Prob. Set on Consumption and Savings

1. Financial aid for colleges (Pell Grants): In determining the amount that the student and his/her family should pay toward college education, the formula used to determine Pell Grants expects students to contribute about 80% of their savings; the remainder is paid out as a grant to the student. That is, if it costs say $1000 to get a college education; then the applicant has to put in 80% of her savings of $1000, to get $200 as a Pell Grant. In your opinion, does such a policy encourage or discourage saving?

2. A consumer has a current real income \((y_1)\) is 90 and future real income \((y_2)\) is 110. His real wealth \(a\) is 20 [just add this to his first period income of \(y_1\)]. And the real interest rate is 10%.

   (a) Find his PVLI
   (b) Write down his budget constraint and draw it (using the #'s given in the question)
   (c) Suppose he wishes to smoothen his consumption perfectly.
      i. how much will he save and consume in the current period?
      ii. how will your answer to the previous question change if his current income went up by 11?
      iii. how will your answer to the previous question change if his future income went up by 11?
      iv. how will his current saving and consumption be affected by an increase of 11 in his initial wealth?

3. Rory Gilmore knows she will live for two time periods. She cares about her young-age consumption \((c_1)\) and her old-age consumption \((c_2)\) in a manner described by the following utility function:

   \[ U(c_1, c_2) = c_1^\alpha c_2^{1-\alpha} \]

   where \(0 < \alpha < 1\). Her income when young is \(w_1 > 0\) and her income when old is \(w_1 > 0\). The interest rate in the economy is \(r\%\) (i.e., if she puts in $1 in a savings account when young, she will get back $ \((1+r)\) when she is old). Assume \(r > 0\). Answer the questions below using all the information provided (i.e., use the specific form of the utility function given). Do not assume a given value for \(\alpha, r, w_1,\) and \(w_2\).

   (a) Write down Rory’s intertemporal budget constraint.
   (b) Will Rory ever choose/want to consume \(c_1 = 0\) and \(c_2 = 0\)? Can she afford to consume \(c_1 = w_1\) and \(c_2 = (1+r)w_1\)? Explain clearly why not.
   (c) What is the maximum amount she can consume during her old age?
   (d) Write down the problem that she should solve in order to compute her utility maximizing consumption levels when young and old.
   (e) Compute her exact utility maximizing choices of \(c_1\) and \(c_2\).
   (f) Grandpa Gilmore cares about Rory and wants to make sure that she consumes at least \(2w_1\) when old. How much inheritance should he leave her (to be available to her when she is old; assume amount will be known to her when she is young) so as to make sure that she consumes exactly \(2w_1\) when old.
   (g) Ignoring the situation in (f): Suppose her young age income were to increase to \(2w_1\) while her old age income remains unchanged. In this case, what would her new utility maximizing choices of \(c_1\) and \(c_2\) be? Compare your answers to (e) above. Explain.