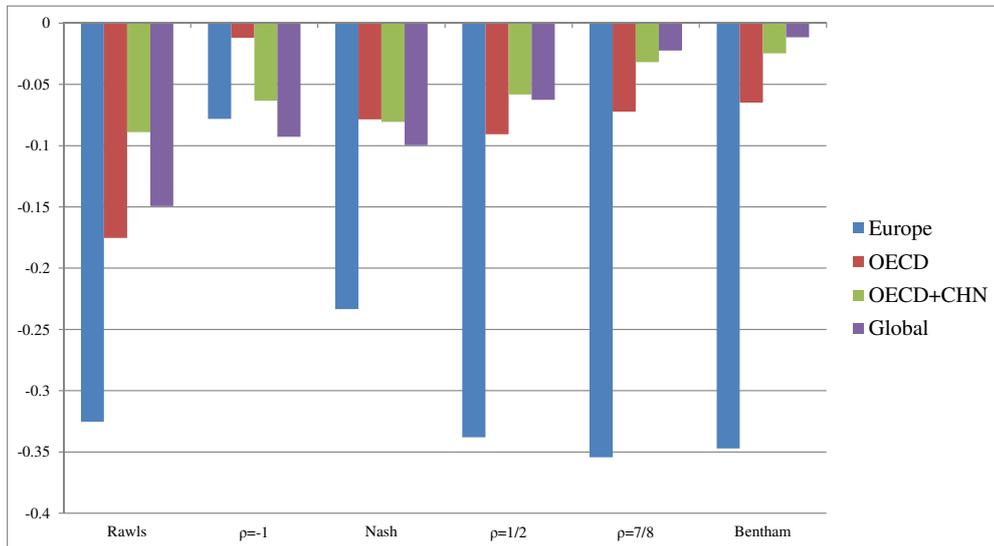


Figure 11: Armington Model Global Welfare Cost of Abatement (%)



pursuing this research agenda.

The methodological focus in some parts of this chapter serves a couple of purposes. The chapter documents our approach to incorporating what is a relatively advanced theory of trade in a CGE model. It may also serve as an initial road map for others interested in these structures. We hope to encourage active research in this area. We have specific questions to ask of the structure, but we find that we often learn as much from observing how others approach similar problems. Our overarching goal is to improve the accessibility of these alternative structures. There are technical hurdles, but hopefully we have lowered them to some degree.

We encourage examination of the structures presented, but we also encourage an examination of a broader set of alternatives. We have adopted some unsettling assumptions along the way. Many of these assumptions are made in analytical presentations simply because without them a closed form solution can be elusive or impossible. Our intent in this chapter is to maintain a degree of proximity to the theoretic presentations, but we feel relaxations of some of these assumptions are relatively easy in a computational setting. For example, we adopt the common restriction that input proportions of fixed and marginal costs are the same. This is just one examples, but there are many more uncomfortable assumptions that undoubtedly matter. We have taken a very clean approach to applying the theory, as it is familiar to trade economists, using this as a point of comparison to a more conventional CGE structure. Generalizations are useful, however, and the application of alternative structures would help extend our understanding.

Our applications do indicate that the additional effort is worthwhile. We see important changes when we incorporate the Melitz (2003) structure of monopolistic competition among heterogeneous firms. Relative to the standard Armington (1969) trade formulation, we see the addition of variety effects and productivity effects. Both of these effects are documented in the empirical trade literature, and we encourage their consideration in computational policy analysis. The initial feature that emerges out of our analysis of subglobal climate action is the

importance of trade diversion. Measured leakage is higher in the Melitz structure, and the trade diversion associated with climate-linked tariffs is larger. Importantly, this is not simply a larger price response; it is a response in the competitive selection of firms. Changes in variety and productivity indicate different margins of impact that we feel are worthwhile exploring in the context of applied policy analysis.

## References

- Anderson, James E., and Eric van Wincoop (2003) 'Gravity with gravitas: A solution to the border puzzle.' *American Economic Review* 93(1), 170–192
- Anderson, James E., and Eric van Wincoop (2004) 'Trade costs.' *Journal of Economic Literature* 42(3), 691–751
- Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare (forthcoming) 'New trade models, same old gains?' *American Economic Review*
- Arkolakis, Costas, Svetlana Demidova, Peter J. Klenow, and Andrés Rodríguez-Clare (2008) 'Endogenous variety and the gains from trade.' *American Economic Review: Papers & Proceedings* 98(2), 444–450
- Armington, Paul S. (1969) 'A theory of demand for products distinguished by place of production.' *Staff Papers-International Monetary Fund* 16(1), 159–178
- Aw, Bee Yan, Xiaomin Chen, and Mark J. Roberts (2001) 'Firm-level evidence on productivity differentials and turnover in taiwanese manufacturing.' *Journal of Development Economics* 66(1), 51 – 86
- Baldwin, Richard E., and Rikard Forslid (2010) 'Trade liberalization with heterogeneous firms.' *Review of Development Economics* 14(2), 161–176

- Balistreri, Edward J., and James R. Markusen (2009) 'Sub-national differentiation and the role of the firm in optimal international pricing.' *Economic Modelling* 26(1), 47–62
- Balistreri, Edward J., and Russell H. Hillberry (2008) 'The gravity model: an illustration of structural estimation as calibration.' *Economic Inquiry* 46(4), 511–527
- Balistreri, Edward J., Russell H. Hillberry, and Thomas F. Rutherford (2010) 'Trade and welfare: Does industrial organization matter?' *Economics Letters* 109(2), 85 – 87
- (2011) 'Structural estimation and solution of international trade models with heterogeneous firms.' *Journal of International Economics* 83(2), 95 – 108
- Bartelsman, Eric J., and Mark Doms (2000) 'Understanding productivity: Lessons from longitudinal microdata.' *Journal of Economic Literature*
- Bernard, Andrew B., and J. Bradford Jensen (1999) 'Exceptional exporter performance: cause, effect, or both?' *Journal of International Economics* 47(1), 1 – 25
- Brown, Drusilla K., Alan V. Deardorff, and Robert M. Stern (1992) 'A north american free trade agreement: Analytical issues and a computational assessment.' *World Economy* 15(1), 11–30
- Burenstam Linder, Staffan (1961) *An Essay on Trade and Transformation* (New York: Wiley)
- Cox, David, and Richard G. Harris (1992) 'North american free trade and its implications for canada: Results from a cge model of north american trade.' *World Economy* 15(1), 31–44
- Dixit, Avinash K., and Joseph E. Stiglitz (1977) 'Monopolistic competition and optimum product diversity.' *The American Economic Review* 67(3), 297–308
- Eaton, Jonathan, and Samuel Kortum (2002) 'Technology, geography, and trade.' *Econometrica* 70(5), 1741–1779

