In-class Numerical Problem on the Two-Period Model

1. Gemma Kreider knows she will live for two time periods. She cares about her young-age consumption ($c_1$) and her old-age consumption ($c_2$) in a manner described by the following utility function:

$$U(c_1, c_2) = \alpha \ln c_1 + (1 - \alpha) \ln c_2$$

where $0 < \alpha < 1$. Her income when young is $w_1 > 0$ and her income when old is $w_2 > 0$. The interest rate in the economy is $r\%$ (i.e., if she puts in $\$1$ in a savings account when young, she will get back $\$1 + r$ when she is old). Assume $r > 0$. Answer the questions below using all the information provided (i.e., use the specific form of the utility function given). Do not assume a given value for $\alpha$, $r$, and $(w_1, w_2)$. [Remember $d \ln (f(x)) \over dx = 1 \over f(x) f'(x)$] where $f'(x)$ is the derivative of the function $f$ with respect to $x$]

(a) Write down Gemma’s per-period budget constraints.

(b) Write down Gemma’s intertemporal budget constraint.

(c) Draw an exact picture of her intertemporal budget constraint keeping $c_2$ on the vertical axis.

(d) Shade in her feasible set of consumption possibilities.

(e) Will Gemma ever choose to consume $c_1 = 0$ and $c_2 = 0$? What about $c_1 = 0$ and $c_2 = w_2$? Explain clearly why not.

(f) What is the maximum amount she can consume during her old age?

(g) Write down the problem that she should solve in order to compute her utility maximizing consumption levels when young and old.

(h) Compute her exact utility maximizing choices of $c_1$ and $c_2$.

(i) Suppose right before Gemma is born, Grandma Kreider dies and leaves her with an inheritance. As a result, Gemma’s old age income increases to $2w_2$ while her young age income remains unchanged. In this case, what would her new utility maximizing choices of $c_1$ and $c_2$ be? Compare your answers to (h) above. Explain.

(j) Write down the expression (in terms of $\alpha$, $r$, $w_1$, and $w_2$) which describes how much Gemma saves ($S$) or borrows when young. [assume her second period income is again just $w_2$]

(k) Under what conditions on the parameters is $S > 0$? Can $S > 0$ hold if $w_2 > w_1$?