The Consumption and Saving Decision

- Why save?
- a) shift income over time (lifecycle motive), and b) precautionary motive
- People trade-off present consumption with future consumption
- How would *Homo economicus* decide?
- Three primary assumptions of the Fisher model:
  1. Only two time periods
  2. Current and future income/wealth given
  3. Agent can choose to borrow/save any amount at the market interest rate.
- Two periods: youth and old age. Real income $y_1$ and real consumption $c_1$ in period 1. Similarly for period 2.

$$c_1 > or < y_1$$

$$S = y_1 - c_1$$

- $S$ positive or negative

- $S$ put in a bank account in period 1 earns an interest “$r$” for the length of the period. Next period, agent has $(S + rS)$. She also has her old-age income of $y_2$.
- Consumption in the second period must satisfy

$$c_2 = (S + rS) + y_2$$

Why?
- Combining the *per-period* budget constraints:

$$\frac{c_1}{1 + r} + \frac{c_2}{1 + r} = y_1 + \frac{y_2}{1 + r}$$

- Intertemporal budget constraint.
The Intertemporal Budget Constraint

- What do the intercepts represent?
- Which points are in? which points are out? the no-borrowing, no-lending point; feasible set of consumption possibilities
- Negative slope implies trade-off between current and future consumption.
- Interpretation of this two-period budget constraint
- The notion of discounting: an item today is worth more than the same item in the future
- Notion of present value: suppose you have to make a payment of $100 to someone one year from now. How much money should you put aside today (or keep in a savings account and not spend) so that you could make that future payment? The answer to this question is the PV of $100.

- Let \( x \) be the amount of money you should put aside today. Then next year you will have \( x + xr \) or \( x(1 + r) \) dollars. Thus, \( x(1 + r) = 100 \). If \( r = 0.1 \), then, \( x(1.1) = 100 \), and then, \( x = 100/1.1 = $90.90 \). So the PV of $100 is $90.90 at the interest rate of 10%.

- How much would an agent have to keep in a bank today to ensure himself a consumption amount of \( c_2 \) tomorrow? How much is that \( c_2 \) tomorrow worth to him today?

- Left hand side of budget constraint = PV of lifetime consumption. RHS = PV of lifetime income \( (Y) \).
Utility and Indifference Curves

- People may have different preferences about whether to consume more when young or to consume more when old.
- The level of utility any combination of current and future consumption brings to a consumer is summarized by the utility function:
  \[ U(c_1, c_2) \]

- Assume \( U \) is strictly increasing in \( c_1 \) and \( c_2 \). More is better!
- Assume: people don’t like consuming in the extremes.
- Example
  \[ U(c_1, c_2) = c_1^{0.4} c_2^{0.6} \]
  Check \( U(20, 1) < U(10, 10) \).
- 3-D picture; to move to 2-D, use indifference curves (IC)
- ICs show all combinations of current and future consumption that yields the same level of utility \( \bar{U} \)
  \[ U(c_1, c_2) = \bar{U} \]
- Figure
- Properties of indifference curves:
  (a) Slope downward from left to right.
  (b) ICs further up and to the right give higher utility to consumer.
  (c) ICs are also bowed toward the origin (wok-shaped).
- Notion of marginal rate of substitution (MRS): rate at which consumer wants to (prefers to) substitute her first period consumption for second period consumption.
• MRS = slope of IC at a point (negative).

• Optimal ("utility maximizing") level of consumption: where MRS = the slope of the budget line. Interpretation.

• Formally,

\[
\max_{(c_1, c_2)} U(c_1, c_2)
\]

subject to the intertemporal budget constraint →

\[
c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}
\]

\[(c_1, c_2) \geq 0\]

• Example:

\[U(c_1, c_2) = \ln c_1 + \beta \ln c_2\]

\[
\max_{c_2} \ln \left[ Y - \frac{c_2}{1+r} \right] + \beta \ln c_2
\]

Solution:

\[c_2^* = (1 + r) \frac{\beta}{(1 + \beta)} \left[ y_1 + \frac{y_2}{1+r} \right]_{\text{PVLI}}\]

\[c_1^* = \frac{1}{(1 + \beta)} \left[ y_1 + \frac{y_2}{1+r} \right]_{\text{PVLI}}\]

\[S^* = \frac{y_1}{(1 + \beta)} - \frac{\beta}{(1 + \beta)} \frac{y_2}{1 + r}\]
• How changes in income affect consumption: income effect

• Key point: regardless of whether the increase in income occurs in the first period or the second period, the consumer spreads it over consumption in both periods. Thus consumption depends on PV lifetime income \( (Y) \) rather than just on current income \( (y_1) \).