- Feenstra, Robert C. (2010) 'Measuring the gains from trade under monopolistic competition.' *Canadian Journal of Economics* 43(1), 1–28
- Hertel, Thomas W. (1997) *Global Trade Analysis: Modeling and Applications* (Cambridge University Press)
- Hummels, David, and Peter J. Klenow (2005) 'The variety and quality of a nation's exports.' *American Economic Review* 95(3), 704–723
- Hunter, Linda, James R. Markusen, and Thomas F. Rutherford (1992) 'Us-mexico free trade and the north american auto industry: Effects on the spatial organisation of production of finished autos.' *World Economy* 15(1), 65–82
- Krugman, Paul (1980) 'Scale economies, product differentiation, and the pattern of trade.' *The American Economic Review* 70(5), 950–959
- Leontief, Wassily W. (1953) 'Domestic production and foreign trade: The american capital position re-examined.' *Proceedings of the American Philosophical Society* 97, 332–349
- Markusen, James R. (1990) 'Derationalizing tariffs with specialized intermediate inputs and differentiated final goods.' *Journal of International Economics* 28(3-4), 375 – 383
- Mathiesen, Lars (1987) 'An algorithm based on a sequence of linear complementarity problems applied to a Walrasian equilibrium model: An example.' *Mathematical Programming* 37, 1–18
- McDougall, Robert (2005) 'The gtap 6 database.' Technical Report, Purdue University
- Melitz, Marc J. (2003) 'The impact of trade on intra-industry reallocations and aggregate industry productivity.' *Econometrica* 71(6), 1695–1725
- Narayanan, Badri, and Betina Dimaranan (2008) 'The gtap 7 database documentation: Chapter 3.' Technical Report, Purdue University, GTAP Resource Number 4164

- Peters, Glen P. (2008) 'From production-based to consumption-based national emission inventories.' *Ecological Economics* 65(1), 13–23
- Peters, Glen P., and Edgar G. Hertwich (2008) 'CO2 embodied in international trade with implications for global climate policy.' *Environmental Science & Technology* 42(5), 1401–1407
- Rutherford, Thomas (2010a) 'Gtappingams, the model.' *Working paper*
- Rutherford, Thomas F. (1997) 'Gtappingams.' Technical Report, University of Colorado
- (2010b) 'Climate-linked tariffs: Practical issues.' *mimeo*: ETH-Zürich, Switzerland
- Trefler, Daniel (2004) 'The long and short of the Canada-U.S. Free Trade Agreement.' *American Economic Review* 94(4), 870–895
- Wyckoff, Andrew W., and Joseph M. Roop (1994) 'The embodiment of carbon in imports of manufactured products: Implications for international agreements on greenhouse gas emissions.' *Energy Policy* 22(3), 187–194

# A Illustrative Partial Equilibrium Trade Models

## A.1 Armington (1969) based model:

\$Title Armington Trade Equilibrium with Iceberg Costs

\*Edward J. Balistreri, Colorado School of Mines (ebalistr@mines.edu)

\*Thomas F. Rutherford, ETH Zurich (tom@mpsge.org).

\*March 2011

Set

    r          countries or regions /R1,R2,R3/  
    j          goods                  /G1/;

Alias (r,s);

Parameters

    sig          elasticity of substitution          /3/,  
    eta          demand elasticity                  /1.5/,  
    mu          supply elasticity                  /0.5/,  
    Q0(j,r)      benchmark aggregate quantity,  
    P0(j,r)      benchmark price index,  
    c0(j,r)      benchmark input cost,  
    Y0(j,r)      benchmark input supply,  
    tau(j,r,s)   iceberg transport cost factor,  
    vx0(j,r,s)  arbitrary benchmark export values,  
    zeta(j,r,s)  bilateral preference weights  
    ;

P0(j,r) = 1;

c0(j,r) = 1;

vx0(j,r,s) = 1;

vx0(j,r,r) = 3;

Q0(j,r) = sum(s, vx0(j,s,r))/P0(j,r);

Y0(j,r) = sum(s, vx0(j,r,s))/c0(j,r);

\* Assume neutral preference weights and calibrate tau

zeta(j,r,s) = 1;

tau(j,r,s) = (vx0(j,r,s)/(c0(j,r)\*Q0(j,s)))\*(1/(1-sig)) \*  
            (zeta(j,r,s)\*P0(j,s)/(c0(j,r)))\*(sig/(sig-1));

\* Alternatively we could specify tau and calibrate zeta

\*tau(j,r,s) = 1;

\*zeta(j,r,s) = c0(j,r)/P0(j,s) \*

\*            (vx0(j,r,s)/(c0(j,r)\*Q0(j,s)))\*(1/sig) \*

\*            tau(j,r,s)\*((sig-1)/sig);

Display zeta,tau;

Positive Variables

    Q(j,r)      Composite Quantity,  
    P(j,r)      Composite price index,  
    c(j,r)      Composite input price (marginal cost),  
    Y(j,r)      Composite input supply (output);

```

Equations
    DEM(j,r)          Aggregate demand,
    ARM(j,r)          Armington unit cost function,
    MKT(j,r)          Market clearance,
    SUP(j,r)          Input supply (output);

DEM(j,r)..  Q(j,r) - Q0(j,r)*(P0(j,r)/P(j,r))**eta =g= 0;

ARM(j,s)..  sum(r,zeta(j,r,s)**(sig) *(tau(j,r,s)*c(j,r))**(1-sig)
             )**(1/(1-sig)) -
    P(j,s) =g= 0;

MKT(j,r)..  Y(j,r) -
             sum(s,tau(j,r,s)*Q(j,s)*
                 (zeta(j,r,s)*P(j,s)/(tau(j,r,s)*c(j,r)))**(sig)
             ) =g= 0;

SUP(j,r)..  Y0(j,r)*(c(j,r)/c0(j,r))**mu - Y(j,r) =g= 0;

model A_1 /DEM.P,ARM.Q,MKT.c,SUP.Y/;

*Set the level values and check for benchmark consistency
Q.l(j,r) = Q0(j,r);
P.l(j,r) = P0(j,r);
c.l(j,r) = c0(j,r);
Y.l(j,r) = Y0(j,r);

A_1.iterlim = 0;
Solve A_1 using MCP;
Abort$(A_1.objval > 1e-6) "Benchmark Replication Failed";

```

## A.2 Krugman (1980) based model:

\$Title Krugman Trade Equilibrium with Iceberg Costs

\*Edward J. Balistreri, Colorado School of Mines (ebalistr@mines.edu)  
\*Thomas F. Rutherford, ETH Z\"{u}rich (tom@mpsge.org).  
\*March 2011

Set

```

    r          countries or regions /R1,R2,R3/
    k          goods                /G1/;
Alias (r,s);

```

Parameters

```

    sig          elasticity of substitution          /3/,
    eta          demand elasticity                  /1.5/,
    mu           supply elasticity                   /0.5/,
    Q0(k,r)     benchmark aggregate quantity,
    P0(k,r)     benchmark price index,

```

```

NO(k,r)      benchmark number of firms,
qf0(k,r,s)   benchmark firm-level quantity,
pf0(k,r,s)   benchmark firm-level pricing (gross of tau),
c0(k,r)      benchmark input cost,
Y0(k,r)      benchmark input supply,
fc(k,r)      fixed costs,
tau(k,r,s)   iceberg transport cost factor,
vx0(k,r,s)   arbitrary benchmark export values
;
c0(k,r)      = 1;
vx0(k,r,s)   = 1;
vx0(k,r,r)   = 3;
Y0(k,r)      = sum(s, vx0(k,r,s))/c0(k,r);
NO(k,r)      = 10;
*           Calibrate the fixed cost based on zero profit
fc(k,r)      = sum(s, vx0(k,r,s))/(sig*NO(k,r)*c0(k,r));
PO(k,r)      = 1;
Q0(k,r)      = sum(s, vx0(k,s,r))/PO(k,r);
pf0(k,r,s)   = (vx0(k,r,s)/(NO(k,r)*Q0(k,s)))*(1/(1-sig));
qf0(k,r,s)   = Q0(k,s)*pf0(k,r,s)*(-sig);
tau(k,r,s)   = (1-1/sig)*pf0(k,r,s)/c0(k,r);
display tau;

```

#### Positive Variables

```

Q(k,r)      Composite Quantity,
P(k,r)      Composite price index,
N(k,r)      Number of firms (varieties)
QF(k,r,s)   Firm-level output in s-market
PF(k,r,s)   Firm-level (gross) pricing in s-market
c(k,r)      Composite input price (marginal cost),
Y(k,r)      Composite input supply (output);

```

#### Equations

```

DEM(k,r)    Aggregate demand,
DS(k,r)     Dixit-Stiglitz price index,
FE(k,r)     Free entry,
DEMF(k,r,s) Firm demand,
MKUP(k,r,s) Optimal firm pricing,
MKT(k,r)    Input market clearance,
SUP(k,r)    Input supply (output);

```

$$\text{DEM}(k,r) \dots Q(k,r) - Q0(k,r) * (P0(k,r)/P(k,r))^{**\text{eta}} =g= 0;$$

$$\text{DS}(k,s) \dots \text{sum}(r, N(k,r) * \text{PF}(k,r,s) ** (1-\text{sig})) ** (1/(1-\text{sig})) - P(k,s) =g= 0;$$

$$\text{FE}(k,r) \dots c(k,r) * \text{fc}(k,r) - \text{sum}(s, \text{PF}(k,r,s) * \text{QF}(k,r,s) / \text{sig}) =g= 0;$$

$$\text{DEMF}(k,r,s) \dots \text{QF}(k,r,s) - Q(k,s) * (P(k,s) / \text{PF}(k,r,s)) ** \text{sig} =g= 0;$$

$$\text{MKUP}(k,r,s) \dots \text{tau}(k,r,s) * c(k,r) - (1 - 1/\text{sig}) * \text{PF}(k,r,s) =g= 0;$$

```

MKT(k,r).. Y(k,r) -
           N(k,r)*(fc(k,r) + sum(s,tau(k,r,s)*QF(k,r,s)))
           =g= 0;

SUP(k,r).. Y0(k,r)*(c(k,r)/c0(k,r))**mu - Y(k,r) =g= 0;

model A_2 /DEM.P,DS.Q,FE.N,DEMF.PF,MKUP.QF,MKT.c,SUP.Y/;

*Set the level values and check for benchmark consistency
Q.l(k,r)   =Q0(k,r)   ;
P.l(k,r)   =P0(k,r)   ;
N.l(k,r)   =N0(k,r)   ;
QF.l(k,r,s)=QF0(k,r,s);
PF.l(k,r,s)=PF0(k,r,s);
c.l(k,r)   =c0(k,r)   ;
Y.l(k,r)   =Y0(k,r)   ;

A_2.iterlim = 0;
Solve A_2 using MCP;
Abort$(A_2.objval > 1e-6) "Benchmark Replication Failed";

```

### A.3 Melitz (2003) based model:

\$Title Melitz Trade Equilibrium with Iceberg Costs

\*Edward J. Balistreri, Colorado School of Mines (ebalistr@mines.edu)  
\*Thomas F. Rutherford, ETH Z\"{u}rich (tom@mpsge.org).  
\*March 2011

Set

```

      r      countries or regions /R1,R2,R3/
      h      goods                /G1/;
Alias (r,s);

```

Parameters

```

      sig      elasticity of substitution      /3.8/,
      eta      demand elasticity              /2/,
      mu       supply elasticity              /0.5/,
      a        Pareto shape parameter         /4.6/,
      b        Pareto lower support           /0.5/,
      Q0(h,r)  benchmark aggregate quantity,
      P0(h,r)  benchmark price index,
      M0(h,r)  benchmark number of entered firms,
      N0(h,r,s) benchmark number of operating firms,
      qf0(h,r,s) benchmark avg firm-level quantity,
      pf0(h,r,s) benchmark avg firm-level pricing (gross),
      phi0(h,r,s) benchmark avg productivity
      c0(h,r)  benchmark input cost,
      Y0(h,r)  benchmark input supply,
      fc(h,r,s) bilateral fixed costs,