• Agent uses saving to “smoothen” consumption over the “life-cycle”; life-cycle motive for saving

• Distinguishing between temporary and permanent changes:

1. **temporary** increase in \( y_1 \) to \( y_1 + \Omega \) with no increase in \( y_2 \)

\[
c_1 + \frac{c_2}{1 + r} = (y_1 + \Omega) + \frac{y_2}{1 + r}
\]

2. **permanent** increase in \( y_1 \) to \( y_1 + \Omega \) and increase in \( y_2 \) to \( y_2 + \Omega \).

\[
c_1 + \frac{c_2}{1 + r} = (y_1 + \Omega) + \frac{(y_2 + \Omega)}{1 + r}
\]

• Permanent increases in income have much stronger effects on consumption than temporary changes.

\[
S_{\text{temp}} = \frac{y_1 + \Omega}{(1 + \beta)} - \frac{\beta}{(1 + \beta)} \frac{y_2}{1 + r}
\]

\[
S_{\text{perm}} = \frac{(y_1 + \Omega)}{(1 + \beta)} - \frac{\beta}{(1 + \beta)} \frac{(y_2 + \Omega)}{1 + r} = S_{\text{temp}} - \frac{\beta}{(1 + \beta)} \frac{\Omega}{1 + r}
\]

\[\Rightarrow S_{\text{perm}} < S_{\text{temp}}\]

• Tax cuts are more likely to have strong effects on consumption if the cut is a permanent one.
Ricardian Equivalence

- Do tax cuts increase consumption?
- Suppose the government cuts taxes today and has no plans to cut spending. Are you better off as a result? Will you consume more?
- The government is “financing” the tax cut by running a budget deficit. In the future, the government will have to raise taxes to pay off the deficit and accumulated interest on its debt. So, the tax cut may actually be temporary. The effect on consumption is likely to be small.
- The situation before any tax-cuts are announced: In period 1, govt. collects taxes $T_1$ and spends $G_1$; similarly in period 2.
- Deficit:
  \[ D_{\text{pre}} = G_1 - T_{1\text{pre}} \]
- Govt. finances deficit $D$ by selling bonds.
- In period 2,
  \[ T_{2\text{pre}} = (1 + r)D_{\text{pre}} + G_2 \]
  or,
  \[ T_{2\text{pre}} = (1 + r)\{G_1 - T_{1\text{pre}}\} + G_2 \]  
  (1)
- After tax-cuts are announced → Suppose govt. announces a tax-cut of $\Delta T$ in the first period. Then, govt. deficit after the policy (denoted $D_{\text{new}}$) is
  \[ D_{\text{post}} = G_1 - \{T_{1\text{pre}} - \Delta T\} = G_1 - T_{1\text{pre}} + \Delta T \]
- Then
  \[ T_{2\text{post}} = (1 + r)D_{\text{post}} + G_2 \]
  or,
  \[ T_{2\text{post}} = (1 + r)\{G_1 - T_{1\text{pre}} + \Delta T\} + G_2 \]
  or,
  \[ T_{2\text{post}} = (1 + r)\{G_1 - T_{1\text{pre}}\} + (1 + r)\Delta T + G_2 \]  
  (2)
- Therefore, taxes will have to be increased by the amount [using (1) & (2)]
  \[ T_{2\text{post}} - T_{2\text{pre}} = (1 + r)\Delta T \]
• Thus a current period tax cut of $\Delta T$ implies a tax hike of $(1 + r)\Delta T$ in the future!

• For the consumer: first period effective income goes up (as a result of the tax cut in the first period) from $y_1$ to $\{y_1 + \Delta T\}$. But second period income (as a result of the tax hike in the second period) goes down from $y_2$ to $\{y_2 - (1 + r)\Delta T\}$.

• What happens to the PVLI?

• The (senior) George Bush experiment; 57% Ricardian tax hike will fall on future generations. Why should you care?

• Altruism; the role of bequests

• bequests in the US: strategic?

\section*{Precautionary Motive}

• life is full of economic risks and uncertainties.

• good and bad outcomes; people don’t like extremes (risk-averse); can save to limit the economic setbacks (buffer); precautionary saving

• suppose second period income is uncertain; can be $y_2 - \Omega$ with probability 0.5 and $y_2 + \Omega$ with probability 0.5

• very risk-averse people focus only (put a lot of weight) on the really bad state.

• Save assuming that their second period income will be $y_2 - \Omega$.

• difference between this and what they would have saved if they knew that their second period income was $y_2$ for sure is called precautionary saving.

• may explain low US saving.