```

```

delt_fs(h,r) annualized sunk cost,
tau(h,r,s) iceberg transport cost factor,
vx0(h,r,s) arbitrary benchmark export values
;

c0(h,r) = 1;
vx0(h,r,s) = 1;
vx0(h,r,r) = 3;
Y0(h,r) = sum(s, vx0(h,r,s))/c0(h,r);
M0(h,r) = 10;
N0(h,r,r) = 9;
N0(h,r,s) = (vx0(h,r,s)/vx0(h,r,r))**2 * N0(h,r,r);
* Calibrate the sunk cost based on free entry
delt_fs(h,r)= Y0(h,r)/M0(h,r) * (sig-1)/(a*sig);
* Calibrate the fixed cost based on zero cutoff profit
fc(h,r,s) = vx0(h,r,s)/(N0(h,r,s)*c0(h,r)) * (a + 1 - sig)/(a*sig);
P0(h,r) = 1;
Q0(h,r) = sum(s, vx0(h,s,r))/P0(h,r);
pf0(h,r,s) = (vx0(h,r,s)/(N0(h,r,s)*Q0(h,s)))**(1/(1-sig));
qf0(h,r,s) = Q0(h,s)*pf0(h,r,s)**(-sig);
phi0(h,r,s)= b * (a/(a+1-sig))**(1/(sig-1)) *
(N0(h,r,s)/M0(h,r))**(-1/a);
tau(h,r,s) = (1-1/sig)*pf0(h,r,s)*phi0(h,r,s)/c0(h,r);
display N0,tau;

```

#### Positive Variables

|            |  |
|------------|--|
| Q(h,r)     | Composite Quantity,                    |
| P(h,r)     | Composite price index,                 |
| M(h,r)     | Number of Entered firms                |
| N(h,r,s)   | Number of Operating firms (varieties)  |
| QF(h,r,s)  | Avg Firm output in s-market            |
| PF(h,r,s)  | Avg Firm (gross) pricing in s-market   |
| PHI(h,r,s) | Avg Firm productivity                  |
| c(h,r)     | Composite input price (marginal cost), |
| Y(h,r)     | Composite input supply (output);       |

#### Equations

|             |                             |
|-------------|-----------------------------|
| DEM(h,r)    | Aggregate demand,           |
| DS(h,r)     | Dixit-Stiglitz price index, |
| FE(h,r)     | Free entry,                 |
| ZCP(h,r,s)  | Zero cutoff profits         |
| DEMF(h,r,s) | Firm demand,                |
| MKUP(h,r,s) | Optimal firm pricing,       |
| PAR(h,r,s)  | Pareto Productivity         |
| MKT(h,r)    | Input market clearance,     |
| SUP(h,r)    | Input supply (output);      |

DEM(h,r).. Q(h,r) - Q0(h,r)\*(P0(h,r)/P(h,r))\*\*eta =g= 0;

DS(h,s).. sum(r,N(h,r,s)\*PF(h,r,s)\*\*(1-sig))\*\*(1/(1-sig)) -

```

P(h,s) =g= 0;

DEMF(h,r,s).. QF(h,r,s) - Q(h,s)*(P(h,s)/PF(h,r,s))**sig =g= 0;

MKUP(h,r,s).. tau(h,r,s)*c(h,r)/PHI(h,r,s) -
(1 - 1/sig)*PF(h,r,s) =g= 0;

FE(h,r).. c(h,r)*delt_fs(h,r) -
sum(s,(N(h,r,s)/M(h,r))*PF(h,r,s)*QF(h,r,s)*(sig-1)/(a*sig))
=g= 0;

ZCP(h,r,s).. c(h,r)*fc(h,r,s) -
(PF(h,r,s)*QF(h,r,s)*(a+1-sig))/(a*sig) =g= 0;

PAR(h,r,s).. PHI(h,r,s) -
b * (a/(a+1-sig))**(1/(sig-1)) * (N(h,r,s)/M(h,r))**(-1/a)
=g= 0;

MKT(h,r).. Y(h,r) - (
delt_fs(h,r)*M(h,r) +
sum(s,N(h,r,s)*(fc(h,r,s) + tau(h,r,s)*QF(h,r,s)/PHI(h,r,s)))
) =g= 0;

SUP(h,r).. YO(h,r)*(c(h,r)/c0(h,r))**mu - Y(h,r) =g= 0;

model A_3 /DEM.P,DS.Q,FE.M,ZCP.N,DEMF.PF,MKUP.QF,PAR.PHI,MKT.c,SUP.Y/;

*Set the level values and check for benchmark consistency
Q.l(h,r) =Q0(h,r) ;
P.l(h,r) =P0(h,r) ;
M.l(h,r) =M0(h,r) ;
N.l(h,r,s) =N0(h,r,s) ;
QF.l(h,r,s)=QF0(h,r,s);
PF.l(h,r,s)=PF0(h,r,s);
PHI.l(h,r,s)=PHI0(h,r,s);
c.l(h,r) =c0(h,r) ;
Y.l(h,r) =Y0(h,r) ;

A_3.iterlim = 0;
Solve A_3 using MCP;
Abort$(A_3.objval > 1e-6) "Benchmark Replication Failed";

```

## B Illustrative General Equilibrium Trade Model

\$Title Mix and Match General Equilibrium with Iceberg Costs

\*Edward J. Balistreri, Colorado School of Mines (ebalistr@mines.edu)

\*Thomas F. Rutherford, ETH Z\"{u}rich (tom@mpsge.org).

\*March 2011

\* This formulation allows for Armington, Krugman, or Melitz

\* trade depending on the user defined subset j(i),k(i),or h(i).  
Option seed = 81567;

Set

```

r      countries or regions  /R1*R3/
f      factors of production /L1*L3/,

i      goods                /G1,G2,G3/,
j(i)   Armington goods     /G1/,
k(i)   Krugman goods       /G2/,
h(i)   Melitz goods        /G3/;
```

Alias (r,s),(f,g);

Parameters

```

alpha   top level elasticity of substitution  /2.0/,
a       Pareto shape parameter              /4.6/,
b       Pareto lower support                /0.5/,
sig_j   industry elasticity of substitution (a+1) /5.6/,
sig_k   industry elasticity of substitution (a+1) /5.6/,
sig_h   industry elasticity of substitution    /3.8/,
vx0(i,r,s) arbitrary benchmark export values
beta(i,r) expenditure weights
gamma(i,f,r) primary factor value shares
lbar(f,r) primary factor supply
ra0(r)   benchmark income
c0(i,r)  benchmark input cost,
y0(i,r)  benchmark input supply,
q0(i,r)  benchmark aggregate quantity,
p0(i,r)  benchmark price index,
m0(h,r)  benchmark number of entered firms,
n0(h,r,s) benchmark number of operating firms,
qf0(i,r,s) benchmark avg firm-level quantity,
pf0(i,r,s) benchmark avg firm-level pricing (gross),
phi0(i,r,s) benchmark avg productivity
fc(h,r,s) bilateral fixed costs,
delt_fs(h,r) annualized sunk cost,
nk0(k,r) benchmark number of (krugman) firms
fck(k,r) krugman fixed costs,
tau(i,r,s) iceberg transport cost factor,
zeta(i,r,s) preference weight parameter
;
```

\* Finite variance restriction

Abort\$(a le (sig\_h - 1))

"Firm size distribution must have a finite variance. a > sig-1";

\* Setup the benchmark with arbitrary data

vx0(i,r,s) = 1;

vx0(i,r,r) = 3;

\* Unit choice

```

c0(i,r)    = 1;
p0(i,r)    = 2;
y0(i,r)    = sum(s, vx0(i,r,s))/c0(i,r);
q0(i,r)    = sum(s, vx0(i,s,r))/p0(i,r);
ra0(r)     = sum(i,q0(i,r)*p0(i,r));
beta(i,r)  = p0(i,r) * (q0(i,r)/RA0(r))**(1/alpha);

*      randomly distribute the factor shares
gamma(i,f,r)=0;
loop(f$(ord(f) ne card(f)),
      gamma(i,f,r) = uniform(0,(1-sum(g,gamma(i,g,r))));
);
gamma(i,f,r)$(ord(f) eq card(f)) = 1-sum(g,gamma(i,g,r));
lbar(f,r)  = sum(i,gamma(i,f,r)*y0(i,r)*c0(i,r));

*---- Melitz model Calibration ----*
M0(h,r)    = 10;
N0(h,r,r)  = 9;
N0(h,r,s)  = (vx0(h,r,s)/vx0(h,r,r))**2 * N0(h,r,r);
*      Calibrate the sunk cost based on free entry
delt_fs(h,r)= y0(h,r)/M0(h,r) * (sig_h-1)/(a*sig_h);
*      Calibrate the fixed cost based on zero cutoff profit
fc(h,r,s)  = vx0(h,r,s)/(N0(h,r,s)*c0(h,r)) * (a + 1 - sig_h)/(a*sig_h);
pf0(h,r,s) = (vx0(h,r,s)/(N0(h,r,s)*q0(h,s)*p0(h,s)))**(1/(1-sig_h));
qf0(h,r,s) = (p0(h,s)*q0(h,s))*pf0(h,r,s)**(-sig_h);
zeta(h,r,s)= p0(h,s)**(1-sig_h);
phi0(h,r,s)= b * (a/(a+1-sig_h))**(1/(sig_h-1)) *
              (N0(h,r,s)/M0(h,r))**(-1/a);
*      Calibrated tau for Melitz sectors
tau(h,r,s) = (1-1/sig_h)*pf0(h,r,s)*phi0(h,r,s)/c0(h,r);
*-----*

*---- Krugman model Calibration----*
NK0(k,r)   = 10;
fcK(k,r)   = sum(s, vx0(k,r,s))/(sig_k*NK0(k,r)*c0(k,r));
pf0(k,r,s) = (vx0(k,r,s)/(NK0(k,r)*q0(k,s)*p0(k,s)))**(1/(1-sig_k));
qf0(k,r,s) = (p0(k,s)*q0(k,s))*pf0(k,r,s)**(-sig_k);
zeta(k,r,s)= p0(k,s)**(1-sig_k);
tau(k,r,s) = (1-1/sig_k)*pf0(k,r,s)/c0(k,r);
*-----*

*---- Armington model Calibration---*
zeta(j,r,s)= 1;
tau(j,r,s) = (vx0(j,r,s)/(c0(j,r)*q0(j,s)))**(1/(1-sig_j)) *
              (zeta(j,r,s)*p0(j,s)/(c0(j,r)))*(sig_j/(sig_j-1));
*-----*

```

```
display tau;
```

```
Positive Variables
```

```
U(r)           Welfare
```

|            |  |
|------------|--|
| E(r)       | True cost of living index              |
| Q(i,r)     | Composite Quantity,                    |
| P(i,r)     | Composite price index,                 |
| M(h,r)     | Number of Entered firms                |
| N(h,r,s)   | Number of Operating firms (varieties)  |
| NK(k,r)    | Number of Krugman firms (varieties)    |
| QF(*,r,s)  | Avg Firm output in s-market            |
| PF(*,r,s)  | Avg Firm (gross) pricing in s-market   |
| PHI(h,r,s) | Avg Firm productivity                  |
| c(i,r)     | Composite input price (marginal cost), |
| Y(i,r)     | Composite input supply (output)        |
| w(f,r)     | Primary factor price                   |
| RA(r)      | Income;                                |

#### Equations

|              |                            |
|--------------|----------------------------|
| EXPFUN(r)    | Unit expenditure function, |
| DEM(i,r)     | Aggregate demand,          |
| PRC_h(h,r)   | Price index,               |
| PRC_k(k,r)   | Price index,               |
| PRC_j(j,r)   | Price index,               |
| FE(h,r)      | Free entry,                |
| FEK(k,r)     | Free entry,                |
| ZCP(h,r,s)   | Zero cutoff profits        |
| DEMF(h,r,s)  | Firm demand,               |
| DEMFK(k,r,s) | Firm demand,               |
| MKUP(h,r,s)  | Optimal firm pricing,      |
| MKUPK(k,r,s) | Optimal firm pricing,      |
| PAR(h,r,s)   | Pareto Productivity        |
| MKT_h(h,r)   | Input market clearance,    |
| MKT_k(k,r)   | Input market clearance,    |
| MKT_j(j,r)   | Input market clearance,    |
| COST(i,r)    | Unit cost functions        |
| LMKT(f,r)    | Primary factor markets     |
| FINAL(r)     | Final demand               |
| BC(r)        | Budget constraint;         |

EXPFUN(r) ..

$$\begin{aligned} & (\text{sum}(i, \text{beta}(i,r)**\alpha * P(i,r)**(1-\alpha))**(1/(1-\alpha)))\$(\alpha \neq 1) \\ & + \text{prod}(i, (P(i,r)/p0(i,r))**\text{beta}(i,r))\$(\alpha \text{ eq } 1) \\ & - E(r) =g= 0; \end{aligned}$$

DEM(i,r) ..

$$Q(i,r) - q0(i,r)*U(r)*(p0(i,r)*E(r)/P(i,r))**\alpha =g= 0;$$

PRC\_h(h,s) ..

$$\begin{aligned} & \text{sum}(r, \text{zeta}(h,r,s)*N(h,r,s)*PF(h,r,s)**(1-\text{sig}_h))**(1/(1-\text{sig}_h)) - \\ & P(h,s) =g= 0; \end{aligned}$$

PRC\_k(k,s) ..

$$\begin{aligned} & \text{sum}(r, \text{zeta}(k,r,s)*NK(k,r)*PF(k,r,s)**(1-\text{sig}_k))**(1/(1-\text{sig}_k)) - \\ & P(k,s) =g= 0; \end{aligned}$$

```

PRC_j(j,s)..
    sum(r,zeta(j,r,s)**(sig_j) *(tau(j,r,s)*c(j,r))**(1-sig_j)
        )**(1/(1-sig_j)) - P(j,s) =g= 0;

DEMF(h,r,s)..
    QF(h,r,s) - zeta(h,r,s)*Q(h,s)*(P(h,s)/PF(h,r,s))**sig_h =g= 0;

DEMPK(k,r,s)..
    QF(k,r,s) - zeta(k,r,s)*Q(k,s)*(P(k,s)/PF(k,r,s))**sig_k =g= 0;

MKUP(h,r,s)..
    tau(h,r,s)*c(h,r)/PHI(h,r,s) - (1 - 1/sig_h)*PF(h,r,s) =g= 0;

MKUPK(k,r,s)..
    tau(k,r,s)*c(k,r) - (1 - 1/sig_k)*PF(k,r,s) =g= 0;

FE(h,r)..
    c(h,r)*delt_fs(h,r) -
    sum(s,(N(h,r,s)/M(h,r))*PF(h,r,s)*QF(h,r,s)*(sig_h-1)/(a*sig_h))
    =g= 0;

FEK(k,r)..
    c(k,r)*fcK(k,r) - sum(s,PF(k,r,s)*QF(k,r,s)/sig_k) =g= 0;

ZCP(h,r,s)..
    c(h,r)*fc(h,r,s) - (PF(h,r,s)*QF(h,r,s)*(a+1-sig_h))/(a*sig_h)
    =g= 0;

PAR(h,r,s)..
    PHI(h,r,s) -
    b * (a/(a+1-sig_h))**(1/(sig_h-1)) * (N(h,r,s)/M(h,r))**(-1/a)
    =g= 0;

MKT_j(j,r)..
    Y(j,r) -
    sum(s,tau(j,r,s)*Q(j,s)*
        (zeta(j,r,s)*P(j,s)/(tau(j,r,s)*c(j,r))**(sig_j)
        ) =g= 0;

MKT_k(k,r)..
    Y(k,r) -
    NK(k,r)*(fcK(k,r) + sum(s,tau(k,r,s)*QF(k,r,s))) =g= 0;

MKT_h(h,r)..
    Y(h,r) - (delt_fs(h,r)*M(h,r) +
    sum(s,N(h,r,s)*(fc(h,r,s) + tau(h,r,s)*QF(h,r,s)/PHI(h,r,s)))
    ) =g= 0;

COST(i,r)..
    c(i,r) - c0(i,r)*prod(f,w(f,r)**gamma(i,f,r)) =g=0;

```

```
LMKT(f,r)..
  lbar(f,r) - sum(i,gamma(i,f,r)*Y(i,r)*c(i,r)/w(f,r)) =g= 0;
```

```
FINAL(r)..  RAO(r)*U(r)*E(r) - RA(r) =g= 0;
```

```
BC(r)..      RA(r) =e= sum(f,w(f,r)*lbar(f,r));
```

```
Model b_1 /
*          GE          Armington      Krugman      Melitz
*          -----      -----      -----      -----
          expfun.U,
          DEM.P,
                PRC_j.Q,      PRC_k.Q,      PRC_h.Q,
                FEK.NK,      FE.M,
                DEMFK.PF,      DEMF.PF,
                MKUPK.QF,      MKUP.QF,
                PAR.PHI,
                MKT_j.c,      MKT_k.c,      MKT_h.c,
          COST.Y,
          LMKT.w
          FINAL.E,
          BC.RA/;
```

```
*Set the level values and check for benchmark consistency
```

```
Q.l(i,r)   = q0(i,r)   ;
P.l(i,r)   = p0(i,r)   ;
M.l(h,r)   = M0(h,r)   ;
N.l(h,r,s) = N0(h,r,s) ;
NK.l(k,r)  = NK0(k,r)  ;
QF.l(h,r,s) = QF0(h,r,s);
PF.l(h,r,s) = PF0(h,r,s);
QF.l(k,r,s) = QF0(k,r,s);
PF.l(k,r,s) = PF0(k,r,s);
PHI.l(h,r,s)= PHIO(h,r,s);
c.l(i,r)   = c0(i,r)   ;
Y.l(i,r)   = y0(i,r)   ;
w.l(f,r)   = 1         ;
U.l(r)     = 1         ;
E.l(r)     = 1         ;
RA.l(r)    = RAO(r)    ;
```

```
b_1.iterlim = 0;
```

```
Solve b_1 using MCP;
```

```
Abort$(b_1.objval > 1e-6) "Benchmark Replication Failed";
```

## C Description of the GTAP 7 based CGE Model

### C.1 Background

The Global Trade Analysis Project (GTAP) is a research program initiated in 1992 to provide the economic research community with a global economic dataset and base CGE model for use in the quantitative analyses of international economic issues. The project's objectives include the provision of a documented, publicly available, global, general equilibrium data base, and to conduct seminars on a regular basis to inform the research community about how to use the data in applied economic analysis. The GTAP version 7.1 database, released in May, 2010, represents global production and trade for 113 country/regions, 57 commodities and 5 primary factors. The data characterize intermediate demand and bilateral trade in 2004, including tax rates on imports and exports and other indirect taxes.<sup>10</sup> The core GTAP data represent a static, multi-regional set of accounts which track the production and distribution of goods in the global economy. In GTAP the world is divided into regions (typically representing individual countries), and each region's final demand structure is composed of public and private expenditure across goods.<sup>11</sup>

We use the GTAP data to calibrate a multiregion CGE model within the GAMS programming language. The model includes the option of structuring trade (for non-energy commodities) consistent with the Melitz (2003) theory of heterogeneous firms. Apart from the Melitz trade structure for specific sectors the model is consistent with standard "GTAPinGAMS" formulations [see Rutherford (2010a) and Rutherford (1997)]. The model is based on optimizing behavior. Consumers maximize welfare subject to a budget constraint with fixed levels of investment and public output. Producers combine intermediate inputs, and primary factors at least cost

---

<sup>10</sup>A guide to what's new in GTAP7 can be found in Narayanan and Dimaranan (2008).

<sup>11</sup>For additional background on GTAP consult the GTAP book, Hertel (1997), and the GTAP version 6 documentation, McDougall (2005). A list of applications based on the GTAP framework can be found at the GTAP home page, <http://www.gtap.org>.

subject to the given technology. The dataset includes a full set of bilateral trade flows with associated transport costs, export taxes and tariffs.

## C.2 Benchmark Data and Accounting Identities

The economic structure underlying the GTAP dataset is illustrated in Figure 12. Symbols in this flow chart correspond to variables in the economic model.  $Y_{ir}$  portrays the production of good  $i$  in region  $r$ ,  $C_r$ ,  $I_r$  and  $G_r$  portray private consumption, investment and public demand, respectively.  $M_{jr}$  portrays the import of good  $j$  into region  $r$ .  $HH_r$  and  $GOVT_r$  stand for representative household and government consumers.

In this figure commodity and factor market flows appear as solid lines. Domestic and imported goods markets are represented by horizontal lines at the top of the figure. Domestic production ( $vom_{ir}$ ) is distributed to exports ( $vxml_{irs}$ ), international transportation services ( $vst_{ir}$ ), intermediate demand ( $vd_{fijr}$ ), household consumption ( $vd_{pm_{ir}}$ ), investment ( $vd_{ipm_{ir}}$ ) and government consumption ( $vd_{gm_{ir}}$ ). The accounting identity on the output side:

$$\underbrace{vom_{ir}}_{\text{Value of Output}} = \sum_s \underbrace{vxml_{irs}}_{\text{Bilateral exports}} + \underbrace{vst_{ir}}_{\text{Transport exports}} + \sum_j \underbrace{vd_{fijr}}_{\text{Intermediate Demand}} + \underbrace{vd_{pm_{ir}} + vd_{ipm_{ir}} + vd_{gm_{ir}}}_{\text{Final Demand (C + I + G)}}$$

The value of output is in turn related to the cost of intermediate inputs, value-added, and tax revenue:

$$\underbrace{vom_{ir}}_{\text{Value of Output}} = \underbrace{\sum_j (vifm_{jir} + vdfm_{jir})}_{\text{Intermediate Inputs}} + \underbrace{\sum_f vfm_{fir}}_{\text{Factor Earnings}} + \underbrace{\mathcal{R}_{ir}^Y}_{\text{Tax Revenue}} \quad (55)$$

Imported goods which have an aggregate value of  $vim_{ir}$  enter intermediate demand ( $vifm_{jir}$ ), private consumption ( $vipm_{ir}$ ) and public consumption ( $vigm_{ir}$ ). The accounting identity on



the output side for these flows is thus:

$$\underbrace{vim_{ir}}_{\text{Value of Imports}} = \underbrace{\sum_j vifm_{ijr}}_{\text{Intermediate Demand}} + \underbrace{vipm_{ir} + vigm_{ir}}_{\text{Final Demand (C+G)}}$$

and the accounting identity relating the value of imports to the cost of associated inputs is:

$$\underbrace{vim_{ir}}_{\text{CIF Value of Imports}} = \underbrace{\sum_s vxmd_{isr} + \sum_j vtwr_{jisr}}_{\text{FOB Exports + Transport Cost}} + \underbrace{\mathcal{R}_{ir}^M}_{\text{Tariffs Net Subsidies}} \quad (56)$$

Part of the cost of imports includes the cost of international transportation services,  $vtwr$ . These services are provided with inputs from regions throughout the world, and the supply demand balance in the market for transportation service  $j$  requires that the sum across all regions of service exports ( $vst_{jr}$ , at the top of the figure) equals the sum across all bilateral trade flows of service inputs ( $vtwr_{jisr}$  at the bottom of the figure):

$$\underbrace{\sum_r vst_{jr}}_{\text{Service } j \text{ Exports}} = \underbrace{\sum_{isr} vtwr_{jisr}}_{\text{Transport Demand for } j} \quad (57)$$

To facilitate the heterogeneous firms formulation we explicitly represent a single Armington aggregation for each commodity in each region. This is slightly different from the standard GTAPinGAMS formulation, which accumulates imported and domestic goods within the final demand and production activities. To hold the value of the Armington composite in a single coefficient let

$$\underbrace{vafm_{ir}}_{\text{Armington Aggregate}} = \underbrace{\sum_j (vifm_{ijr} + vdfm_{ijr})}_{\text{Intermediate Inputs}} + \underbrace{vdpm_{ir} + vdim_{ir} + vdg m_{ir} + vipm_{ir} + vigm_{ir}}_{\text{Final Demand (C + I + G)}} \quad (58)$$

Carbon emissions associated with fossil fuels are represented in the GTAP database through a satellite data table ( $eco2_{igr}$ ) constructed on the basis of energy balances from the International Energy Agency. These emissions are proportional to fossil fuel use (commodities OIL, GAS, and COL). Given detailed emissions associated with fossil fuel use, we can calculate direct carbon emissions associated with the production of good  $g$  in region  $r$  as:

$$\underbrace{co2e_{gr}}_{\text{Aggregate Carbon}} = \sum_i \underbrace{eco2_{igr}}_{\text{Sum of Carbon in Fuel Inputs}}$$

where  $eco2_{igr}$  is the IEA-based statistics describing carbon emissions associated with the input of fuel  $i$  in the production of good  $g$  in region  $r$ .

### C.3 The General Equilibrium Model

Variables which define a general equilibrium model based on GTAP 7.1 are summarized in the Tables 11 – 13. Table 11 defines the various dimensions which characterize an instance of the model, including the set of sectors/commodities, the set of regions, the set of factors of production. Set  $g$  combines the production sectors  $i$  and private and public consumption demand (indices "c" and "g") and investment demand (index "i"). Tables 12, 13, and 14 display the concordance between the variables and their GAMS equivalents.

Table 12 defines the primal variables (activity levels) which define an equilibrium. The model determines values of all the variables except international capital flows, a parameter which would be determined endogenously in an intertemporal model. Table 13 defines the relative price variables for goods and factors in the model. As is the case in any Shoven-Whalley model, the equilibrium conditions determine *relative* rather than *nominal* prices. One market equilibrium condition corresponds to each of the equilibrium prices. The heterogeneous firms variables and their GAMS correspondence are presented in Table 14. The heterogeneous firms

Table 11: Set Indices

|        |   |
|--------|---|
| $i, j$ | Sectors and goods, an aggregation of the 55 sectors in the GTAP 7 database. The subset index $h$ designates sectors and goods conforming to the Melitz (2003) heterogeneous firms theory.                           |
| $g$    | The union of produced goods $i$ , private consumption "c", public demand "g" and investment "i"   |
| $r, s$ | Regions, an aggregation of the 113 regions in the GTAP 7 database   |
| $f$    | Factors of production (consisting of <i>mobile factors</i> , $f \in m$ , skilled labor, unskilled labor and capital, and specific factors corresponding to crude oil, natural gas and coal resources) <sup>12</sup> |

Table 12: General Equilibrium Activity Levels

| Var      | Description                  | GAMS Variable | Bmk value     |
|----------|------------------------------|---------------|---------------|
| $Y_{ir}$ | Production                   | $Y(i, r)$     | $vom(i, r)$   |
| $C_r$    | Aggregate consumption D      | $Y("c", r)$   | $vom("c", r)$ |
| $G_r$    | Aggregate public D           | $Y("g", r)$   | $vom("g", r)$ |
| $I_r$    | Aggregate investment D       | $Y("i", r)$   | $vom("i", r)$ |
| $Q_{ir}$ | Aggregate Armington activity | $ARM(i, r)$   | $vafm(i, r)$  |
| $YT_j$   | Intl. transp. services       | $YT(j)$       | $vtw(j)$      |

Table 13: General Equilibrium Prices

| Var        | Description   | GAMS Variable | Bmk value |
|------------|---|---------------|-----------|
| $p_r^C$    | Consumer price index  | P("c", r)     | 1         |
| $p_r^G$    | Public provision price index                                      | P("g", r)     | 1         |
| $p_{ir}^I$ | Investment price index  | P("i", r)     | 1         |
| $c_{ir}$   | Supply price, unit cost of output                                 | P(i, r)       | 1         |
| $P_{ir}$   | Armington price index   | PA(i, r)      | 1         |
| $p_j^T$    | Marginal cost of transport services                               | PT(j)         | 1         |
| $p_{fr}^F$ | Factor prices for labor, land and resources                       | PF(f, r)      | 1         |
| $p_{ir}^S$ | Price of the sector-specific primary factor for CRU, GAS and COL. | PS(i, r)      | 1         |

Table 14: Variables Associated with Heterogeneous-firms Goods

| Var                     | Description  | GAMS Variable |
|-------------------------|--|---------------|
| $\tilde{r}_{hrs}$       | Firm-level Revenues ( $\tilde{p}_{hrs}\tilde{q}_{hrs}$ ) | RFT(h, r, s)  |
| $N_{hrs}$               | Number of operating firms                                | NN(h, r, s)   |
| $M_{hr}$                | Total number of entered firms                            | MM(h, r)      |
| $\tilde{\varphi}_{hrs}$ | Firm-level Productivity                                  | PHIT(h, r, s) |
| $\tilde{p}_{hrs}$       | Firm-level Price   | PFT(h, r, s)  |

model and the decomposition method that we use to solve the model are discussed extensively in the text. We proceed here with a documentation of the other parts of the model.

Our model departs from the conventional GTAP framework with the explicit representation of energy demand and supply elasticities. Thus, while the basic equilibrium conditions (market clearance, zero-profit and income balance) are more or less identical to the GTAP7inGAMS model Rutherford (2010a), there are several differences in the nesting structure of sectoral production and private consumption where explicit substitution between energy and non-energy composites has been introduced. The energy goods included in the model include:

|     |                      |
|-----|----------------------|
| CRU | Crude oil            |
| OIL | Refined oil products |
| COL | Coal                 |
| GAS | Gas                  |
| ELE | Electricity          |

Two of these are *secondary* energy goods (refined oil and electricity), both of which are produced subject to constant returns to scale with inputs of capital, labor, energy and materials. Oil products are refined from crude, and electricity is produced with inputs of coal, natural gas and oil. Variations in dispatch of different generating units are approximated through a Cobb-Douglas aggregation of gas, coal and oil inputs.

Primary factors in the model correspond to skilled and unskilled labor, capital and energy resources. Capital and labor are intersectorally mobile whereas crude oil, gas and coal resources are sector-specific. Given specific factors, the primary fossil fuels, crude oil, coal and natural gas, are produced subject to decreasing returns to scale. Given resource rental shares ( $\theta_{ir}$ ) from the database, the elasticity of substitution between resources and other inputs to primary energy production are calibrated to match assumed price elasticities of supply, denoted  $\epsilon_i$ , for

these three fossil fuels. The calibrated substitution elasticities are given by:

$$\sigma_{ir} = \epsilon_i \frac{1 - \theta_{ir}}{\theta_{ir}},$$

where we assume the following elasticities of supply:  $\epsilon_{\text{COL}} = 1$ ,  $\epsilon_{\text{CRU}} = 0.5$ , and  $\epsilon_{\text{GAS}} = 0.25$ .

Our equilibrium framework is based on the assumption of optimizing atomistic agents, and applies for both producers and consumers. Each sector is assumed to minimize unit cost subject to technical constraints. For any sector  $Y_{ir}$  we characterize input choices as though they arose from minimization of unit production costs.

Underlying production function are represented by a nested constant-elasticity-of-substitution (CES) form in which the top-level substitution describes energy demand and a Cobb-Douglas aggregate describes trade-offs between electricity, natural gas, oil and coal. Non-energy intermediates enter as fixed-coefficients (Leontief) nest with capital-labor value-added composite in which capital, skilled and unskilled labor are substitutable with elasticity  $\sigma_g^{KL}$ .

Bilateral trade flows are either determined by an Armington or Melitz structure as described in the text. To maintain proximity with the GTAPinGAMS model (and most other GTAP based models) sectors that are characterized by Armington trade include an nested Armington aggregation. In the top-level nest domestic goods trade off with a composite import, and in the lower-level nest the import varieties trade off with other import varieties. Numeric values of the Armington elasticities are drawn from the GTAP 7.1 database except for the elasticity of substitution for GAS, which we reduce to 10 (the same value as is adopted for crude oil).

Private consumption (final demand), like production, introduces substitution between an energy composite and a non-energy composite. At the second level non-energy goods are substitutable according to a Cobb-Douglas substitution function. Finally, international transportation services are provided as a Cobb-Douglas aggregation of transportation services exported from countries throughout the world, and both public consumption and investment demands

are fixed. This formulation introduces substitution at the second level between domestic and imported inputs while holding sectoral commodity aggregates constant.

At the time of this writing the GAMS code is in the process of being cleaned up for release. It will be posted on the web at <http://inside.mines.edu/~ebalistr> by mid April 2011.